A Capability-based Measure of Economic Working Gas Capacity

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Introduction

Definitions of measures of working gas capacity seem indistinct when one looks at available work. A recent review of capacity measures is contained in the EIA report published in September 2006. In general, the size of the space is measured without reference to the ability of a facility to flow to and from the possible inventory levels in the available cycle time. This note uses simple models of rate capability to define two normative measures of working gas capacity derived from the capability of a facility to cycle its capacity. One measure is purely physical and finds the largest capacity that a facility can repeatedly cycle. The other measure uses the machinery of the physical but adds cost and profit optimization to arrive at a definition of economic working gas capacity which accounts for the costs and revenues related to the capability of a facility to cycle its capacity.

Three sections describing the set-up, solution and use of the model follow. A summary ends this note. The first section describes the model set up and highlights results which lead to definition of the maximum physical working gas that can be repeatedly cycled through a storage facility. The second section uses the results of the physical optimization and extends them with a simplified profit maximization analysis. With the aid of a constructed example and the mathematical results previously presented, illustration of the relationship between the measures is illustrated and discussed in the third section. Finally, a summary and conclusion reiterates key points and indicates further research that could build on this analysis.

A. Working Gas Capacity Derived From Gas Storage Flow Capability

This section describes a method for determining the physically maximal working gas capacity starting with a simple model of rate capability for a gas storage facility. Equations for injection and withdrawal rates as functions of opening inventory of a period are transformed into inventory levels as functions of time, yielding a conversion of the performance boundary from capability to capacity. From these a solvable linear system provides expressions for the lowest and highest inventory levels that can repeatedly reached in the cycle times allowed. The difference yields a measure of the maximum long-run working gas capacity for a facility derived from the model describing the capability to cycle the capacity. Using standard definitions of injection and withdrawal seasons, the “Maximum Cycling Working Gas” (MCWG) is easy to quantify. Throughout this work it is assumed that the capability is fully utilized during time periods that flow is occurring; this condition restricts this model to a selection of the number of periods to operate rather than the intensity of utilization over a longer cycle time.

The gas physics underlying the rate models result in the down and upward sloping functions which form a naturally bounded and linked system, in terms of quantities and flows over set time spans. Because of these capability functions, the time available for an inventory cycle is the main source of limitation on potential working gas capacity of the facility. The simplest models are straight lines – in (inventory, rate) coordinates – which, with increasing levels of inventory, decrease for injection and increase for withdrawal. Using a’s for withdrawal function parameters, b’s for injection parameters, R’s for rates and I’s for inventory levels the flow-rate model is simply:
$R_W = a_I l_{t-1} - a_0$, for withdrawal \hspace{1cm} (1)
$R_I = b_0 - b_I l_{t-1}$, for injection \hspace{1cm} (2)

With these equations, substituting in an opening inventory, between $I_{Min}$ and $I_{Max}$, gives the largest possible change in inventory over a unit of time. Chaining together a series of rates for one equation gives the inventory change over the time span of the series by accumulating the changes. The mathematics allows arbitrary inventory-cycle spans. Therefore, with a cycle-time assumption, the maximum size of working gas for the facility can be found from the solution of the rate equations. The model set-up is illustrated with the following schematic diagram of capability at inventory for a simplified storage facility.

For a stable-state system, the starting and ending points of injection and withdrawal have to match their respective turnover points – from $BG$ to $I_{Full}$ and back to $BG$ – in order to complete a full inventory cycle in the time available. We assume here that the flows take place along the performance boundaries and thus define the working gas which maximizes capacity utilization of the facility. $I_{DMin}$ and $I_{DMax}$ are illustrations of possible facility design limits on the inventory range. The diagram is intended to illustrate an operating range engineered to have some extra capacity.

Because withdrawal slows as inventory decreases, more can be withdrawn in a fixed time by starting at a higher level. Similarly, more can be injected over a fixed time if the flow starts at as low a level of inventory as possible. In the times allowed, the largest inventory level that can be attained from $BG$ is $I_{Full}$, while $BG$ is the smallest inventory that can be reached from $I_{Full}$. Thus the maximum cycle-able inventory (over $T_I + T_W$) is $I_{Full} - BG = WG$, which is the maximum working gas possible in the cycle time while fully utilizing capability. We assume $BG$ and $I$ are not constrained for reasons other than cycle time and the engineering limits.

With the objective in sight, we derive formulas from the rate equations that allow us to find these boundary values and, therefore, the maximum working gas for the cycle-span. The first task is to determine the (time, inventory) solutions to the rate equations. Standard solutions are available\(^1\) for simple transformations of the equations above and allow us to simply state the resulting forms:

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\(^1\) See Tu for eg.
The first equation pertains to withdrawal and simply states that by using the full rate capability, $R_{Wt}$, at time $T$ starting at $t$, the inventory at the end will be $I_T$; $I_T$ will be less than $I_t$. Inventory at the end of $T$ periods of consecutive maximal injections can be found from the second equation where $I_T$ will be greater than $I_t$. Here we assume $t=0$ for each leg.

This abstract notation is meant to handle general situations. In order to derive the desired formulas for inventory endpoints, and interpret the results, the notation is specialized to our case. Each specialization implies an assumption which bounds values of parameters and fixes a part of the interpretation. First, we define the time intervals as $T_I$ and $T_W$ such that each is, respectively, the lengths of time for semi-cycles of continuous injection and withdrawal.

The remaining inventory after a withdrawal period will be considered to be the base gas, denoted $BG$. Starting at $BG$, continuous injection along the rate boundary yields the largest inventory in a cycle and is denoted with $I_{Full}$. At the end of the fill cycle the initial level for withdrawal becomes fixed at the prior $I_{Full}$ while the initial level of the injection semi-cycle will be the prior $BG$. A useful observation here is that the solution of the rate equations contain formulas for the mathematical maximum and minimum inventory levels: from injection ($b_0/b_1 = I_{Max}$) and from withdrawal ($-a_0/a_1 = I_{Min}$). Finally, to simplify notation, set $(1-a_1) = a$ and $(1-b_1) = b$. Making these substitutions into the last pair of equations gives the model to be optimized:

$$BG = a^{T_W} [I_{Full} - I_{Min}] + I_{Min}$$  

$$I_{Full} = b^{T_I} [BG - I_{Max}] + I_{Max}$$

Clearly, the total volume moved on each leg must be equal to have a WG inventory which can be repeatedly cycled in the time available for each. The second part of defining $MCWG$ is to fix the time available on each leg. For the measure “maximum cycling working gas,” these times span are set to $T_I = 214$ days and $T_W = 151$ days to match the conventional gas-industry injection and withdrawal semi-cycle spans in a “gas year.” Marking the optimal endpoint values as $BG*$ and $I_{Full}$*, the dynamic model is:

$$BG^* = a^{T_W} [I_{Full}^* - I_{Min}] + I_{Min}$$  

$$I_{Full}^* = b^{T_I} [BG^* - I_{Max}] + I_{Max}$$

Since we now have two linear equations in two unknowns, solving for $I_{Full}^*$ and $BG^*$ generates an expression giving the optimal value for each, the difference of which is the $MCWG$, the maximum cycling working gas available from the facility in the given time spans. Following this procedure, the optimal end-point values are found to be:

$$BG^* = [(a^{-T_W} - 1)I_{Min} + (1 - b^{T_I})I_{Max}]/(a^{-T_W} - b^{T_I})$$  

$$I_{Full}^* = b^{T_I} [(a^{-T_W} - 1)I_{Min} + (1 - b^{T_I})I_{Max}]/(a^{-T_W} - b^{T_I}) + (1 - b^{T_I})I_{Max}$$

The difference $I_{Full}^* - BG^* = MCWG$ gives the largest possible inventory that can be repeatedly cycled through the facility in the conventional semi-cycle periods of the gas-industry year. After simplifying the difference, the formula for the measure $MCWG$ is found to be:

$$MCWG = (1 - b^{T_I}) (1 - a^{-T_W}) (I_{Max} - I_{Min})/(1 - a^{-T_W} b^{T_I})$$

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2 Obviously, the count of cycles could be important to starting and ending values for this system, however, the object here is to determine the optimal “steady-state” solution where a cycle means that the volume delivered is first injected, that all that is injected is then delivered, and that this volume is the maximum possible. Thus, multi-cycle terms do not appear since we assume a full cycling of the inventory.
Therefore, starting with a linear approximation of a facility’s rate functions and applying standard solution techniques, the physically maximal capability-based measure of long-run working gas capacity, $MCWG$, can be expressed as a reasonably simple analytic function of the constants of the capability or rate equations and two set time spans. While the purely physical measure yields the biggest capacity, finding the maximum profit yields a measure of the best working gas capacity.

\section*{B. Economic Working Gas Capacity}

By employing the preceding results for the physical model and incorporating variables for costs and revenue, a capability-based measure of capacity that generalizes the $MCWG$ can be derived. We propose this as a measure of “economic working gas capacity” (EWGC) since the resulting model solves for a capability-based profit maximizing working gas capacity, given the costs of flow and revenues to capacity. The starting point is to first solve the state equations for time as the dependent variable. Then, a per period cost variable for each leg is added to the model in order to define a total cost function for a cycle which has working gas volume as the independent variable. If we assume that the storage facility operator receives constant long-run revenue per unit of capacity, $MR$, then total revenue is a simple linear function increasing in working gas. Of course, total revenue less total cost gives the profit on the capacity cycle. The purpose of this section is to describe optimization of the profit function and define a capability-based measure of economic working gas capacity.

Starting with equations (3) and (4), solution of these state equations for the time variables is a simple algebraic task. The total cycle time is then just the sum of the times taken for each leg, as functions of the same $BG$ and $WG$. If $WG < MCWG$, there will be an infinitude of leg-timing combinations that will cycle a given $WG$. However, because of the nature of the time functions – increasing in continued flow on the leg – there will be one combination that minimizes the total time to cycle a given $WG$. Forming the sum of the time functions, differentiating with respect to $BG$, setting the derivative equal to zero, and solving for $BG$ yields the starting inventory which minimizes the time required to cycle the selected $WG$. Specifically,

$$\frac{dT_{WGC}}{dBG} = \frac{dT_I}{dBG} + \frac{dT_W}{dBG} = 0 \tag{10}$$

Substituting in the re-ordered state equations and solving (10) for $BG$ generates a function that minimizes the time to cycle the dependent variable, $WG$.

If we assume a form of cost function for each leg that increases at an increasing rate as $WG$ rises, then, like the total time minimization, there will be a combination of times for each leg that minimizes the total cost of cycling the given $WG$. If $c_I$ and $c_W$ represent per-time-period costs of cycling the $WG$ over, respectively, the injection and withdrawal times, then the total cost of flowing each leg is just $TC_I = c_I*T_I$ and $TC_W = c_W*T_W$. With this form of total cost function for the flow legs, the marginal cost function is the period cost divided by a declining flow rate as the end-points of the inventory cycle are widened to accommodate a larger $WG$ capacity. Obviously total cost of the cycle is then $TC_{WGC} = c_I*T_I + c_W*T_W$. Assuming the cost variables do not change with $WG$, following the same procedure as minimizing the time to cycle yields a function of $WG$ which gives the $BG$ that minimizes the cost of cycling the selected $WG$.

Using the definition of the cost-minimized working gas cycle, a short extension takes us to profit maximization and the economically optimum working gas capacity. If we further assume the operator of the storage facility is a competitive price taker who knows the long-run expected capacity price, $MR$, we have a revenue model that is a simple multiple of $WG$. Subtracting the minimized total cost for the $WG$ from $TR = MR*WG$ defines the maximized total profit function, given the $WG$: $P_{WGC} = TR - TC_{WGC}$. Differentiating this with respect to $WG$, setting the result to zero, and solving for $WG$ yields the working gas that maximizes profit, under the condition that we select the least-cost inventory window to cycle this working gas volume. In particular, the working gas that satisfies the following equation is the maximum economically efficient working gas capacity, or, as promised above, it is a capability-based measure of economic working gas capacity since the cost functions are determined by the flow capability of the facility.
The next section reviews an example to make the concepts more obvious and to show how this measure of economic working gas capacity can indicate when cost and revenue conditions result in the operator offering less than their MCWG to the market.

C. Example Solution for Economic Working Gas Capacity

The example reviewed in this section uses the modeling results described above and populates them with arbitrary parameter, cost, and revenue assumptions which are specifically chosen to put the resulting numbers into orders of magnitude that may seem vaguely familiar to a reader who works with gas storage. Two charts are presented and discussed to illustrate the concepts described above. First, however, the fictitious parameter and constant values must be listed.

<table>
<thead>
<tr>
<th>Rate Model Parameters yielding ( MCWG = 10,000,000 ) units:</th>
<th>Profit function assumptions for Economic Working Gas Capacity:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 = 45,400 )</td>
<td>( c_I = c_W = $7,200/day )</td>
</tr>
<tr>
<td>( a_1 = 0.0210 )</td>
<td>( MR = $0.50 ) per unit of WG</td>
</tr>
<tr>
<td>( a = 0.9790 )</td>
<td></td>
</tr>
</tbody>
</table>

Substituting these values into the formulas described above generates the charts that follow. The first chart displays this specific example in the form of the schematic in the first section. The second chart with total cost, revenue and profit as a function of WG illustrates the same model. The cost and revenue relationship here creates the circumstance where a profit maximizing operator would offer less capacity to the market than the MCWG window.
In addition to portraying the numeric example for the rate model, the first chart also shows the relationship between the assumed design range, the MCWG and EWGC. The marginal costs of each leg are also plotted. Given the cost assumptions here, it appears we would rarely visit the edges of the design envelope. The difference between the two maximized windows is that it is more economical to operate the facility at a higher turnover rate and smaller capacity than to make the most of the time and space. If the market or regulator increases the MR enough then the economic measure will increase to equal the MCWG.

The final chart shows revenue, minimum cost and profit as functions of working gas over the high half of the range. The measure of economic working gas capacity is the point of maximum total profit, which is about 5% greater than the profit at the MCWG (10,000,000) but uses less than 92% of the capacity. Increasing the slope of the TR function, by increasing MR, to equal the slope of the Total Cost function puts the maximum profit at the MCWG and therefore makes the economic and physical measures equal. If MR is lowered, the profit maximizing capacity will also fall. Increasing costs would also lower the economically efficient working gas while decreasing costs will increase EWGC until the slope of the cost function matches the slope of the revenue function at which point equality of measures occurs. Although quite simple, this model and points to the importance of economic conditions and prices in estimating fleet capacity.

<table>
<thead>
<tr>
<th>Working Gas Capacity (Units)</th>
<th>Total Cost</th>
<th>Total Revenue</th>
<th>Total Profit</th>
<th>Maximum Profit</th>
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<tbody>
<tr>
<td>5,000,000</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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<tr>
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<td>9,148,729</td>
<td>10,000,000</td>
<td>2,495,504</td>
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</tr>
</tbody>
</table>

D. Summary and Conclusions

This paper employs simple linear capability models for injection and withdrawal rates in gas storage capacity modeling. The solutions to transformations of the rate equation model allow us to define and use measures of capacity that are based on the capability of a facility to cycle its working gas. Here we offer a purely physical measure of “Maximum Cycling Working Gas”, being the largest capacity a facility could cycle an indefinite number of times, and a measure derived from profit maximization of the capability derived working gas referred to as “Economic Working Gas Capacity”. Illustration with a numeric example shows how design capacity encloses the MCWG which in turn encloses the EWGC, the most profitable capacity level to cycle. The principle conclusion of this work is that capability-based measures of gas storage space are relatively easy to construct and are much more informative since the constraint of cycle time and the costs and benefits of doing the business are incorporated in sizing the capacity to be offered from a facility. In the case of the example considered here, some general principles of the relationship between the physical and economic measures could be discerned. In particular, it is easy to see what conditions are needed for the facility to offer up its full long-run cycling capability at its MCWG.
Although simple, it is easy to generalize this approach to model non-linear performance boundaries with piece-wise continuous linear approximations. All the same principles apply with some extra accounting burden being the difference. Possible topics for future research extending this line of work includes the role of base gas levels and returns-to or costs-of carry in determining optimal working gas capacity and window position, the aggregation of capability-based capacity models into fleets, and the matching of observed flows and utilized capacity to an underlying capability model.

References

