An Efficiency Analysis of the Firm Energy Obligations Auction

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Abstract

In this paper we analyze the efficiency of the firm energy obligations auction (descending clock auction), implemented in the Colombian electricity market in 2008. Efficiency is achieved when the winning bidder is the one with the lowest costs. It is important in the allocation of public services like electricity, since its absence can end in the allocation of electricity generation to agents with high costs, affecting not only the competitiveness of the market at scarcity periods, but also affecting consumers due to high prices resulting from inefficient auctions. In this analysis the Nash-Bayesian equilibrium is verified taking into account different valuation distributions. Additionally, according to approximate data valuations we calculate the optimal bids and compare them with the actual bids. As a result we find that for lower-cost bidders it is optimal to over-bid and for average and high-cost bidders it is optimal to shade their bids, all this in different magnitudes, which can lead to inefficient allocations.

Keywords: Multi-unit Auctions, Efficiency, Nash-Bayesian equilibrium, Electricity Markets.

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1 Introduction

Colombian electricity market is divided into three main sources of income for electricity generators: the spot market, bilateral contracts and the firm energy market. For the latter, an auction was held to allocate Firm Energy Obligations (OEF), as part of a new\(^1\) proposal by CREG\(^2\), to ensure the reliability of electricity supply in Colombia in a long term. This scheme called “Cargo por Confiabilidad” seeks to promote the expansion by compensating energy projects for the firm energy - Energy capable of generating during scarcity periods.

In this article we analyze the descending clock auction, implemented by CREG and XM, proposed by Cramton & Stoft (2007), in order to verify its efficiency. In the efficiency of this auction lies the adequate electricity service during scarcity periods at low prices (i.e lower generation costs). This is because electricity in Colombia comes from a large proportion of hydro generation (about \(\frac{3}{4}\) of the actual installed capacity)\(^3\), reason why the hydrological and constant changes in climate can generate great volatility in spot market prices. This affects not only existing generators and possible new investors, who face a high risk, but also affects market competitiveness, since it encourages the participation of agents with higher costs, and leads efficient agents to inflate their costs. Additionally, it can directly affect consumers, who have to pay high prices for energy during critical events like “El Niño”; having to pay even higher prices with the inefficiency of the auction. All this makes it relevant to have electricity generation plants with firm energy, which replaces the energy generated by hydropower at

\(^1\) The previous proposal implemented was “Cargo por Capacidad.”
\(^2\) “Comisión de Regulación de Energía y Gas”: Colombian energy and gas market regulator; more information in www.creg.gov.co/html/1_portals/index_ingles.php
\(^3\) Information taken from XM S.A E.S.P: market operator and administrator; more information in www.xm.com.co/english/Pages/default.aspx
efficient prices, thereby satisfying the demand.

In order to review the impact of this mechanism in the Colombian electricity market we verify the auction efficiency by characterizing a model, taking into account the different bidders’ characteristics, and then we perform an analysis of the Nash-Bayesian equilibrium. Later, we analyze bidders’ behavior and the effect on results by comparing the optimal bid, given bidders’ approximate valuations, with the actual bids of the auction.

The theoretical analysis conducted implies that the equilibrium is inefficient, which can lead to inefficient allocation of the auction. Besides, the analysis of the Nash-Bayesian equilibrium, suggests that in most cases it is optimal for participants to over-bid, bidding an amount greater than their true costs, thus obtaining a high closing price. Also, there are cases in which is optimal for the bidders to bid below their true costs to be awarded with a greater number of units, obtaining positive benefits if the closing price is higher than these costs, or incurring in the risk of having a negative benefit if it is not. Additionally, empirical experiments conducted suggest that for bidders with lower-cost power plants, including hydro technology, is optimum to over-bid in different magnitudes; and for bidders with middle and high costs, of gas and coal technology, it is optimal to bid below their costs in different magnitudes, which in many cases leads to an inefficient allocation. Besides the fact that actual bids tend to be higher than optimal bids, it can lead to a high closing price.

Thus, we conclude that there is evidence of the inefficiency of the descending clock auction equilibrium in cases in which agents bid below and over their true costs in different magnitudes. That is why the remuneration of the generating agents might have been different than it should, ending in a surplus transfer between consumers and generators.
The paper is organized as follows. Section 2 provides a brief description of the auction and its results. Section 3 explains the methodology implemented. Section 4 characterizes the model and additionally verifies the existence of the Nash-Bayesian equilibrium with Monte Carlo simulations of the same. Section 5 provides an empirical verification with the data of firms in the electricity market, comparing the theoretical bids with the results of the auction of 2008. Finally, the last section concludes.

2 Firm Energy Obligations (FEO) Auction

This is a multi-unit auction as it sells many units of a homogeneous good, in the case of electricity each MWh or MWh lot can be considered as a separate unit. It is a descending clock auction in which agents participate with 2 types of power plants\(^4\). A generating agent may have new and existing power plants, i.e the bid is for each plant. The auction is different for the two types of power plants: For existing power plants, the agents have a passive participation in the auction. They send their bids as a supply function compromising their full generation capacity, and have limited opportunities to drop out the power plants at prices established before the auction. The closing price is the same as in a uniform multi-unit auction. For new power plants, the agents have an active participation. They participate on a dynamic descending clock auction format, with the same uniform closing price of existing power plants. Before the auction, the agents know the total number of participants, the price ceiling, the price floor and the market demand curve\(^5\).

\(^4\)Plant refers to an active power generation, which may be of different technologies, such as: thermal, hydro, coal, wind, solar and others.

\(^5\)Note that this information is what the agents really receive and is not part of any model assumption.
In the first round of the FEO auction the auctioneer announces the starting price (price ceiling) and the round closing price, the agents with new power plants must submit their bids for quantities and prices to which they are willing to offer these quantities between the round opening and closing price. These bids must be consistent with a upward slope supply curve, i.e. bidders can only maintain or reduce the quantities as the price falls. These bids are not made public, only the auctioneer and the bidder know them. Bidders can drop out the new power plants as the price falls, at any level in any round, if the price is very low and does not cover the costs of installing capacity. None of the other agents are aware of how many participants withdraw in each round, or at what price. Subsequently, the auctioneer determines and announces the excess supply and the next round closing price, and so on until the excess supply is zero or minimum.

The closing price is given where the market supply and demand are equal; the corresponding bid price for the last agent assigned with a FEO or the highest winning bid. The information disclosure round after round makes this auction behave like a hybrid between an inverse English multi-unit auction\(^6\) (with uniform closing price) and a uniform auction for new power plants; despite the closed bid format.

In this auction implemented for the first and only time in May 2008, FEO were assigned between December 2012 and November 2013. It had a reserve price of two times the “cost of new entry” (CONE)\(^7\) and a price floor of \(\frac{1}{2}\)CONE. 8 participants took part with 10 new power plants.

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\(^6\)In a multi-unit Inverse English clock auction, the auctioneer announces an opening price (i.e price ceiling). This price is decreased in previously established decrements, and bidders withdraw when the sale price is too low for them. That is, agents passively bid, the real bid is where they decide to drop out. This continues until there is a price that matches the remaining passive bids with the number of units for purchase, setting this as the auction closing price. All bidders left are assigned to units for which they bid (passively) at a price equal to the highest winning bid.

\(^7\)The cost of new entry is the average marginal cost of production of the marginal plant from the uniform.
plants (1 with 3 new power plants) and 16 participants with 47 existing power plants, for a total of 18 participants (6 participants with new and existing power plants), for only 2 new entrants in the wholesale electricity market (MEM). As a result, FEO were assigned to all existing power plants and 3 new power plants, of which only one was a new participant in the MEM. The auction closing price was 13.998 $US/MWh; greater than the “efficient” price (CONE), given by the regulator (13.045 $US/MWh), so they may have been a higher remuneration of what it is considered efficient. Therefore this high price can be seen as an extraction of consumer surplus by generators.

2.1 The Auction in the Literature

To the knowledge we have, this particular auction theoretically differs from others studied in the literature, since no other has such features together: interdependent values (De Castro & Riascos (2009), Milgrom & Weber (1982) and McAfee & McMillan (1987)), unknown number of bidders (McAfee & McMillan (1987)), risk aversion (Milgrom & Weber (1982)), and efficiency in multi-unit auctions (Uniform/Inverse English) (Kagel & Levin (2005) and Ausubel & Cramton (2002)). However, in this paper we relax the interdependent valuations, unknown number of bidders and risk aversion assumptions due to the theoretical difficulty.

At first glance, open auctions differ from sealed-bid auctions, specifically in the way they are implemented in real life, the open auctions generally require that participants are in the same place, while in the sealed-bid format the bidders can be anywhere emitting their bids. In an open auction bidders can extract information from the bids and behavior of others, while in

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8Information taken from XM S.A E.S.P.
sealed-bid auctions this does not happen. However, under private valuations assumption, there is strategic equivalence\(^9\) or outcome equivalence between open and sealed-bid auctions. There is also equivalence between static and dynamic auctions, the first makes reference to auctions in which there are multiple rounds, and the later to auctions with only one round.

Given the equivalence between static and dynamic auctions, and between closed and sealed-bid auctions, we find the Nash-Bayesian equilibrium of a dynamic hybrid (sealed-bid/open) auction from its equivalent in static sealed bid auctions, specifically, the equilibrium of the FEO auction of the Colombian electricity market.

Note that a dynamic auction can be seemed as a sequence of multiple static auctions, except that in a dynamic auction, as in the open auction, bidders can get some useful insights of the behavior and bids of other agents, which may affect their strategic behavior. However, the payoff received by agents will not change from one auction to other, because the decision of when to withdraw the power plant is static.

On the other hand, if we consider that English multi-unit auction is the opposite of the inverse English multi-unit auction, and that the ascending uniform auction is the opposite of the descending Uniform auction, then, under private valuations, as the ascending uniform auction has weak outcome equivalence\(^{10}\) with the English multi-unit auction, we can state that there would be outcome equivalence between the Inverse English multi-unit auction and the descending Uniform auction. This is the reason why the payoff received by bidders in the descending clock auction can be modeled as a uniform auction.

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\(^{9}\)Krishna (2009) explains in detail the strong and weak equivalences between open and sealed-bid auctions, in one and multiple units.

\(^{10}\)Note that under interdependent valuations this equivalence does not hold.
Below, we characterize the auction model, which is based on assumptions like: risk neutrality, known number of bidders and private valuations.

3 The Model

In this model we assume that there are $N$ bidders participating in the auction; where $N$ is the set of generating agents, considering a bidder $i \in N$. There are $K$ units auctioned, where $K > 1$, being a multi-unit auction in which each MWh can be viewed as an independent unit and perfect substitute. We assume that bidders are risk neutral, have no budget constraint and there are zero entry and transaction costs.

Each bidder $i$ participates with $m_i \in \mathbb{Z}_{++}$ power plants, with generating capacity $\alpha_i = (\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^{m_i})$, $\alpha_i \in \mathbb{Z}_{++}^{m_i}$, where $\alpha_i^z$ is the generating capacity of the $z \in \{1, \ldots, m_i\}$ power plant of bidder $i$. These power plants can be of 2 different types: New ($U$) or existing ($E$); determined by the set $S = \{U, E\}$ and of one of four different generating technologies: Hydro ($H$), Gas ($T$), Coal ($C$) or Liquids ($L$); determined by the set $R = \{H, T, C, L\}$.

Although there are discussions whether the valuations in this market are interdependent or private, we assume them to be private. This is assumed first of all because we consider that the costs of installing capacity are known only by the bidder itself, as they know exactly the field where the power plants are or would be located and the equipment that the power plant requires to function, this is why these costs are private information of each bidder. Secondly, if we also consider that each bidder knows how their power plants behave during scarcity periods, the costs of generating electricity at this time are private as well.

We define the value of bidder $i$ for his $j$ power plant for the $k_{th}$ unit as $V_{i,j,k}^k$; it is a function
of $C_i$, which includes the production and capacity investment costs, independently and identically distributed according to the distribution function $F(C_i)$. The cost of bidder $i$ for its $j$ power plant for the $k_{th}$ unit is defined as $C_{ik}^j$, which is a function of the $j$ power plant type, $(S)$, of the technology $(R)$ and of the generation capacity $\alpha_i^j$, determining $v_{ik}^j(C_{ik}^j(S,R,\alpha_i^j))$ as the marginal value of bidder $i$ for its $j$ power plant for the $k_{th}$ unit. Finally we define the value of bidder $i$ for its $j$ power plant for the $k_{th}$ unit as $V_{ik}^j = v_{ik}^j(C_{ik}^j(\cdot))$.

In ascending multi-unit auctions, valuations are not increasing, so, as we have a descending multi-unit auction, we equivalently assume that valuations are not decreasing, i.e:

$$V_{i(k)}^j \leq V_{i(k+1)}^j \quad \forall k \in \{1, ..., K - 1\}.$$  

Bidders have the strategy to bid depending on the features previously mentioned, this is why we define the bidder’s $i$ bid for his $j$ power plant for the $k_{th}$ unit as $\beta_{ik}^j(v_{ik}^j(C_{ik}^j(\cdot))) \equiv \beta_{ik}^j(\cdot); \forall i \in N, j \in \{1, ..., m_i\}, k \in \{1, ..., K\}$, with $\beta_{ik}^j(\cdot) \in \mathbb{R}^{++}$ dividing it into a price bid and a quantity bid. In ascending multi-unit auctions, bids are not increasing, so, as we have a descending multi-unit auction, we equivalently assume that bids are not decreasing, i.e: $\beta_{i(k)}^j(\cdot) \leq \beta_{i(k+1)}^j(\cdot) \quad \forall k \in \{1, ..., K - 1\}$. Additionally we assume that bids are not decreasing among valuations, because as costs increase bidders should increase or remain their bids constant to win more units, i.e:

$$\frac{\partial \beta_{i(k)}^j(\cdot)}{\partial V_{i(k)}^j} \geq 0 \quad \forall i \in N, j \in \{1, ..., m_i\}, k \in \{1, ..., K\}$$

Based on all mentioned above we carry out a theoretical verification of the efficiency:
3.1 Theoretical Efficiency Verification

Theoretically, there are two major aspects to evaluate auction performance, efficiency and optimality. In the first it is expected that the bidders with the lowest costs are allocated with the units. In the latter it is expected to get the lowest costs, i.e minimize the costs, or equivalently obtain the lowest possible clearing price. The FEO auction’s aim is to purchase utilities (electricity generation), in which the main concern may be efficiency rather than optimality as it wants efficient generators to win the auction to ensure Colombia’s power system reliability, although indirectly reducing the clearing price.

Efficiency in a multi-unit auction has two implications, first, optimal bids must be separable, i.e the bid for a $k_{th}$ unit does not depend on the valuation by other $l_{th}$ unit, with $l \neq k$. Second, the bid must be symmetric across bidders, units and plants; regardless of whether the bid is from another bidder, plant, or unit, the optimal function form should not change. The following theorem explains this in a greater detail:

**Theorem 1** The equilibrium in a multi-unit auction is efficient, if optimal bids are separable and symmetric across bidders, units and plants\textsuperscript{11}:

\[
\frac{\partial \beta_{i(k)}^j(\cdot)}{\partial V_{i(l)}^j} = 0, \forall i \in N, j \in \{1, ..., m_i\}, k, l \in \{1, ..., K\}, l \neq k
\]

\[
\beta_{i(k)}^j(\cdot) = \beta_{x[l]}^s(\cdot), \forall i, x \in N, j, s \in \{1, ..., m_{i,x}\}, k, l \in \{1, ..., K\}
\]

Analyzing the results of Krishna (2009) and Ausubel & Cramton (2002), we observe that in an ascending uniform auction, when a bidder has a multiple unit demand, he has incentives

\textsuperscript{11}This is an original result proposed by Krishna (2009, Proposition 13.3) applied to FEO auction.
to reduce his bid or what is equivalent, shade his true valuation - theoretically called demand reduction-. This can lead the uniform auction equilibrium to be inefficient, because if the bidder with the higher value shades his bid in a greater magnitude than other, for a particular unit, then the bidder with the highest valuation may not be allocated this unit.

From the above it follows Proposition 1. If there is an increase and/or reduction of the supply, or equivalently if bidders over-bid or shade their bids, in different magnitudes, for any particular unit; it can lead to an inefficient allocation in the FEO auction. This is because, if any bidder bids below its costs and another over-bids, or both over-bid, in different magnitudes, then, this unit could be allocated to a bidder not necessarily with the lowest costs.

**Proposition 1**  *In a multi-unit auction, if for two bidders \( i, x \in N \) which has multiple demand, with not decreasing bids, is optimal to bid, for the \( k_{th} \) unit, an amount different than their valuation, in different magnitudes, then the equilibrium reached can be inefficient.*

**Lemma 1**  *In a descending uniform auction, for any bidder \( i \), it is a weakly dominated strategy to shade his bid for the first and last \( (K_{th}) \) unit: \( \beta_{ik}^j \geq V_{ik}^j \forall i \in N, j \in \{1, ..., m_i\}, \) for \( k = \{1, K\} \)\(^{12}\).*

This auction differs from the one studied in Krishna (2009) (ascending uniform), where it is a weakly dominant strategy to bid the value truthfully for the first unit; this is explicitly given by the market clearing price rule, where the price is the highest losing bid. If instead the price was the lowest winning bid, the strategy would be to shade the bid, or equivalently in an descending auction to over-bid, as shown in De Castro & Riascos (2009).

\(^{12}\)For other units, the result is different, as seen on Proposition 2. The proof of Lemma 1 is available if requested.
On the other hand, Krishna (2009) states that it is a weakly dominant strategy to shade the bid for the last unit; equivalently to over-bid. In the FEO auction there are cases in which bidders prefer not to over-bid for the last unit, as this is a dynamic hybrid (open/sealed bid) auction, and they know the number of units available (excess supply as a block of units) as rounds go on. Let us explain this with an explicit case in which there are few units available: if the bidder over-bids, there is a positive probability that he losses the allocation of this unit, which is the one that he values the most. Now, if the bidder bids his value truthfully, the winning probability would increase and he would be sure to be allocated with all the units, since this is the highest bid. All this at a lower clearing price than if the bidder over-bid and was allocated with this unit, but maximizing his revenue (it depends on which strategy gives a higher profit).

**Proposition 2** In the FEO auction, for units different from the first or the last, it can be a weakly dominant strategy both to over-bid or to shade the bid: \( \beta_{ik}^j \geq V_{ik}^j \lor \beta_{ik}^j \leq V_{ik}^j \forall i \in N, j \in \{1, \ldots, m_i\}, k \in [2, K - 1] \).

According to Krishna (2009), in the ascending uniform auction it is a weakly dominant strategy to shade the bid for units different than the first to set a lower clearing price; or equivalently to over-bid. However, as the FEO auction is dynamic and hybrid between open and sealed-bid, and as the quantity of remaining units for adjudication is revealed (excess supply), the uncertainty is lower than in a classic uniform auction.

Due to this information disclosure, bidders tend to risk more to earn more units, since the probability of being adjudicated with units increases as the revealed excess supply is lower. This is why if bidders over-bid, they may not win any unit, but if they shade their bid they
ensure the adjudication of these units, at a potentially high price. Furthermore, this bidder may also over-bid with other plants, or other bidder will, to set a high clearing price, winning one way or another.

With the above we can observe that bidders have incentives both to shade the bid and to over-bid. On one hand, they may shade their bid to be awarded with a greater number of units, and on the other hand, for other units, bidders may over-bid to set a high clearing price for the units for which they shade their bid. Where does the bid tend to, i.e above or below the costs? It depends on the valuation distribution function, the bidders’ risk aversion, the number of power plants \( (m_i) \) per each bidder \( i \), the number of bidders \( (N) \) and the amount of auctioned units \( (K) \).

### 3.1.1 Some Additional Remarks

In the literature some authors have studied auctions similar to the FEO auction, among these are Cramton (2004) and Kagel & Levin (2005). Below we make a comparison between the results of these and those found in this article.

Cramton (2004) studies a uniform descending auction and concludes that bidding above the marginal cost of production is a result of profit maximization by each bidder. This behavior\(^\text{13}\) is equivalent to over-bid, which may this type of auctions to become inefficient depending on the magnitudes in which bidders over-bids.

The analysis in this paper is consistent with Cramton (2004), where bidders over-bid in order to set a higher closing price. But in the case of the FEO auction, as it also acts as an

\(^\text{13}\)However, the author states that “Bidding above marginal costs is and should be the competitive norm in uniform price electricity auction markets” . . . “so long as there is some probability that the supplier’s bid may affect the clearing price, the profit maximizing bid curve exceeds its marginal cost curve.”
inverse English auction, bidders know with more certainty when they may over-bid in a greater magnitude to be able to set the clearing price, or in a lower magnitude, to be awarded with the last units.

In this article we mention the existence of a tradeoff between shading the bid and over-bidding. This also happens in Cramton (2004), where bidders have a tradeoff between offering a low price to ensure the adjudication of more units, and to offer a high price but winning fewer units at a higher price. But, what does this depend on? Cramton (2004) argues that this depends on whether the marginal gain of over-bidding in the last units exceeds the marginal loss of not being awarded with them. This is another explanation of why in the descending clock auction, there is no clear strategy between over-bidding or bidding the value truthfully for the last unit. However, in the FEO auction bidders have less uncertainty than in the uniform auction discussed in Cramton (2004) as this auction has characteristics of an open auction.

Additionally, Kagel & Levin (2005) studies the uniform ascending auction and the ascending clock auction, and finds similar and equivalent results to those found in this article: In equilibrium, (1) bidders shade their bid for lower valuations. (2) Bidders over-bid in a big magnitude for high valuations to ensure winning these units, this is determined as “exposure problem” because it can be a risky strategy in which bidders can incur in potential losses. (3) For middle valuations, there is a counterbalance between the two previous forces, where in the uniform auction bidders over-bid, but in a small magnitude, and in the clock auction, they over-bid in a big magnitude, depending on prices at which rivals withdraw.

The results of Kagel & Levin (2005) adjust in an equivalent way (as they are ascending auctions) to what we find in this article. We have the following comparison of these results:
(1) Bidders over-bid for lower valuations. (2) Bidders shade their bid for higher valuations to be awarded with these units, having the same problem of getting negative revenue. (3) Bidders shade their bid for middle valuations in the descending clock auction, but as this is a hybrid auction (and opposite) of the two auctions studied in Kagel & Levin (2005), bidders shade their bid in a medium magnitude. That is, bidders shade their bid in a greater magnitude than in a uniform auction, but in a lower magnitude than in an inverse English auction. This is because in the FEO auction bidders know the excess supply, or remaining amount of units, but they ignore how many bidders withdraw and the withdrawal prices.

3.2 Payoff Function

To continue to propose the payoff function of bidder $i$ we need the following definitions: Let $X^j_i = (V^j_{i1}, V^j_{i2}, V^j_{i2}, ..., \sum_{k=1}^{K} V^j_{ik})$ be the aggregated valuations $K$-vector of bidder $i$ for his $j$ power plant; with $X^j_{il} = \sum_{k=1}^{l} V^j_{ik}$ as the $l_{th}$ component of vector $X^j_i$. Let $\gamma^j_{-i}$ be the $K$-vector of competitive bids which bidder $i$ faces with his $j$ power plant. We obtain this vector by rearranging, in increasing order, the other $K \cdot \left( \sum_{i=1}^{N} m_i - 1 \right)_{14}$ bids; this is, the other $(N - 1)$ bidders’ bids for their $m_f$ power plants, for $f \neq i$, for all the $K$ units, and from bidder $i$ for his power plants different from $j$, selecting the first $K$ components. For example, $\gamma^j_{-i_1}$ is the lowest bid from bidders different from $i$ and from the bids from bidder $i$ with his power plants different from $j$, $\gamma^j_{-i_2}$ is the second lowest bid from bidders different from $i$ and from the bids from bidder $i$ with his power plants different from $j$, and so on.

We observe that equivalently to the uniform ascending auction treated in Krishna (2009),

\[ K \cdot m_1 + K \cdot m_2 + ... + K \cdot m_{N-1} + K \cdot (m_i - 1) = K \cdot \sum_{i=1}^{N} m_i - K = K \cdot \left( \sum_{i=1}^{N} m_i - 1 \right). \]
in the descendent auction, a bidder \(i\) wins exactly 1 unit with his \(j\) power plant if: \(\beta_{i1}^j < \gamma_{-ik}^j\) \(\wedge\beta_{i2}^j > \gamma_{-iK-1}^j\). In a more general case, a bidder \(i\) wins exactly \(k\) units with his \(j\) power plant if: \(\beta_{ik}^j < \gamma_{-iK-k+1}^j\) \(\wedge\beta_{i(k+1)}^j > \gamma_{-iK-k}^j\). The price in this uniform auction is given by the highest winning bid, then if bidder \(i\) wins exactly \(k\) units with his \(j\) power plant the market clearing price is:

\[
P = \max \{ \beta_{ik}^j, \gamma_{-iK-k}^j \}.
\]

We now denote the marginal distributions \(H_l \forall l \in \{1, K\}\), corresponding to each competitive bid \(\gamma_{-il}^j\) with densities \(h_l\). We have that \(H_l (\beta_{ik}^j) = \Pr [\gamma_{-il}^j < \beta_{ik}^j]\); also \(H_0 (\cdot) = 1 \wedge \gamma_{-i0}^j = 0\). With this, we define the probability that bidder \(i\) wins \(k\) units with his power plant \(j\) as:

\[
H_{K-k} (\beta_{i(k+1)}^j) - H_{K-k+1} (\beta_{ik}^j)
\]

Taking into account that the price of this auction is given by (1), and that we need to differentiate this function, then we define the probability that the price is equal to \(\beta_{ik}^j\), given that bidder \(i\) wins \(k\) units with his \(j\) power plant, as:

\[
H_{K-k} (\beta_{ik}^j) - H_{K-k+1} (\beta_{ik}^j)
\]

From the above, we continue to write the payoff function\(^\text{15}\) of bidder \(i\), where we take into account the revenue for each of his \(m_i\) power plants, as well as the probabilities of winning each one of the \(K\) units, and that the price is the bid \(\beta_{iK}^j\), according to equations (2) and (3),

\(^\text{15}\)The proof of the payoff function is available if requested.
obtaining the following:

\[
\pi_i(\cdot) = \sum_{j=1}^{m_i} \sum_{l=1}^{K} l \cdot \left( [H_{K-l}(\beta^j_{il}) - H_{K-l+1}(\beta^j_{il})] \cdot \beta^j_{il} + \int_{\beta^j_{il}}^{\beta^j_{il(l+1)}} \gamma^j_{iK-l} \cdot h_{K-l}(\gamma^j_{iK-l}) \, d\gamma^j_{iK-l} \right) \\
- [H_{K-l}(\beta^j_{i(l+1)}) - H_{K-l+1}(\beta^j_{il})] \cdot X^j_{il}
\]

\[\pi_i(\cdot) = \sum_{j=1}^{m_i} \sum_{l=1}^{K} l \cdot \left( [H_{K-l}(\beta^j_{il}) - H_{K-l+1}(\beta^j_{il})] \cdot \beta^j_{il} + \int_{\beta^j_{il}}^{\beta^j_{il(l+1)}} \gamma^j_{iK-l} \cdot h_{K-l}(\gamma^j_{iK-l}) \, d\gamma^j_{iK-l} \right) \\
- [H_{K-l}(\beta^j_{i(l+1)}) - H_{K-l+1}(\beta^j_{il})] \cdot X^j_{il}
\]

3.3 Nash-Bayesian Equilibrium Verification

Based on the payoff function, we proceed to find the optimal bids \( \beta^j_{ik} \forall k \in \{1, K\} \). For this, we differentiate the function with respect to each one of the bids for each unit to obtain the following first order conditions (F.O.C):

\[
\frac{\partial \pi_i(\cdot)}{\partial \beta^j_{il}} = l \cdot [H_{K-l}(\beta^j_{il}) - H_{K-l+1}(\beta^j_{il})] + h_{K-l+1}(\beta^j_{il}) \cdot (V^j_{il} - \beta^j_{il}) = 0 \quad \forall l \in \{1, K\}
\]

(5)

It is worth mentioning that each of the \( K \) conditions depends specifically on the variable for which we are differentiating, i.e the optimal bid for each unit is separable. Finally, for \( l = K - 1 \) and \( l = K \), we obtain the equations (6) and (7), which represent the optimal bids \( \beta^j_{i(K-1)} \) and \( \beta^j_{iK} \) respectively:

\[
\frac{\partial \pi_i(\cdot)}{\partial \beta^j_{i(K-1)}} = (K - 1) \cdot \left[ H_1(\beta^j_{i(K-1)}) - H_2(\beta^j_{i(K-1)}) \right] + h_2(\beta^j_{i(K-1)}) \cdot (V^j_{i(K-1)} - \beta^j_{i(K-1)}) = 0
\]

(6)

\[
\frac{\partial \pi_i(\cdot)}{\partial \beta^j_{iK}} = K \cdot \left[ 1 - H_1(\beta^j_{iK}) \right] + h_1(\beta^j_{iK}) \cdot (V^j_{iK} - \beta^j_{iK}) = 0
\]

(7)

\[\text{16The proof of the F.O.C is available if requested.}\]
Lemma 2 Despite the optimal bid is separable in the valuations, the FEO auction is inefficient since the optimal bid is not symmetric among bidders, nor among the power plants, nor among the units\textsuperscript{17}.

We have already proven that the equilibrium of this auction is theoretically inefficient. Now, with the purpose of observing if this auction can lead to inefficient allocations, we follow to analyze the behavior of optimal bids by Monte Carlo simulations of the F.O.C:

3.3.1 Monte Carlo Simulations of the Equilibrium

Based on equation (7)\textsuperscript{18} we perform Monte Carlo simulations for the $\beta_{iK}$ optimal bid. For this we assume four different valuation distributions $F(C_i^j)$ with their respective parameters: 
Exponential (1), Uniform (0,5) and Normal (17.5,5)\textsuperscript{19}. Based on these distributions we use two methodologies.

The first one is to have a fixed number of bidders $N$: we create $N$ vectors of size $K$ distributed as mentioned above and compute $\gamma_{-i_1}^j$, the lowest competitive bid. Now for the same $K$, we perform this same procedure 1000 times. We estimate the distribution\textsuperscript{20} of the 1000 components of $\gamma_{-i_1}^j$. If for example this data is distributed Uniform with parameters (2,4), then $H_1(\beta_{iK}) = \frac{\beta_{iK} - 2}{4 - 2}$. We now replace this distribution function in equation (7), and for a range of $V_{iK}^j \in (0,5)$, we compute and graph, for various $K$, each optimal bid on the same figure for

\textsuperscript{17}The proof is available if requested. Note that $H_1(\beta_{iK})$ changes if we modify the bidder, the power plant or the unit, as $\gamma_{-i}^j$ varies.

\textsuperscript{18}We choose the F.O.C of the $K_{th}$ unit first of all because it is more simple as we only have to estimate one distribution function ($H_1$), and secondly because this is the marginal unit in which the clearing price would be set.

\textsuperscript{19}These distributions were chosen taking into account since Exponential, Uniform and Normal are among the most used. We also performed simulations with Rayleigh distribution obtaining similar results.

\textsuperscript{20}This is done through the Kolmogorov-Smirnov hypothesis test.
this fixed $N$, as shown later. For the second methodology we perform this same procedure but now for a fixed number of units $K$, varying $N$. Next we show some of the simulations performed:

*Exponential (Fixed $N$)*:

![Figure 1: Optimal Bid: Fixed $N$, Exponential Distribution](image1)

*Fixed $K$*:

![Figure 2: Optimal Bid: Fixed $K$, Exponential Distribution](image2)

For the *Exponential* distribution, we can see that in most cases the optimal bid is above the valuation, i.e. bidders over-bid, and in few cases the optimal bid is below the valuation, i.e. bidders shade their bid; but it is never optimal to bid the valuation truthfully. We can also observe that as the valuation increases, bidders over-bid even more. This is why when the valuations follow this distribution an inefficient equilibrium can be reached. However, the optimal bid doesn’t have any relationship in $K$ or $N$, since it is apparently decreasing in $N$. 
and increasing in $K$, but some cases contradict this. We note that the optimal bid is linear and increasing in the valuations, and additionally, when $K$ is fixed, it is convex in $K$.

**Uniform (Fixed $N$):**

![Figure 3: Optimal Bid: Fixed $N$, Uniform Distribution](image1)

**Fixed $K$:**

![Figure 4: Optimal Bid: Fixed $K$, Uniform Distribution](image2)

For the *Uniform* distribution, as in the previous distribution, in most cases it is optimal to over-bid, and in few cases it is optimal to shade the bid, but it is never optimal to bid the valuation truthfully. We can also observe that as the valuation increases, bidders over-bid even more. This is why when the valuations follow this distribution an inefficient equilibrium can be reached. However, the optimal bid doesn’t have any relationship in $K$ or $N$, since it is apparently increasing in $K$ and $N$, but some cases contradict this. We note that the optimal
bid is linear and increasing in the valuations, and additionally, when $N$ is fixed, it is convex in $N$.

Normal (Fixed $N$):

![Figure 5: Optimal Bid: Fixed $N$, Normal Distribution](image)

(Fixed $K$):

![Figure 6: Optimal Bid: Fixed $K$, Normal Distribution](image)

For the Normal distribution it is optimal to over-bid in all cases. We can also observe that as the valuation increases, bidders over-bid even more. This is why when the valuations follow this distribution an inefficient equilibrium can be reached. However, the optimal bid doesn’t have any relationship in $K$ or $N$, since it is apparently decreasing in $N$, but some cases contradict this. We note that the optimal bid is linear and increasing in the valuations, although is sometimes oscillates.
In general, we remark that in most cases is optimal for bidders to over-bid, and in other cases to shade their bid, therefore the auction can reach an inefficient equilibrium, so the auction may be theoretically inefficient. In addition, simulations of Nash-Bayesian equilibrium corroborate the lemmas, propositions and results of section 4, as, if bidders over-bid and/or shade their bid in different magnitudes an inefficient allocation can be reached. Thus there is evidence of the inefficiency of the equilibrium of the descending clock auction implemented.

4 Equilibrium Empirical Verification

Previously we had found the optimal bid function for a random range of valuations. Now we perform an empirical analysis, with approximations of bidder’s valuations, to first find an optimal bid function for the range of costs we have, and then to find the optimal bid that would have been observed if bidders had behaved according to the model. All this in order to compare the results with the actual bids, and to check if this leads to inefficient allocations.

In this analysis we only use the production costs, although this mechanism aims to compensate the bidders to cover their fixed costs, of installing capacity, these are a good proxy due to economies of scale presented in this market\textsuperscript{21}.

4.1 Data

The data we use consists of a weekly time series of estimated production costs from January 1, 2006 until August 1, 2007, for different MEM power plants, including 45 existing plants of 16 bidders from the FEO auction. We convert these data in $COP/KWh to $US/MWh using

\textsuperscript{21}In the appendix we show a robustness test with installation capacity costs.
TRM\textsuperscript{22} for each of the dates in matter, prices are inflated to the second quarter of 2007, and averaged to obtain the average cost for each plant. Through these costs and marginal costs reported by the program MPODE\textsuperscript{23} we interpolate and calculate the approximate costs for the other bidders for all their power plants, depending on geographical location, technology and generation capacity\textsuperscript{24}. These estimated data of the 18 bidder for the 57 power plants can be discriminated by power plants, or bidders (averaging the plants of each agent), or by new and existing power plants; descriptive statistics are shown below:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Cost $\text{US}/\text{MWh}$ By Plants</td>
<td>17.278</td>
<td>4.843</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>By Bidders</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17.477</td>
<td>3.693</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>New</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>19.988</td>
<td>4.759</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Existing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.701</td>
<td>4.710</td>
<td>47</td>
</tr>
</tbody>
</table>

Table 1: Descriptive statistics of used data. Source: Superservicios & MPODE

4.2 Experiments

In the experiments we carry out, we suppose that the approximate costs are the true costs of producing energy (i.e. valuations). The aim of these experiments is to demonstrate how inefficient allocations can or could be reached since for bidders is an optimal strategy to over-bid or shade their bid in different magnitudes.

In the first experiment proposed we assume that all existing plants are allocated\textsuperscript{25} with FEO. For this we aggregate the supply of these plants, having an excess demand, to be covered

\textsuperscript{22}Exchange rate from $\text{COP}$ to $\text{US}$.

\textsuperscript{23}Modelo de Programación Dinámica Dual Estocástica (MPODE): used to predict rainfall by which production costs for hydroelectric and thermal plants are estimated.

\textsuperscript{24}To estimate the cost of a power plant we take into account data from other power plants in the same region, the same technology, and in a similar range of installed capacity (MW).

\textsuperscript{25}This is because in the auction, existing plants could only withdraw (if desired) at prices below 0.8 times the CONE, plants which were actually allocated with FEO.
by the new plants. We assume that bidders behave like in a Vickrey auction, where bidders bid
their costs truthfully. So we sort the costs from new plants, in an increasing order, and add up
the individual supply one by one to the aggregate supply of existing plants until supply equals
demand, or the excess supply is the minimum. Finally we obtain the efficient allocation (i.e
lower-cost) of the auction, with 3 new plants allocated efficiently: Termocol, Tasajero 2 and
Amoyá. From this result we can appreciate that 2 of the 3 power plants allocated with FEO
in this experiment were allocated in the actual auction. While the power plant not assigned in
the 2008 auction was Tasajero 2, in its place the power plant Gecelca 3 of bidder Gecelca was
assigned. This shows that this auction was not entirely efficient because it adjudicated FEO to
non lower-cost bidders.

In the second experiment, we find that the costs are distributed: \( F(C_i^j) : GEV \) and
\( Normal \). Later, knowing that \( N = 18 \) and \( K = 66 \ TWh/\text{Año} \), we apply the same methodology
as the one in Monte Carlo simulations to find \( H_1(\beta^j_{iK}) \), and finally we obtain the optimal bid
function \( \beta^j_{iK} \) for \( V^j_{iK} \in (0, 40) \). The resulting graphs are shown below:

This experiment shows similar results both for \( F(C_i^j) : Normal \) and \( GEV \), as \( H_1(\beta^j_{iK}) \) is
distributed Generalized Pareto \( (GP) \) with similar parameters, and as the shape of the optimal
bid is linear and almost equal. It is important to appreciate that for small valuations bidders tend to over-bid, while for medium and large valuations bidders tend to shade their bid in a great amount\textsuperscript{26}.

In the last experiment we compute the optimal bids based on the model’s equation (7) in order to compare them with the actual bids\textsuperscript{27}, and to analyze the individual efficiency.

As a previous result of this experiment, comparing the actual bids with the approximated costs, we have that from the 57 plants, 5 of them have their bid above their costs (2 new and 3 existing), the lower-cost, 4 of which are of hydro technology (not necessarily with the lowest capacity); meanwhile all other power plants have their bid below their costs.

To find the optimal bids $\beta^j_{ik}$ we perform an iterative methodology which consists of estimating the distribution $H_1$ with Kolmogorov-Smirnov statistic: ($Log\text{Normal}, GEV, GP$). First with $H_1^{Initial}$: GEV and GP, with the parameters found in the second experiment. Below we summarize the descriptive statistics of the optimal bids computed:

<table>
<thead>
<tr>
<th>$H_1^{Initial}$</th>
<th>$H_1$</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEV</td>
<td>LogNormal</td>
<td>5.447</td>
<td>0.502</td>
<td>6.750</td>
<td>4.751</td>
</tr>
<tr>
<td>GEV</td>
<td>GEV</td>
<td>28.242</td>
<td>0.048</td>
<td>28.337</td>
<td>28.168</td>
</tr>
<tr>
<td>GP</td>
<td>GP</td>
<td>12.819</td>
<td>1.379</td>
<td>21.816</td>
<td>12.022</td>
</tr>
<tr>
<td>GP</td>
<td>LogNormal</td>
<td>8.986</td>
<td>0.609</td>
<td>10.529</td>
<td>7.551</td>
</tr>
<tr>
<td>GP</td>
<td>GEV</td>
<td>9.692</td>
<td>0.591</td>
<td>10.865</td>
<td>8.558</td>
</tr>
<tr>
<td>GP</td>
<td>GP</td>
<td>9.534</td>
<td>0.332</td>
<td>11.749</td>
<td>9.448</td>
</tr>
</tbody>
</table>

Table 2: Descriptive Statistics of the Optimal Bids

The following table shows more detailed results about the quantity of power plants for which

\textsuperscript{26}This result is consistent with the results of Kagel & Levin (2005) mentioned in section 4.1.1.

\textsuperscript{27}For the actual bids from the new power plants which didn’t drop out (were adjudicated) we assume that the actual bids are equal to the clearing price (13.998 $US/MWh). For the actual bids from the existing power plants we assume that the actual bids are equal to 0.8 times the CONE, as this could only drop out below this price.
the actual bid is higher than the optimal bid, and the quantity of power plants for which the optimal bid is higher than the estimated costs. Additionally, we observe if there is an efficient allocation for the new plants.

<table>
<thead>
<tr>
<th>$H_{Initial}^1$</th>
<th>$H_1$</th>
<th>Actual $&gt;$Optimal</th>
<th>Optimal $&gt;$Costs</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEV</td>
<td>LogNormal</td>
<td>57</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>GEV</td>
<td>GEV</td>
<td>0</td>
<td>57</td>
<td>Yes</td>
</tr>
<tr>
<td>GP</td>
<td>GEV</td>
<td>9</td>
<td>12</td>
<td>No</td>
</tr>
<tr>
<td>GEV</td>
<td>GP</td>
<td>56</td>
<td>0</td>
<td>No</td>
</tr>
<tr>
<td>GP</td>
<td>GEV</td>
<td>53</td>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>GP</td>
<td>GP</td>
<td>56</td>
<td>3</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 3: Actual/Optimal Bids and Costs Results

From the previous table we notice that when $H_1$ is LogNormal, the actual bid is higher than the optimal bid for almost all power plants, and additionally, bidders shade their bid in different magnitudes for all power plants, ending in an inefficient allocation. When $H_{Initial}^1$ and $H_1$ are GEV we note that the actual bid is always lower than the optimal bid, and furthermore, bidders always bid above their costs in different magnitudes, however, this time, it reaches an efficient allocation.

In the case where $H_{Initial}^1$ is GEV and $H_1$ is GP, we observe that for 9 of the 10 new power plants the actual bid is higher than the optimal bid, with the highest-cost power plant being the only one with an optimal bid higher than the actual bid. Moreover, 12 of the lower-cost power plants over-bid in different magnitudes, of which 11 are of Hydro technology; meanwhile the remaining power plants shade their bids in different magnitudes. This last result ends in an inefficient allocation, since the lowest-cost new power plant was not allocated because it over-bid, therefore only the two following lower-cost power plants were allocated.

\[\text{In none of the cases the actual bid is equal to the optimal bids, neither the optimal bid and estimated costs.}\]
Now, in the case where $H_1^{\text{Initial}}$ is GP and $H_1$ is GEV or GP we obtain similar results, in which the real bid is higher than the optimal bid for most of the power plants. In addition, bidders tend to shade their bid for most of their plants in different magnitudes, with only the 3 lower-costs power plants (of Hydro technology) over-bidding. Despite this, one case ends in an efficient allocation and the other does not.

In general, we can summarize these experiments by affirming that lower-cost power plants tend to over-bid in different magnitudes, being these of hydro technology; while the other remaining power plants tend to shade their bids in different magnitudes. This result and the fact that the actual bids tend to be higher that the optimal bids can lead to inefficient allocations as stated in Proposition 2. This is because lower-cost power plants have incentives to over-bid in order to obtain a higher closing price, taking the risk of not being adjudicated with FEO, while the medium and high cost power plants shade their bids to be assigned with units, taking the risk of being adjudicated with FEO at a price which generates losses.

5 Conclusions

In this paper we show evidence of the FEO auction inefficiency, since as this auction is theoretically equivalent to a hybrid between a descendent uniform auction and an inverse English clock auction, for bidders is an optimal strategy to bid above and/or below their costs in different magnitudes. For this reason, by implementing this auction, the compensation given to the bidders might had been different than it should; noting the fact that the clearing price was higher than the “efficient” price.

Through the theoretical model, we proved that bidders have as an optimal strategy to
over-bid to set a higher clearing price, and to shade their bid to be awarded with more units. The difference between the magnitudes which bidders bid above or below their costs, makes it possible to reach an inefficient allocation, as seen on the empirical experiments performed.

Most of the Monte Carlo simulations performed show how for different distributions the optimal bid is higher than the costs, while few cases show that the optimal bid is lower than the costs. Furthermore, experiments conducted show that lower-cost power plants, mostly of Hydro technology, over-bid in different magnitudes, while the remaining power plants shade their bid in different magnitudes. Additionally, the actual bids tend to be higher than the optimal bids, which may lead to a higher clearing price than what is considered efficient. All these facts indicate that there is wide evidence that the FEO auction held is theoretically inefficient, therefore the 2008 auction may had reached an inefficient allocation.

Theoretically there may be efficient multi-unit auctions to replace the current format of the FEO auction: The discriminatory auction, in which bidders can over-bid in a lower proportion than in an uniform auction, as wells as they may not be an inefficient allocation as shown in Swinkels(1999), where as the number of bidder increases the inefficiency in this auction tends toward zero. The Vickrey auction, which is theoretically efficient. Both types of auctions can lead not only to an efficient allocation but also to a lower clearing price, improving the revenue of efficient bidders and consumers, as well as contributing the market competitiveness in times of scarcity.

The auctions are usually a good allocation mechanism, which are helpful when there are information asymmetries. However, the design of an auction must adjust to the market being applied, taking into account its characteristics; as “auction design is not one size fits all and
must be sensitive to the details of the context” (Klemperer (2002)). So when implementing a new format to FEO auction, Colombian electricity market must be well known to achieve a suitable design that leads to an efficient allocation.

6 Appendix: Robustness Test

In this appendix we take approximate costs of installing capacity in $US/KW, converting them to hourly payments, first in a period of 15 years with a WACC of 6.22% and with different power plant factors depending on the technology. As existing plants have already covered their fixed costs, i.e of installing capacity; we assume that all these are adjudicated with FEO. This is why we only take into account new power plants, assuming that the bids of existing power plants don’t affect the bids of new power plants. The costs calculated have a mean of 14.591$US/MWh and a standard deviation of 4.044 for the 10 new power plants. Below we implement the same procedure as in the last experiment of Section 5 for the distributions showed in the table:

<table>
<thead>
<tr>
<th>$H_{init}$</th>
<th>$H_1$</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Actual&gt;Optimal</th>
<th>Optimal &gt;Costs</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEV</td>
<td>LogNormal</td>
<td>12.076</td>
<td>0.521</td>
<td>10</td>
<td>3</td>
<td>No</td>
</tr>
<tr>
<td>GEV</td>
<td>GEV</td>
<td>10.428</td>
<td>0.605</td>
<td>10</td>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>GP</td>
<td>GP</td>
<td>10.706</td>
<td>0.525</td>
<td>10</td>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>LogNormal</td>
<td>LogNormal</td>
<td>12.545</td>
<td>0.697</td>
<td>9</td>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>GEV</td>
<td>GEV</td>
<td>12.269</td>
<td>1.858</td>
<td>10</td>
<td>4</td>
<td>No</td>
</tr>
<tr>
<td>GP</td>
<td>GP</td>
<td>12.092</td>
<td>0.595</td>
<td>10</td>
<td>3</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 4: Actual/Optimal Bids and Costs Results

Results are in line with the ones shown on Section 5 as they follow a general trend: the actual bid is higher than the optimal bid for almost all power plants, and additionally, lower-cost power plants over-bid in different magnitudes, while the high-cost shade their bid in different magnitudes, leading in 5 of the 6 cases to inefficient allocations. In this case, lower-cost plants are not necessarily of Hydro technology, Gas instead, since production costs are lower for Hydro, but costs of installing capacity are lower for Gas. However, the empirical result is the same,
lower-cost plants have incentives to over-bid to obtain a higher closing price, while the high and average cost plants shade their bid to be assigned to units.

References


