Emissions Trading in Forward and Spot Markets of Electricity

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Abstract

In recent years there has been growing discussion regarding market designs of emissions allowances trading. This paper develops an endogenous model to analyze the interaction of emissions allowances markets with forward and spot markets of electricity under the framework of Cournot competition. The problem is defined as an equilibrium problem with equilibrium constraints (EPEC). Stylized numerical examples show that allocating initial emissions allowances more to the “clean” generator than to the “dirty” generator may improve efficiency, increasing power supply and decreasing power price.

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1 Introduction

There has been a growing concern about emissions trading scheme such as pioneering European Union Emissions Trading Scheme (EU ETS). Specifically, an emerging issue has been the interaction between oligopolistic electricity markets and emissions allowances markets.

Chen and Hobbs (2005) examine a complementarity approach to simulate the interaction of emissions allowances markets with electricity markets, using a conjectural variation model for the permits market. Although they consider forward contracts for electricity, the contract amount is given as an exogenous variable. Chen et al. (2006) investigate a Stackelberg game in which the largest producer can manipulate both electricity and emissions allowances markets. However, they do not incorporate forward contracts for electricity into their model. Mansur (2007) discusses that, in the Pennsylvania-New Jersey-Maryland (PJM) electricity market, only 10 to 15 percent of supply comes from the spot market, while approximately 30 percent is from forward (short and long-term) contracts and 53 to 59 percent is self-supplied. It should be noted that forward contracts for electricity play an important role in reality.

On the other hand, Fowlie (2009) investigates two-period forward and spot markets of electricity with emissions leakage by extending the model of Allaz and Vila (1993). However, she assumes that the allowances price is exogenously determined. As far as the author knows, a two-period model in which the allowances price is endogenously determined has not been fully developed.

The current paper develops an endogenous model to analyze the interaction of emissions allowances markets with forward and spot markets of electricity under the framework of Cournot competition. The problem is defined as an equilibrium problem with equilibrium constraints (EPEC). How will Cournot firms use forward contract? Will initial allocation of allowances have effects on efficiency? We will address these questions in this paper.

The paper is organized as follows: In Section 2, we present a two-period model with endogenous emissions allowances trading. In Section 3, we consider stylized numerical examples to illustrate the interaction of emissions allowances markets with forward and spot markets of electricity. Section 4 summarizes our results.
2 The Model

2.1 The Two-Stage Game

Allaz and Vila (1993) show that a Cournot duopolist has a strategic incentive to trade forward, seeking for the first mover advantage. However, when both do so, a prisoner’s dilemma makes them worse off. They show that the forward market leads the firms to behave more competitively in the spot market, which reduces spot prices and increases social welfare. Bushnell (2007) examines a two-period Cournot model with multiple symmetric firms and simulates the PJM market under the symmetric assumption. Su (2007) proves the existence of two-period market equilibrium in the more general case.

We extend the model of Allaz and Vila (1993) and Bushnell (2007) to incorporate the emissions allowances market. Specifically, we consider a two-period model of electricity markets with endogenous emissions allowances trading. In the first period, firms can sell forward contracts for electricity. Then, firms generate electricity in the second period spot market. At the same time, firms can trade CO2 emissions allowances in the second period. It is worth noting that the allowances price is endogenously determined. We analyze this two-stage game using backward induction. The notation used in this paper is summarized below.

\[ i, j \] Indices of firms.
\[ I \] Total number of firms.
\[ q_i \] Power generated by firm \( i \). \( q_i \geq 0 \).
\[ q_i^f \] Generation capacity of firm \( i \).
\[ P(Q) \] Spot price of power (inverse demand function).
\[ P^f \] Forward price of power.
\[ r_i \] CO2 emissions rate of firm \( i \).
\[ e_i \] Initial allowances of CO2 emissions owned by firm \( i \).
\[ P^e \] Alliances price of CO2 emissions.
2.2 Spot Transaction (Second Period)

Since firm \( i \) has already sold \( q_i^f \) in the forward market, it can only sell \( q_i - q_i^f \) in the spot market. Given the forward position \( q_i^f \), firm \( i \) earns the revenue \( P(Q)(q_i - q_i^f) \) from the spot market. The power generation cost \( C_i(q_i) \) is subtracted from the revenue.

\( r_iq_i - e_i \) represents the number of tradable CO2 emissions allowances purchased (positive) or sold (negative) by firm \( i \). Multiplied by the allowances price \( P^e \), \( P^e (r_iq_i - e_i) \) represents the net expense of allowances.

Considering the nonnegativity constraint and generation capacity constraint, Cournot firm \( i \) solves the following maximization problem concerning the spot transaction in the second period:

\[
\max_{q_i} P(Q)(q_i - q_i^f) - C_i(q_i) - P^e (r_iq_i - e_i) \tag{1}
\]

\[
\text{subject to}
\]

\[
q_i \leq \bar{q}_i, \tag{2}
\]

\[
0 \leq q_i. \tag{3}
\]

Define the Lagrangian as

\[
L_i = P(Q)(q_i - q_i^f) - C_i(q_i) - P^e (r_iq_i - e_i) + \rho_i (\bar{q}_i - q_i), \tag{4}
\]

where \( \rho_i \geq 0 \) is the dual variable associated with the generation capacity constraint. Then, the Karush–Kuhn–Tucker (KKT) conditions for the firm \( i \)'s problem in the second period are given by

\[
0 \leq q_i \perp - \frac{\partial L_i}{\partial q_i} \geq 0, \tag{5}
\]

\[
0 \leq \rho_i \perp \bar{q}_i - q_i \geq 0, \tag{6}
\]

where \( \perp \) denotes complementarity. For example, \( 0 \leq a \perp b \geq 0 \) means that \( 0 \leq a, 0 \leq b, \) and \( ab = 0. \)
2.3 Emissions Trading (Second Period)

We assume that not only the firms in the electric power industry but also a lot of firms in various industries participate in the CO2 allowances market. Thus, these firms would be price takers in the allowances market. Following Sartzetakis (1997), we here assume that the total number of allowances in the electric power industry does not change after trading, and hence the equilibrium allowances price can be derived by examining this sector alone, which would make the problem more tractable.1

The market clearing condition for tradable allowances is expressed as a complementarity condition:

\[ 0 \leq P^e \sum_i (e_i - r_i q_i) \geq 0. \] (7)

If there are excess allowances in the market, the allowances price \(P^e\) will be zero. If the demand for allowances equals the available supply, \(P^e\) can be positive.

2.4 Forward Contract (First Period)

Following Allaz and Vila (1993), we assume no uncertainty and perfect foresight. In other words, the forward price of power \(P^f\) equals the correctly anticipated spot price \(P\). Noting this, the profit of firm \(i\) including the revenue from the forward sales can be written as follows:

\[
P^f q^f_i + P(Q)(q_i - q^f_i) - C_i(q_i) - P^e (r_i q_i - e_i)
= P(Q)q_i - C_i(q_i) - P^e (r_i q_i - e_i). \] (8)

We then embed both the KKT conditions for the spot transaction and the complementarity condition for the allowances trading in the second period as a set of constraints in firm \(i\)’s profit maximization problem in the first period. The optimization problem of firm \(i\) concerning the forward contract in the first period can be expressed as a Mathematical Program with Equilibrium Constraints (MPEC):

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1Daxhelet (2008) examines a model that incorporates non-electric industries more explicitly by considering the net balance of allowances exchange between electric and non-electric industries.
max \( P(Q)q_i - C_i(q_i) - P^e (r_i q_i - e_i) \)  \( i \) subject to

\begin{align*}
0 & \leq q_j - \frac{\partial L_j}{\partial q_j} \geq 0, \quad \forall j, \\
0 & \leq \rho_j \frac{\partial q_j}{q_j} - q_j \geq 0, \quad \forall j, \\
0 & \leq P^e \sum_i (e_i - r_i q_i) \geq 0, \\
0 & \leq q^f_i, 
\end{align*}

where \( q = (q_1, q_2, \ldots, q_I) \) and \( \rho = (\rho_1, \rho_2, \ldots, \rho_I) \).

Define the Lagrangian as

\[
L^f_i = P(Q)q_i - C_i(q_i) - P^e (r_i q_i - e_i) \\
+ \sum_j \left( \mu_{i,j}^1 q_j \frac{\partial L_j}{\partial q_j} - \mu_{i,j}^2 q_j \frac{\partial L_j}{\partial q_j} \right) \\
+ \sum_j \left( \nu_{i,j}^1 \rho_j + \nu_{i,j}^2 (\bar{q}_j - q_j) + \nu_{i,j}^3 \rho_j (\bar{q}_j - q_j) \right) \\
+ \phi_i^1 P^e + \phi_i^2 \sum_j (e_j - r_j q_j) + \phi_i^3 P^e \sum_j (e_j - r_j q_j),
\]

where \( (\mu_{i,j}^1 \geq 0, \mu_{i,j}^2 \geq 0, \mu_{i,j}^3 \geq 0), (\nu_{i,j}^1 \geq 0, \nu_{i,j}^2 \geq 0, \nu_{i,j}^3 \geq 0), \) and \( (\phi_i^1 \geq 0, \phi_i^2 \geq 0, \phi_i^3) \) are the dual variables associated with the complementarity conditions (10), (11), and (12), respectively. The KKT conditions for the firm \( i \)'s problem in the first period are then given by

\begin{align*}
0 & \leq q^f_i - \frac{\partial L^f_i}{\partial q^f_i} \geq 0, \quad (15) \\
\frac{\partial L^f_i}{\partial q_j} & = 0, \quad \forall j, \quad (16) \\
\frac{\partial L^f_i}{\partial \rho_j} & = 0, \quad \forall j, \quad (17)
\end{align*}
\[ \frac{\partial L_i^f}{\partial P^e} = 0, \]  
(18)

\[ 0 \leq q_j \perp \mu_{i,j}^1 \geq 0, \quad \forall j, \]  
(19)

\[ 0 \leq -\frac{\partial L_j}{\partial q_j} \perp \mu_{i,j}^2 \geq 0, \quad \forall j, \]  
(20)

\[ -q_j \frac{\partial L_j}{\partial q_j} = 0, \quad \forall j, \]  
(21)

\[ 0 \leq \rho_j \perp \nu_{i,j}^1 \geq 0, \quad \forall j, \]  
(22)

\[ 0 \leq \bar{q}_j - q_j \perp \nu_{i,j}^2 \geq 0, \quad \forall j, \]  
(23)

\[ \rho_j (\bar{q}_j - q_j) = 0, \quad \forall j, \]  
(24)

\[ 0 \leq P^e \perp \phi_i^1 \geq 0, \]  
(25)

\[ 0 \leq \sum_j (e_j - r_j q_j) \perp \phi_i^2 \geq 0, \]  
(26)

\[ P^e \sum_j (e_j - r_j q_j) = 0. \]  
(27)

### 2.5 An EPEC Approach

Putting together the MPECs of all firms yields an Equilibrium Problem with Equilibrium Constraints (EPEC). Following Hu and Ralph (2007) and Su (2007), we reformulate the EPEC as the collection of KKT conditions for all firms. We then solve the system of these KKT conditions numerically using PATH solver (Dirkse and Ferris, 1995).

\[ 0 \leq q_i^f \perp -\frac{\partial L_i^f}{\partial q_i^f} \geq 0, \quad \forall i, \]  
(28)

\[ \frac{\partial L_i^f}{\partial q_j} = 0, \quad \forall i, j, \]  
(29)
\[
\frac{\partial L^f_i}{\partial \rho_j} = 0, \quad \forall i, j, \quad (30)
\]
\[
\frac{\partial L^f_i}{\partial P^e} = 0, \quad \forall i, \quad (31)
\]
\[
0 \leq q_j - \mu_{i,j}^1 \geq 0, \quad \forall i, j, \quad (32)
\]
\[
0 \leq -\frac{\partial L^f_j}{\partial q_j} - \mu_{i,j}^2 \geq 0, \quad \forall i, j, \quad (33)
\]
\[
- q_j \frac{\partial L^f_j}{\partial q_j} = 0, \quad \forall j, \quad (34)
\]
\[
0 \leq \rho_j - \nu_{i,j}^1 \geq 0, \quad \forall i, j, \quad (35)
\]
\[
0 \leq q_j - q_j - \nu_{i,j}^2 \geq 0, \quad \forall i, j, \quad (36)
\]
\[
\rho_j (q_j - q_j) = 0, \quad \forall j, \quad (37)
\]
\[
0 \leq P^e - \phi^1_i \geq 0, \quad \forall i, \quad (38)
\]
\[
0 \leq \sum_j (e_j - r_j q_j) - \phi^2_i \geq 0, \quad \forall i, \quad (39)
\]
\[
P^e \sum_j (e_j - r_j q_j) = 0. \quad (40)
\]

3 Numerical Examples

3.1 Setup

We consider stylized examples of non-identical Cournot duopolists. Firm 1 has a low CO2 emissions rate, but its marginal cost of power generation is high (e.g., natural gas-fired generator). In contrast, Firm 2 has a high CO2 emissions rate, but its marginal cost of power generation is low (e.g., coal-fired generator).

The CO2 emissions rates of Firm 1 and Firm 2 are given by 0.4, 0.8
(kg-CO2/kWh), respectively. The marginal costs of Firm 1 and Firm 2 are \( C_1'(q_1) = 0.01q_1 + 1 \) and \( C_2'(q_2) = 0.005q_2 + 1 \), respectively. Moreover, the inverse demand function is given by \( P(Q) = 0.03Q + 110 \). We assume that the generation capacity of each firm is sufficient (i.e., capacity constraints would not be binding).

### 3.2 Benchmark Case without Emissions Regulation

Before turning to the analysis of allowances trading scheme, we first examine a benchmark case in which no emissions regulation is in place. Base Case assumes that firms engage in Cournot competition in forward and spot markets without caring about their CO2 emissions. As shown in Table 1, both firms indeed trade forward in equilibrium seeking for the first mover advantages, which is consistent with the results of Allaz and Vila (1993).

<table>
<thead>
<tr>
<th>Spot market only</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power generated (MWh)</td>
<td>1,045 (100%)</td>
<td>1,195 (100%)</td>
</tr>
<tr>
<td>Power price (yen/kWh)</td>
<td>43</td>
<td>32</td>
</tr>
<tr>
<td>Consumer surplus (1,000 yen)</td>
<td>75,246</td>
<td>101,073</td>
</tr>
<tr>
<td>Producer surplus (1,000 yen)</td>
<td>84,609</td>
<td>68,775</td>
</tr>
<tr>
<td>Social surplus (1,000 yen)</td>
<td>159,855</td>
<td>169,848</td>
</tr>
</tbody>
</table>

Table 2. Power generation, price and surplus in Base Case
3.3 Introduction of Emissions Regulation

We now compare two cases in which emissions regulation, specifically cap-and-trade system, is introduced. Firms are issued tradable CO2 emissions allowances.

First, Case 1 assumes that initial allowances are allocated in proportion to historical emissions (grandfathering). We assume that the cap is set at 95% of the emissions simulated in Base Case (i.e., 5% reduction of emissions for each firm): initial allowances are 452 tons and 1,068 tons for Firm 1 and Firm 2, respectively.

Tables 3 and 4 summarize allowances trading, forward contracts, and spot sales in Case 1. Compared to Base Case without emissions regulation, Firm 1 generates less power and hence reduces emissions, while Firm 2 generates more power and hence increases emissions in Case 1. As a result, Firm 1 sells the excess allowances to Firm 2 at the equilibrium allowances price of 51 yen/kg. It is worth noting that, in comparison with Base Case, Firm 2 increases power generation in the second period by highly committing to the forward contract in the first period, although it turns to be a purchaser of power in the spot transaction.

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial allowances (ton)</td>
<td>452</td>
<td>1,068</td>
</tr>
<tr>
<td>Emissions (ton)</td>
<td>319</td>
<td>1,201</td>
</tr>
<tr>
<td>Allowances sold (ton)</td>
<td>133</td>
<td>-133</td>
</tr>
<tr>
<td>Allowances price (yen/kg)</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>Profit from allowances trading (1,000 yen)</td>
<td>6,813</td>
<td>-6,813</td>
</tr>
</tbody>
</table>

Table 3. Allowances trading in Case 1

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward and spot markets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward</td>
<td>413</td>
<td>1,784</td>
</tr>
<tr>
<td>Spot</td>
<td>385</td>
<td>-282</td>
</tr>
<tr>
<td>Total</td>
<td>798</td>
<td>1,502</td>
</tr>
</tbody>
</table>

Unit: MWh

Table 4. Forward contracts and spot sales in Case 1

Table 5 reports producer surplus in Case 1. In comparison with Base Case, Firm 1 reduces surplus from power trading, while it obtains positive
surplus by selling excess allowances. In contrast, Firm 2 increases surplus from power trading, while surplus from allowances trading is negative. It should be noted that, in total, each firm can increase its surplus compared to Base Case. Therefore, the strategic behavior of each firm in Case 1 is profitable, and hence rational.

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer surplus in Base Case</td>
<td>29,962</td>
<td>38,814</td>
</tr>
<tr>
<td>Change in surplus associated with power trading</td>
<td>-1,220</td>
<td>+15,634</td>
</tr>
<tr>
<td>Change in surplus associated with allowances trading</td>
<td>+6,813</td>
<td>-6,813</td>
</tr>
<tr>
<td>Producer surplus in Case 1</td>
<td>35,554</td>
<td>47,634</td>
</tr>
</tbody>
</table>

Unit: 1,000 yen

Table 5. Producer surplus in Case 1

Next, we examine Case 2 in which initial allocation of allowances between firms is changed while the total number of allowances is kept unchanged. Thus, Case 2 is different from Case 1 in the sense that initial allocation of allowances is not exactly proportional to historical emissions. We here assume that initial allowances of Firm 1, the “clean” generator, are increased by 100 tons compared to Case 1 (i.e., 452+100=552 tons), while initial allowances of Firm 2, the “dirty” generator, are decreased by 100 tons compared to Case 1 (i.e., 1,068-100=968 tons). Note that total initial allowances and hence total emissions in Case 2 remain the same as in Case 1.

Tables 6 and 7 summarize allowances trading, forward contracts, and spot sales in Case 2. Compared to Case 1, Firm 1 increases power generation from 798 MWh to 968 MWh, and hence emissions from 319 tons to 387 tons. In contrast, Firm 2 reduces power generation from 1,502 MWh to 1,417 MWh, and hence emissions from 1,201 tons to 1,133 tons. Firm 1 still sells the excess allowances to Firm 2 although the allowances price falls down from 51 yen/kg to 34 yen/kg. Note that commitment of Firm 2 to the forward contract is still high but weaker than that in Case 1.
### Table 6. Allowances trading in Case 2

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial allowances (ton)</td>
<td>552</td>
<td>968</td>
</tr>
<tr>
<td>Emissions (ton)</td>
<td>387</td>
<td>1,133</td>
</tr>
<tr>
<td>Allowances sold (ton)</td>
<td>165</td>
<td>-165</td>
</tr>
<tr>
<td>Allowances price (yen/kg)</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Profit from allowances trading (1,000 yen)</td>
<td>5,672</td>
<td>-5,672</td>
</tr>
</tbody>
</table>

### Table 7. Forward contracts and spot sales in Case 2

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward and spot markets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward</td>
<td>501 (52%)</td>
<td>1,321 (93%)</td>
</tr>
<tr>
<td>Spot</td>
<td>467 (48%)</td>
<td>95 (7%)</td>
</tr>
<tr>
<td>Total</td>
<td>968 (100%)</td>
<td>1,417 (100%)</td>
</tr>
</tbody>
</table>

Unit: MWh

Table 7. Forward contracts and spot sales in Case 2

Table 8 reports producer surplus in Case 2. In comparison with Case 1, producer surplus of Firm 1 increases, whereas that of Firm 2 decreases. This is because initial allocation of allowances in Case 2 is in favor of Firm 1. However, note that each firm’s surplus is still larger than that in Base Case.

### Table 8. Producer surplus in Case 2

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producer surplus in Base Case</td>
<td>29,962</td>
<td>38,814</td>
</tr>
<tr>
<td>Change in surplus associated with power trading</td>
<td>+1,618</td>
<td>+9,224</td>
</tr>
<tr>
<td>Change in surplus associated with allowances trading</td>
<td>+5,672</td>
<td>-5,672</td>
</tr>
<tr>
<td>Producer surplus in Case 1</td>
<td>37,252</td>
<td>42,365</td>
</tr>
</tbody>
</table>

Unit: 1,000 yen

Table 8. Producer surplus in Case 2

### 3.4 Implications of Changing Initial Allocation of Allowances

Figures 1 and 2 compare the power generation and CO2 emissions in Case 2 with those in Case 1. As shown in Figure 1, the increase (+68 tons) in Firm
1’s emissions is exactly cancelled out by the reduction (-68 tons) in Firm 2’s emissions since total emissions are regulated to be constant, i.e., 1,520 tons. Nevertheless, total power generation increases from 2,300 MWh to 2,385 MWh, as illustrated in Figure 2. This is because the increase (+170 MWh) in power generation by Firm 1, the “clean” generator, is greater than the decrease (-85 MWh) in power generation by Firm 2, the “dirty” generator.

Table 9 compares the price and surplus in Case 2 with those in Case 1. The increase in total power generation leads to the decline in power price from 41 yen/kWh to 38 yen/kWh. Consequently, social surplus is greater in Case 2 than in Case 1. We find that allocating emissions allowances more to the “clean” generator than to the “dirty” generator may improve efficiency, increasing power supply and decreasing power price, under the assumption of Cournot competition in forward and spot markets of electricity.

![Figure 1. CO2 emissions in Cases 1 and 2](image-url)
Figure 2. Power generation in Cases 1 and 2

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power generated (MWh)</td>
<td>2,300</td>
</tr>
<tr>
<td>Power price (yen/kWh)</td>
<td>41</td>
</tr>
<tr>
<td>Consumer surplus (1,000 yen)</td>
<td>79,327</td>
</tr>
<tr>
<td>Producer surplus (1,000 yen)</td>
<td>83,188</td>
</tr>
<tr>
<td>Social surplus (1,000 yen)</td>
<td>162,515</td>
</tr>
</tbody>
</table>

Table 9. Power generation, price and surplus in Cases 1 and 2

## 4 Concluding Remarks

We have developed an EPEC model to analyze the interaction of emissions allowances markets with forward and spot markets of electricity under the framework of Cournot competition. Our numerical simulation shows that allocating emissions allowances more to the “clean” generator than to the “dirty” generator may improve efficiency, increasing power supply and decreasing power price.

Further work should aim to incorporate other important elements such as diverse technologies, emissions leakage, and transmission capacity con-
straints. Another avenue for future research is to extend the framework to conduct simulation based on real market data.

References


