Climate Effects of Carbon Taxes, Taking into Account Possible Other Future Climate Measures

Abstract

The increase of fuel extraction costs as well as of temperature will make it likely that in the medium-term future technological or political measures against global warming may be implemented. In assessments of a current climate policy the possibility of medium-term future developments like backstop technologies is largely neglected but can crucially affect its impact. Given such a future measure, a currently introduced carbon tax may more generally mitigate climate change than recent reflections along the line of the Green Paradox would suggest. Notably, the weak and the strong version of the Green Paradox, related to current and longer-term emissions, may not materialize. Moreover, the tax may allow the demanding countries to extract part of the resource rent, further increasing its desirability.

JEL-Code: Q54, Q31, Q38, Q41, Q42.

Keywords: climate change policy, greenhouse gas tax, carbon tax, Green Paradox, anticipation effects, exhaustible resources, fossil fuels market, backstop technology, uncertainty, resource rent.

Florian Habermacher
University of St. Gallen
Swiss Institute for International Economics and Applied Economic Research (SIAW-HSG)
Bodanstrasse 8
9000 St. Gallen
Switzerland
florian.habermacher@unisg.ch

Gebhard Kirchgässner
University of St. Gallen
Swiss Institute for International Economics and Applied Economic Research (SIAW-HSG)
Leopoldina and CESifo
Bodanstrasse 8
9000 St. Gallen
Switzerland
gebhard.kirchgaessner@unisg.ch

22 March 2011 – Thanks to Robert King, Thomas Davoine and Oliver Schenker for helpful remarks and suggestions.

1 Introduction

In his seminal contribution, Pearce (1991) discussed conveniences of a carbon tax as an efficient policy instrument to reduce carbon dioxide emissions. He solely considered the demand side, implicitly assuming a fixed, exogenous energy supply function.

Today a large fraction of climate economics research still exhibits the same limitation, reducing the supply side of the energy market to a static process. However, at least since the contribution of Sinn (2008), there exists growing awareness that supply side effects can be crucial for the assessment of carbon emission reduction strategies. Along the claim which Sinn entitled 'Green Paradox', a realistic carbon tax introduced at a low initial level but rapidly increasing over time might be counterproductive for the climate, by primarily accelerating exploitation of the limited resources rather than delaying or reducing their combustion. This is the conclusion he derives from a model in which owners of limited stocks of fossil fuels optimise their sales over time. They anticipate in early periods that the tax will in future be higher, inducing them to sell more of their fuels today rather than on the highly taxed future markets. While controversial, Sinn's analysis has impressively demonstrated the importance of supply side effects for the assessment of greenhouse gas policies.

There exists a growing literature that tries to assess the possibility for the mentioned counterproductive effects of climate protection policies to occur in specific situations. Focusing on alternative technologies rather than on a carbon tax, Gerlagh (2011) examines the impact of suppliers' anticipation on the climate benefits from cheaper future backstop technologies. A similar direction is taken by Van der Ploeg and Withagen (2010). While the latter also show that in some cases a specific, not rapidly increasing tax could be beneficial for the climate, they do not discuss effects of other, non-optimal taxes. Polborn (2011) concludes that intensifying research on carbon capture and storage has the advantage of reverting the negative anticipation effects that research on backstop technologies would have in terms of near-term carbon emissions.

The analyses by Sinn and subsequent contributors assumed a world in which the debated policy would be the only potential relevant climate measure, valid from today on throughout the entire future. But abstaining from a carbon tax today will not imply that neither a carbon tax, nor any alternative climate relevant development may materialize in the future. Rather, without substantial measures today, the unlimited growth of the climate threat may increase the necessity of future measures to be taken, implying even more stringent future measures than if the carbon tax would have been introduced today. This point is likely to be relevant for the desirability of a current carbon tax as the resource owners may not only anticipate a rapidly increasing tax but also other potential future measures.

In a recent contribution, Hoel (2010) has taken some account for this. He has been the first to explicitly model the fact that to purposely avoid the introduction of a current tax influences only the probability to have a certain tax in the medium or long-term future, rather than implying that the current abstention could necessarily prevent any potential future tax. He considered a stylized two-period model with the carbon tax in the second period being endogenous, and found that the impossibility of long-term commitments of current politics increases the desirability of the introduction of a carbon tax today.

Finally, Van der Ploeg and Withagen (2011) discuss the effect of dirty and clean backstops on optimal carbon taxation. However, they focus on backstops that are already available today and, more importantly, consider only the choice of optimized or prohibitive tax paths, ignoring the possibility of the arbitrarily increasing taxes that are inherently part of the argumentation of the Green Paradox.

In this paper, we model the impact of an – eventually rapidly increasing – carbon tax on global medium and long-term emissions, taking future climate measures into account: we assess the impact of current carbon taxes given the fact that even if a tax is currently avoided, other climate measures, such as backstop technologies, global fuel demand cartels à la Kyoto, carbon capture and storage systems, or, last but not least, alternative carbon taxes, may be introduced at some point in the future. In order to keep the model tractable, we assume these future measures to be introduced independently of the current tax, although in many cases taking into account the endogeneity of such measures could even strengthen our findings.

The analysis is based on a dynamic multi-period model of the behaviour of forward looking resource owners. They seek to maximize their present discounted revenues by optimally rationing the sales of their resources over time. In order to derive rather general results, we leave the exact nature of the modelled market as open as possible. We do neither assume a specific functional form for the extraction cost curve nor use specific assumptions about the tax path or the time-varying demand function. Finally, we consider both cases, where the suppliers act monopolistically or competitively. In addition, we investigate the case where a carbon tax is only introduced regionally, i.e. in some part(s) of the world.

In the presence of an anticipated future regime change such as the introduction of a backstop technology, any presently implemented positive tax path bridging the time until the future measure becomes effective unambiguously reduces cumulative emissions not only in the long, but already in the medium-term, suggesting that the strong version of the Green Paradox may not hold.[1] This generally holds for a backstop technology becoming worldwide effective at a specific future time. At least for limited tax levels, the

[1] Following Gerlagh (2011) we use the notions of weak and strong versions of the Green Paradox to differentiate between the increase of current emissions (weak) resp. of net present value of cumulative emissions (strong) due to the anticipation of cheaper clean energy.

results remain valid in the case of regional taxes. The exact type of the future scheme does not affect our findings. According to recent estimates the warming effect of emissions in the current century will remain almost unchanged over at least the next 1'000 years (Solomon et al., 2009). This suggests that, as far as we restrict the attention to medium-term emissions, primarily the *cumulative* emissions matter and the exact path of the emissions across the decades is only of limited additional importance. Thus, reducing medium-term emissions, the tax is very likely favourable for the climate.

Giving up the assumption that an alternative measure is implemented at a fixed point of time, we further consider a case where the time of the introduction of the backstop is stochastic. Even under these conditions, the weak version of the Green Paradoxes' claim, i.e. that taxes increasing at a rate faster than the real interest rate lead to increased current emissions, does not necessarily hold; taxes increasing at rates higher than the real interest rate can not only reduce cumulative emissions for some future period, but reduce current and near term emissions as well.

This analysis has important implications for climate policy assessment in general. There exist numerous assessments of different climate policy measures, but these studies typically compare scenarios with the measure in question to a business as usual scenario containing no alternative climate policy measures. There is, however, no reason to believe that the decision about a particular climate policy will be decisive for every other potential climate measure as well. Taking the possibility of alternative climate measures into account might often be necessary to prevent strongly biased results.

The remainder of the paper is organized as follows: Section 2 describes the model for the resource owners' intertemporal decision problem. In Section 3, we show how the anticipation of a backstop implemented in the medium-term future affects the resource suppliers' behavior in the business as usual scenario without any present tax. We explain that the anticipation of the future regime change induces a situation that is comparable to a future high tax, implying even according to the anticipation effects pointed out by Sinn (2008) that it becomes especially urgent to introduce a present tax.

Section 4 shows that a presently introduced tax bridging the time up to the introduction of the backstop will unambiguously reduce cumulative medium-term emissions. Section 5 explains how the analytical derivation in the previous section extends to the case of alternative future schemes. In Section 6, we discuss possible extensions of the model and show the robustness of our analysis to a tax that may be applied only regionally. Also, in this section, we show that a stochastic time of introduction of the backstop implies that taxes can unambiguously reduce short- and medium-term emissions even if they increase faster than the maximal rate which, according to the proposition of the Green Paradox, is compatible with a reduction of carbon emissions. We also shortly discuss the possible endogeneity of the future scheme switch.

Section 7 provides a short discussion of the importance of the future regime switch to our results, as well of how the tax can lead to a shift of part of the resource rent to the consumer countries. Finally, Section 8 concludes.

2 The Model

Lumping the different categories of fossil fuels into one considered *resource*, we assume a world where consumers' instantaneous demand rate r_t, which equals the extraction rate, is a continuous, strictly decreasing, and potentially time-varying function of its price, p_t. Thus, we have the demand curve, $r_t(p_t)$, as well as its inverse, $p_t(r_t)$, as two strictly decreasing functions, $r_t'(\cdot) < 0$, $p_t'(\cdot) < 0$, where the strict inequalities may only not apply if the values of r or p reach their respective upper or lower boundaries, should these exist.

Instantaneous extraction rates integrate to cumulative extractions, A_t, which are normalized to zero at the starting time, $A_0 \equiv 0$, $A_t = \int_0^t r_s ds$. Extraction costs, c, are assumed to be strictly increasing in the cumulative extractions: $c'(A) > 0$. This implies that the most easily extractable resources are extracted first - a standard assumption which has been shown to be a necessary condition for the potential optimality of an extraction path (Herfindahl, 1967).[2]

We model the resource owners' problem of maximizing their present value of expected total net revenues, applying a positive discount rate ρ.

Given a tax path τ_t, the revenue flow for a specific seller i at time t is $r_{t,i} \cdot (p_t - c_t - \tau_t)$, where $r_{t,i}$ is seller i's extraction rate, and the suppliers' maximization problem can thus be written as

$$\begin{aligned} U_i &= \max_{r_{t,i}} \int_{t=\underline{t}}^{\bar{t}} \mathrm{e}^{-\rho t} r_{t,i} \cdot (p_t(r_t) - c(A_t) - \tau_t)\, dt \qquad (1) \\ &\text{s.t.} \quad \dot{A}_t = r_t \text{ and } A_0 = 0, \text{ i.e. } A_t = \int_{s=0}^{t} r_s ds, \text{ and } r_t = \sum\nolimits_i r_{t,i}. \end{aligned}$$

In the competitive (comp) case, suppliers' individual rates are so small that each considers the market price as given independently of his own supply, while the monopolistic (mono) supplier will take the effect of his extraction rate onto prices into account, since the total rate equals his own supply rate, $r_t \equiv r_{t,i}$. Defining P_t as the considered rate of change of

[2] For positive real interest rates it is actually straightforward to see that this must hold.

the term $r_t \cdot \mathrm{p_t}(r_t)$ in Eq. (1), we thus have in the two considered variants of the model:

$$\begin{aligned} P_{t,\mathrm{mono}}(r_t) \equiv \left.\frac{\partial\left[r_{t,i}p_t(r_t)\right]}{\partial r_{t,i}}\right|_{\mathrm{mono}} &= p_t(r_t) + r_t p_t'(r_t) \\ P_{t,\mathrm{comp}}(r_t) \equiv \left.\frac{\partial\left[r_{t,i}p_t(r_t)\right]}{\partial r_{t,i}}\right|_{\mathrm{comp}} &= p_t(r_t) \end{aligned}$$

Taking this into account in the current-value Hamiltonian,

$$\mathcal{H} = r_t \cdot (p_t(r_t) - c(A_t) - \tau_t) - \lambda_t r_t, \tag{2}$$

we arrive at the following two first order conditions:

$$\frac{\partial \mathcal{H}}{\partial r_t} = 0: \quad P_t(r_t) = c(A_t) + \tau_t + \lambda_t \tag{3}$$

$$\dot{\lambda}_t = \rho\lambda_t + \frac{\partial \mathcal{H}}{\partial A_t}: \qquad \dot{\lambda}_t = \lambda_t\rho - \dot{c}_t, \tag{4}$$

where we defined $c_t \equiv \mathrm{c}(A_t)$, and where λ_t is the shadow value at time t for a marginal unit of resource stock, after the cumulative extraction of A_t previous units. This multiplier λ_t is a non-negative value, as with a larger resource stock, the producer's future extraction costs will be reduced and therefore the future achievable profit potentially higher and never lower.

The backward resp. forward looking explicit solution for the multiplier in Eq. (4) become, for any $\underline{t} < t < \bar{t}$,

$$\begin{aligned} \lambda_t &= \mathrm{e}^{\rho(t-\underline{t})}\lambda_{\underline{t}} - \int_{s=\underline{t}}^{t} \mathrm{e}^{\rho(t-s)}\dot{c}_s ds \\ \lambda_t &= \mathrm{e}^{\rho(t-\bar{t})}\lambda_{\bar{t}} + \int_{s=t}^{\bar{t}} \mathrm{e}^{\rho(t-s)}\dot{c}_s ds. \end{aligned} \tag{5}$$

The primary assumptions on which we will base our analysis of the supply behavior implicitly defined with the maximization problem are the following:

- Property 1: $p(0) > c(0)$, i.e. in the absence of a tax there will be a strictly positive extraction rate at least at the starting time.
- Property 2: $p(0) < \infty$, i.e. the choke-price is finite. This is an intuitive assumption notably as surrogates such as renewable wood or plant oils lend themselves as natural substitutes.
- Property 3: $c(A) < p(0) \Rightarrow 0 < c'(A) < \infty$, i.e. as long as some resources are profitably extractable, the rate of increase of the extraction costs is strictly positive and finite.

- Property 4: $\lim_{r\to\infty} p(r) = 0$, i.e. when the supply rate tends to infinity, the demand price becomes zero.

- Property 5: Single-crossing in the first order conditions for the monopolistic supplier: the marginal revenue of a monopolist's resource sales at a specific period is falling in the current rate of extraction, i.e. $\frac{\partial[p(r)+p'(r)r]}{\partial r} < 0$ holds in the case for the globally homogenous market, and $\frac{\partial[p(r,\tau)+p'(r,\tau)r]}{\partial r} < 0$ in the case of the regional tax.[3]

3 Future Regime Change in the BAU

In our analysis, the business as usual scenario (BAU) simply refers to the case in which no present tax is introduced. However, we generally assume it to contain a relevant future regime switch. As this is a major difference to previous studies, this section compares this BAU to the case where no future regime change would take place.

Ruling out taxes, the suppliers' maximization problem can be represented by the Hamiltonian formulation from Eqs. (2) through (4), by simply assuming τ_t to be zero everywhere.

Introducing a backstop at time T prevents future sales and thus implies that the value of the remaining resources at that time are zero. In this case, we can use $\bar{t} = T$ and $\lambda_T = 0$ in Eq. (5), yielding

$$\lambda_t = \int_{s=t}^{T} \mathrm{e}^{\rho(t-s)} \dot{c}_s ds.$$

This shows that λ_t is positive and approaches zero for $t \to T$.

If, on the other hand, *no backstop is introduced*, we know that $\lim_{t\to\infty} \lambda_t \mathrm{e}^{-\rho t} = 0$. Using thus $\bar{t} = \infty$ and $\lim_{\bar{t}\to\infty} \mathrm{e}^{-\rho\bar{t}} \lambda_{\bar{t}} = 0$ in Eq. (5), we get

$$\lambda_t = \int_{s=t}^{\infty} \mathrm{e}^{(t-s)\rho} \dot{c}_s ds.$$

Thus, for any time t prior to the extraction of the last unit, the multiplier λ_t will take on a strictly positive value in the BAU variant without backstop. This will notably be the case for the time of the implementation of the backstop in the other BAU scenario, i.e. at T: defining the backstop-scenario as a case where the backstop is *relevant*, implies that it would be introduced at a time before the resource extraction would otherwise have stopped. We thus have

$$\lambda_T \begin{cases} = 0 & \text{for} \quad \mathrm{BAU}_{\text{backstop}} \\ > 0 & \quad\quad\;\; \mathrm{BAU}_{\text{no backstop}}. \end{cases} \tag{6}$$

[3]This assumption seems largely unproblematic; an extended note on it is provided in Part A of the Annex.

It is obvious that the introduction of the backstop at time T affects the resource owners' optimization problem exactly in the same manner as a tax introduced at T would if the tax rate were to be high enough for preventing any oil sales from time T onwards. This leads already to the primary mechanism by which we will find that in expectation of alternative future schemes it is rather urgent than counterproductive to introduce stringent present carbon dioxide taxes: given the future schemes, the suppliers anticipate a future tax-resembling measure - if no tax is introduced today, the situation for the suppliers will correspond to one with a high future tax but with none today, which corresponds exactly to the case in which the Green Paradox would - in this case righteously - predict counterproductive effects. A tax introduction today is thus even more urgent the more anticipation effects drive the resource owners.

As is emphasized in Proposition 1 and proven in Part B of the Annex, Eq. (6) ultimately implies that the anticipated introduction of the backstop in the BAU scenario increases the pre-T emissions.

Proposition 1. *An anticipated increase (decrease) of the marginal value of the unexploited resources at a specified future time will lead to a decrease (increase) in cumulative extraction during the period up to that future time.*

4 Introducing a Tax before the Backstop

Here we consider the case for a present tax when a backstop technology is introduced in the medium-term future at time T. The Hamiltonian formulation with the corresponding first order conditions for the dynamic problem is given in Eqs. (2) through (4) in Section 2.

Recall that the multiplier becomes zero at the time of the introduction of the backstop, $\lambda_T = 0$. This strong assumption, which has been used in earlier literature as well (see e.g. Dasgupta and Heal 1974), may not necessarily have to be as far away from reality as it may seem at first sight: given that a backstop will substitute the fossil fuels in all major energy related applications, the residual demand for them, dedicated to, e.g., chemical applications, will only amount to a limited fraction of prior consumption, drastically reducing the expected achievable resource rent. Note that the smaller demand would limit the scope for monopolies as even owners of small stocks could become relevant competitors. We will consider the case for a residual value λ_T for the post-T period that is non-zero and can vary with the amount of resources left at time T, in the next section.

As the tax generally reduces the possible net revenues from resource sales, it seems intuitive that positive tax rates will lead to reduced *cumulative* extractions.

Proposition 2. *If at a specific time $T > 0$ an alternative climate measure that fixes the marginal value $\lambda_T(A_T) = \lambda_T = const$ is introduced, any scheme of positive carbon taxes up to time T leads to a reduction of cumulative emissions up to time T.*

Proposition 2 is proven in Part D of the Annex.

If a regime change such as the introduction of a backstop technology is anticipated, a carbon tax thus yields a decrease of total consumption, independently of the form of the tax path or of the demand and production cost structure. According to our argumentation above, reducing cumulative medium-term emissions is of primordial importance compared to the exact path of the emissions, as long as relatively limited time-spans are considered. Thus under the assumption of a future backstop quite any path of nonnegative tax rates is beneficial for the climate.

5 Extension to Alternative Future Schemes

In Proposition 2 we have shown that any (continuous) path of positive tax rates reduces cumulative resource use during the period from the introduction of the tax until its replacement by a different climate change mitigation measure, given that the value of the marginal remaining resource unit for post-T sales is independent of the size of the stock of remaining resources, i.e. $\lambda_T(A_T) = \text{const}$. This condition may not necessarily be given in reality: rather, the marginal value of additional reserves, λ_T, depends on the cumulative exploitations at time T for example if the post-T scheme is a demand cartel or an extremely high tax - only with a (perfect) backstop completely substituting the fossils would λ_T not vary with the residual resource stock. This section shows that the argument for the tax to be reducing cumulative extractions extends to the case of a flexible multiplier, $\lambda_T = \lambda_T(A_T)$.

If the post-T regime does not prohibit all lucrative sales of the resource, an increase of the remaining stock of resources can affect λ_T in either direction: satiation and the higher discounting of future sales tend to decrease the shadow value of additional resources on one hand, but the lower extraction costs for the additional unextracted resources can also increase the additional units' value. Without further assumptions about the exact nature of the post-T resource market framework or about extraction costs or the demand function it cannot a priori be known which effect dominates. Still, using Proposition 1, we show that the economics of the problem implies that one can rule out one possible case for the relationship between λ_T and A_T in the region of the optimally chosen amount of cumulative extractions, A_T^*. Then, an illustration why the derived restriction on the relationship between λ_T and A_T implies that Proposition 2 extends to cases with a flexible final multiplier $\lambda_T(A_T)$ follows.

First, some clarifications about the relationships between the marginal value and the amount of cumulative exploitations at the time of the introduction of the new regime, λ_T and A_T. The function $\lambda_T(A_T)$ indicates the value of a marginal additional unexploited unit of resource at time T available for the post-T period, defined as the additional (expected) profit the resource owner can make in the post-T future if he has a marginally increased stock of remaining exploitable resources at time T. Conversely, function $A_T(\lambda_T)$ designates the cumulative amount of pre-T sales the resource owner chooses for a given final marginal multiplier λ_T. It therefore corresponds to the amount of pre-T sales for which the sale of an additional marginal unit in the pre-T period would yield exactly λ_T additional corresponding units of pre-T profits (ignoring the influence on the post-T situation). Optimizing his overall profits, the resource owner will choose an amount A_T^* of pre-T sales for which the marginal additional pre-T profit for another sold marginal unit in the pre-T period just equates the marginal foregone profit from post-T sales due to the increase of the pre-T exploitations. With other words, if A_T^* denotes the choosen (optimal) amount of pre-T sales, and λ_T^* the corresponding final multiplier, the following condition is satisfied

$$\lambda_T(A_T^*) \stackrel{!}{=} A_T^{\text{inv}}(A_T^*),$$

where $A_T^{\text{inv}}(\cdot)$ is the inverse function of $A_T(\lambda_T)$.

For the following be $\lambda_T^{\text{pre}}(A_T) \equiv A_T^{\text{inv}}(A_T)$, whose simple interpretation is the marginal pre-T profit from additional pre-T sales given A_T units sold until T. For clarity then, call $\lambda_T^{\text{post}}(A_T) \equiv \lambda_T(A_T)$.

Recall from Proposition 1 that $A_T(\lambda_T)$ is decreasing in λ_T. Thus, for the optimal amount of cumulative exploitations A_T^* the condition

$$\frac{\partial \lambda_T^{\text{pre}}(A_T^*)}{\partial A_T} \le \frac{\partial \lambda_T^{\text{post}}(A_T^*)}{\partial A_T} \tag{7}$$

must hold, as otherwise it would be lucrative for the resource owner to increase A_T^*: the change in overall discounted profits, $\Pi = \Pi^{\text{pre}} + \Pi^{\text{post}}$ can be approximated as

$$\begin{aligned}
\Pi(A_T^* + \varepsilon) - \Pi(A_T^*) &= \Pi^{\text{pre}}(A_T^* + \varepsilon) + \Pi^{\text{post}}(A_T^* + \varepsilon) - \Pi(A_T^*) \\
&\approx \varepsilon \lambda_T^{\text{pre}}(A_T^*) + \frac{\varepsilon^2}{2}\frac{\partial \lambda_T^{\text{pre}}(A_T^*)}{\partial A_T} - \varepsilon \lambda_T^{\text{post}}(A_T^*) - \frac{\varepsilon^2}{2}\frac{\partial \lambda_T^{\text{post}}(A_T^*)}{\partial A_T} \\
&\approx \frac{\varepsilon^2}{2}\left[\frac{\partial \lambda_T^{\text{pre}}(A_T^*)}{\partial A_T} - \frac{\partial \lambda_T^{\text{post}}(A_T^*)}{\partial A_T}\right] \qquad (8)
\end{aligned}$$

for small deviations from A_T^*. Clearly, if Eq. (7) does not hold, Eq. (8) would imply profits that increase for any small value of ε, i.e. A_T^* would not be a profit-maximizing choice. Graphically, this is illustrated in Fig. 1, where the pluses indicate regions in which it would be optimal for the resource owner to increase pre-T sales, and minuses where it would be optimal for him to decrease sales.

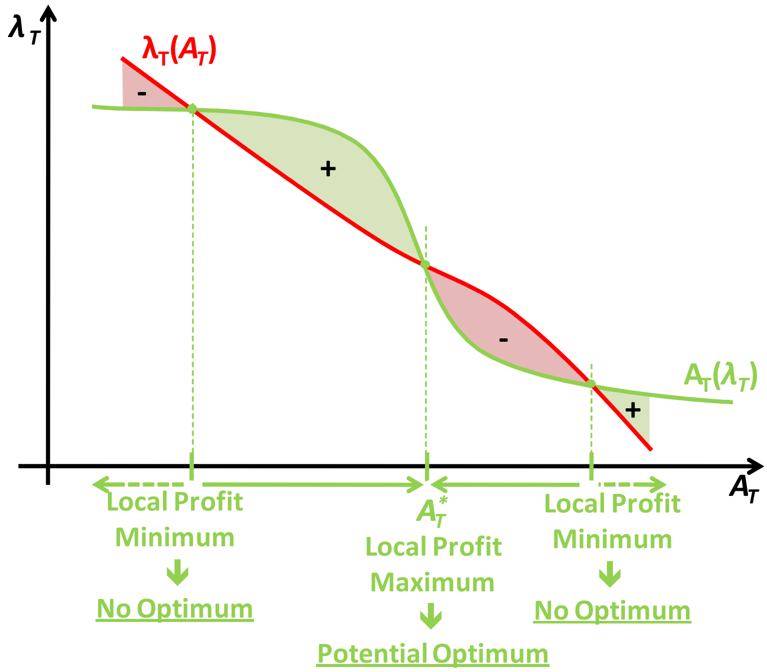

Figure 1: Possible equilibrium situations with flexible $\lambda_T(A_T)$

As a second point, recall, from Proposition 2, that the tax unambiguously reduces pre-T sales for any given fixed λ_T. As the function $A_T(\lambda_T)$ remains the same here as when $\lambda_T^{\text{post}}(A_T)$ was constant, we thus know that in a diagram with A_T on the horizontal axis, $A_{T,\text{tax}}$ must lie strictly on the left of $A_{T,\text{no}}$ in all relevant ranges, as is shown in Fig. 2.

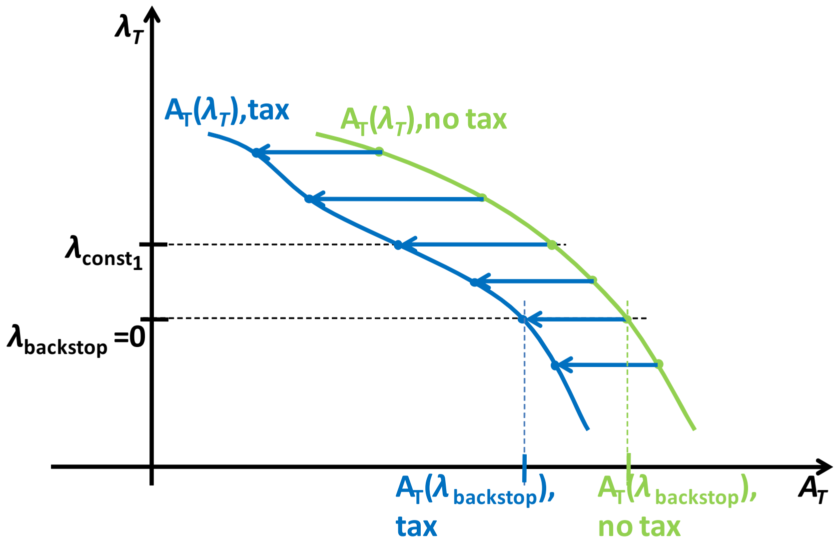

Figure 2: Tax reduces pre-T emissions A_T for constant λ_T

Considering the case where $\lambda_T^{\text{post}\prime}(A_T^*) > 0$, it is straightforward to see that this implies that the tax reduces the optimal amount of pre-T sales A_T^*. This is illustrated in Fig. 3.

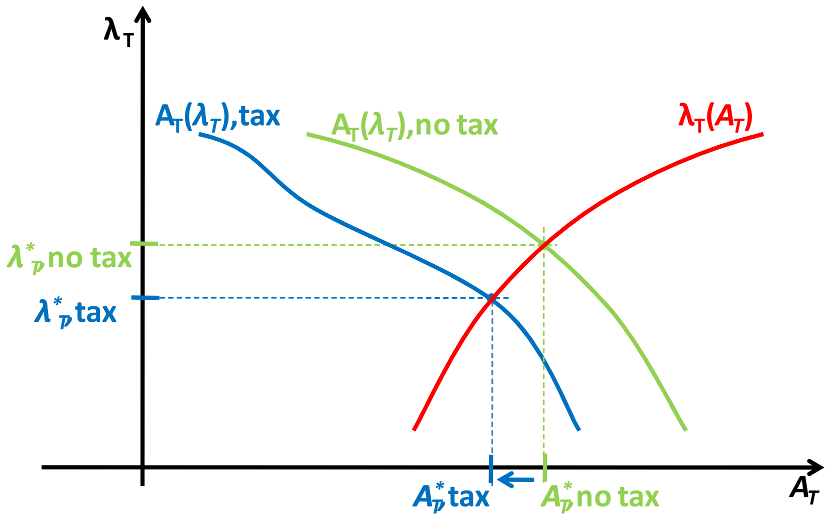

Figure 3: Tax reduces pre-T emissions A_T for flexible $\lambda_T(A_T)$ when $\lambda_T'(A_T) > 0$

As $\lambda_T^{\text{post}\prime}(A_T^*) > 0$, $\lambda_T^{\text{pre}\prime}(A_T) < 0$ and $A_T^{\text{tax}}(\lambda_T) < A_T^{\text{no tax}}(\lambda_T)$, we have $A_{T,\text{tax}}^* < A_{T,\text{no}}^*$. By a similar argument and using Eq. (7) it becomes clear that even if $\lambda_T^{\text{post}\prime}(A_T) < 0$, $A_{T,\text{tax}}^* < A_{T,\text{no}}^*$ holds. This situation is depicted in Fig. 4.

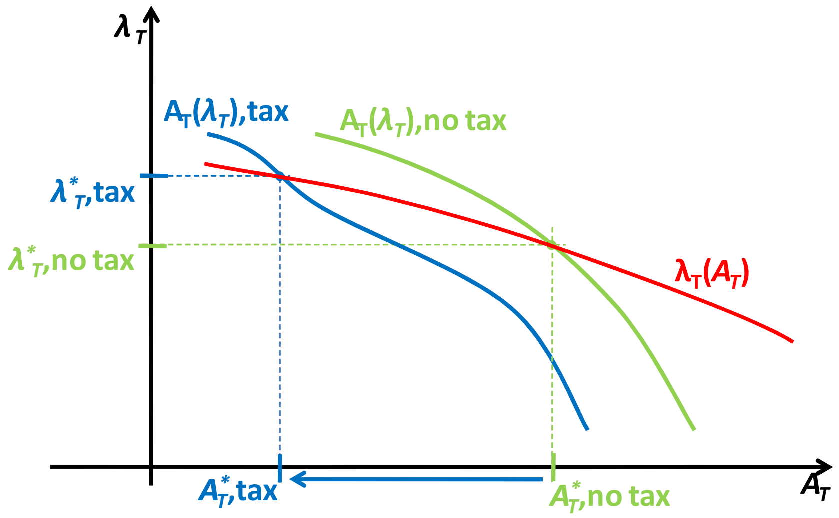

Figure 4: Tax reduces pre-T emissions A_T for flexible $\lambda_T(A_T)$ when $\lambda_T'(A_T) < 0$

Therefore the proposition from the previous section extends to the case of a flexible final multiplier, $\lambda_T^* = \lambda_T(A_T^*)$, which we summarize in Proposition 3.

Proposition 3. *If at a specific future time T an alternative climate measure is introduced that fixes the marginal value to a continuous differentiable function of the cumulative extractions up to T, $\lambda_T = \lambda_T(A_T)$, any scheme of positive CO_2 taxes up to time T leads to a reduction of cumulative emissions up to time T.*

6 Further Extensions

6.1 A Regional Tax

So far, experiences with climate protection discussions suggest that, should in the close future some international carbon tax be introduced, not all countries may be willing to participate in such a treaty. We therefore examine the effect of a bridging tax which remains limited to a part of the world. Analytically, this implies that the world, respectively its demand for fossil fuels, is split in two regions: Region 1 which imposes a tax on its carbon emissions, and Region 2 which will not take any comparative regulatory action in the close future. In our model for this divided world we assume in a first step that the ratio by which the worldwide demand is split is - for a price that is the same in both regions - fixed and constant over time. We explain at the end of the modeling part why the conclusions derived extend to the case where the fractions of the two regions of the world are changing over time. This last point may be of relevance as the parts of the world that have been revealed as the leaders resp. the laggards in the current political climate debate do not exhibit only distinguished climate intensities but also different rates of growth of their respective demand.

Demand structure A demand for fossil fuels split into two fixed regions implies that the demand resulting for a specific price in one region can be expressed as a multiple of the corresponding demand in the other region. Accordingly, we introduce the variable x as the following ratio:

$$r_2(p) = x \cdot r_1(p),$$

i.e. x indicates which multiple of the demand in Region 1, r_1, corresponds to demand in Region 2, r_2.

The worldwide demand is the sum of both regions' demands,

$$r = r_1 + r_2.$$

When a tax is levied in Region 1, the *consumption price* for the resource, p_1, will be the sum of the *consumption price* of Region 2, p_2, and the tax level, τ. The price p_2 corresponds to the *sales price* for the resource owner, p_R:

$$\begin{aligned} p_1 &= p_2 + \tau = p_R + \tau \\ p_2 &= p_R \end{aligned}$$

The demands of the two regions, $r_1(p_1)$ and $r_2(p_2)$ can thus be expressed as $r_1(p_1) = r_1(p_R + \tau)$ and $r_2(p_2) = r_2(p_R)$. Thus, as shown in Fig. 5, the total demand for a given sales price and tax rate is

$$r(p_R, \tau) = r_1(p_1) + r_2(p_2) = r_1(p_R + \tau) + r_2(p_R). \tag{9}$$

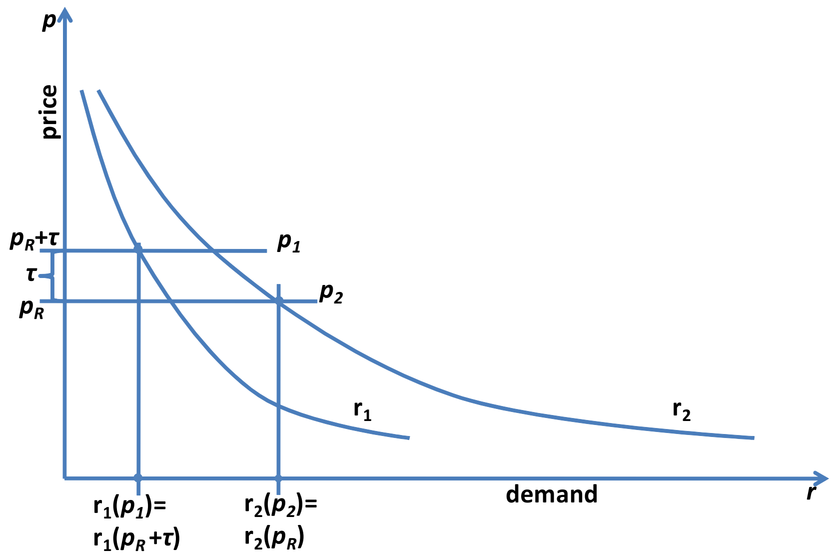

Figure 5: Regional demand with tax in Region 1

Effect of the tax on the demand The demand curves of both regions are assumed to be continuous and strictly decreasing. Eq. (9) implies thus that the current worldwide demand decreases as well in the current sales price p_R as in the current tax rate τ. Therewith the inverse demand curve, here the sales price which yields a specific demand, $p_R(r, \tau)$, is strictly decreasing in r.

Given this new demand structure, the optimality condition Eq. (3) becomes

$$p(r_t, \tau_t) + r_t \frac{\partial p(r_t, \tau_t)}{\partial r} = c(A_t) + \lambda_t. \tag{10}$$

While it eventually seems intuitive, without any further analytical inspection it seems not necessarily clear whether the LHS of the supplier's adapted FOC, Eq. (10), decreases unambiguously in τ_t in the case of the regional tax. It is therefore proven and stated as a general result in Lemma 1 (the lemma is stated below Proposition 4 and its proof given in Part E of the Annex), at least for limited tax levels.

Thus, according to Lemma 1, a regional tax levied on Region 1's consumption at time t reduces worldwide consumption at the same time t for a given multiplier λ_t and extraction costs c_t. Given this result, it is straightforward to see that the proof for Proposition 2 extends to the case of the regional tax - Lemma 1 ensures that Eq. (A.12) holds in the proof.

Thus, also a regional tax leads to a reduction in cumulative emissions up to time T, which is emphasized as Proposition 4. Note that, while we are not aware of any particular reasons for which the statement should not extend to larger taxes as well, the proven validity of our analytical derivations for Lemma 1 and thus Proposition 4 is restricted to certain smoothness conditions for the tax as well to tax rates that are not too large.

The analysis remains valid in the case where the demand-ratio between the regions, x, varies over time: Lemma 1 is not affected at all, and the proof of Proposition 2 allows for time-varying $p_R(r,\tau)$.

We thus emphasize the following result:

Proposition 4. *If an alternative climate measure is introduced at a specific future time T, any scheme with positive carbon taxes covering a (eventually non-constant) fraction of the world's demand up to time T leads to a reduction of cumulative worldwide emissions up to T, at least for limited tax rates.*

Lemma 1. *If an interior solution to the profit maximization problem with the first order condition*

$$p(r_t,\tau_t) + r_t \frac{\partial p(r_t,\tau_t)}{\partial r} = c(A_t) + \lambda_t$$

exists, then a current regional tax at time t levied on Region 1's consumption reduces current worldwide consumption for a given multiplier λ_t and extraction costs c_t, at least for not too large tax rates.

The proof of Lemma 1 is provided in Part E of the Annex.

6.2 Stochastic Introduction of the Future Scheme

It cannot be predicted with certainty which future development in terms of climate change mitigation may once prevent all carbon stored in fossil fuels to be released into the atmosphere. It would be even more unrealistic to pretend knowing when exactly such a breakthrough will occur. Also, the change may come gradually over several years rather than at one specific point in time. What's more, the uncertainty about the time of the future regime change may even be large. Finally, it could also be the case that no real regime change will happen at all. To account for these uncertainties, a stochastic model has to be considered, eventually considerably complicating the analysis.

An analytical investigation of the stochastic case may be possible to a certain extent, especially with a backstop, at the introduction of which the resources left underground at time T will loose all their value. In such a case, the stochastic end time can readily be accounted for by augmenting the discounting rate ρ by an appropriate term ψ_t and otherwise using the deterministic model, as has been shown by Dasgupta and Heal (1974).

For simplicity, we here consider the case where the probability of the introduction of a backstop, conditional on no prior occurrence (further called periodic probability), is constant. The additional discounting factor, ψ, which equals this periodic probability, inherits this constancy, i.e. $\psi_t = \psi$. This implies that the analytical structure of the model does not differ from the deterministic case at all. Note that the underlying (unconditional) probability density for the introduction of the backstop at date t is then $f(t) = \psi e^{-\psi t}$.

The additional discount factor due to the possible introduction of the backstop alters the conclusion about the taxes' impact on the emissions. While in the case where no backstop was considered the Green Paradox would hold up to a certain extent, implying that a tax rising more rapidly than with the real interest rate would lead to larger current emissions, this finding is not valid anymore in the case of the possible backstop: in this case, taxes that exponentially rise at any rate lower than $\rho + \psi$ imply reductions of current emissions and lower cumulative emissions at any future time period. We emphasize this claim with Proposition 5 - the analytical proof is given in Part F of the Annex.

Proposition 5. *Any positive tax exhibiting a rate of increase, θ, that figures between 0 and the sum of the real interest rate, ρ, and the periodic probability of the introduction of a backstop technology, ψ, leads to a reduction of the expectancy of the cumulative emissions and notably does for no period yield increased potential cumulative emissions.*

6.3 Endogenous Future Regime Change

The introduction of a carbon tax changes on one hand the consumption prices for conventional energy and therewith incentives for the development of alternative technologies. On the other hand, it directly affects the carbon emissions and the climate and thus the political pressure to work on additional measures. It is clear that a present tax may therefore influence the likelihood resp. the timing of the implementation of future measures. Assuming the latter to be perfectly exogenous is thus a simplification of reality and it seems important to address the possible endogeneity of the future climate regime. But this is beyond the scope of the present work. It can, however, be foreseen that the direction of the effect on the expected results is ambiguous: lower political pressure due to eventually tax induced emissions reductions could lower the probability of early measures. But technological development boosted by the eventually higher carbon prices could imply earlier development of substitute technologies.

7 Discussion

Arguments questioning the relevance of the Green Paradox have already been raised prior to this paper. Yet, the possibility of an exogenous future regime switch has been neglected so far in the literature on that topic. Taking this additional element into account in the modeling of the effects of a carbon tax renders the predictions more accurate and shows that a carbon tax may be more desirable than previous studies have suggested overall. This is important notably as some other points in favor of the tax raised in literature do not necessarily invalidate all aspects of the Green Paradox. Hoel (2010) argues that any positive tax rate would reduce overall, i.e. long-term, emissions anyway, which can intuitively be understood, given smoothly increasing extraction costs together with a demand-price limited by a finite choke-price: while without any tax the last unit of fuel exploited would be the one for which extraction costs correspond to the choke-price, this price would be reduced by any positive tax. Due to the increasing extraction cost curve this would imply that total extractions would decrease as well. Due to two reasons this insight may not in every case be considered as a decisive argument in favor of a carbon tax: depending on the form of the demand function and the extraction costs curve defining the 'available' resource quantities, if no future regime switch were available, the time when the last unit of the resource would be exploited may theoretically lie far enough in the future that the timing of the emissions could not anymore be considered as of subordinate importance compared to the absolute emissions. Even more importantly, if alternative technologies are not developed well enough, the choke-price of the demand may be large enough for the extraction cost curve to be rather steep at the corresponding point already: given that the total amount of the physically existent fossil reserves is a limited quantity, of which on one hand an important fraction is exploitable at rather low costs but on the other hand the last drops somewhere deep in the ground would be exploitable only at very high costs, it may seem plausible that the cost curve in the region of the choke-price may be rather steep, implying that the change in the overall exploited quantity may vary only to a small extent as a reaction to some limited tax.

Beyond the above analysis, there is an additional reason why the anticipation effects could increase the desirability of a carbon tax rather than reduce it. Without the external climatic effects of the combustion of carbon containing fuels, one may generally depart from the assumption that an eventual carbon tax could be associated with negative economic effects on the taxed region. This negative effect on the economy may increase with the level of the tax, and only the negative climatic externalities may justify an eventual carbon tax: for a fixed net fuel price and in a first approximation the optimal compromise between climate protection and economic activity should be achieved by a tax level that corresponds to the level of the marginal climate costs of an emitted unit of carbon. In this case, the demand for fuels should be reduced to the point from which on an additional reduction would yield economic costs that exceed the additional benefit from increased climate protection[4]. Now, the analysis of the profit maximizing behavior of the resource owners shows that they reduce the net price they demand for their goods if a climate tax is introduced. Therewith, the previously described 'optimal' climate tax would reduce the consumption by less than the climate policy maker may have expected, should he have neglected this behavioral adaptation: the gross price does not increase by the full amount of the tax rate, but only by part of it. In this sense one could at first sight be tempted to consider the tax as inefficient. The reduction of the demanded net sales price from the side of the resource owners could, however, also be utilized to fix the tax rate so much above the 'optimal'[5] tax rate until the gross price exceeds the net price from the no-tax scenario by the value of the 'optimal' tax rate.[6] In this case the originally located demand reduction and the originally mentioned economic costs result: despite the higher than originally described tax rate, the costs for the economy increase only by the originally targeted value. At the same time, however, the taxing region generates higher extra tax revenues, which corresponds to a transfer of parts of the resource rent from the resource owners to the consumer countries. The tax induced behavioral adaptation of the resource owners can thus be used to the advantage of the fuel importing countries and increases the economic attractivity of such a tax. In this sense the profit-expectation-reductions related to the anticipatory effects of the resource owners, and the associated attenuation of the impact of the tax on the sales price and the demand, should not be considered as an efficiency problem of the tax. Rather, they should be understood as a possible means to reduce, in an efficient manner, at the same time the cumulative demand as well as the import costs of the oil.

8 Conclusions

The claim that carbon taxes with rapidly increasing tax rates would exacerbate the climate problem rather than alleviating it, cannot be sustained as generally as it has been suggested with the Green Paradox.

This paper indicates two primary reservations against the claims brought forward with

[4]It is beyond the scope of this paper to address the numerous practical problems of the introduction of such a tax - while very crucial for any project of a carbon tax in general, they seem to be of minor importance to our specific argumentation.

[5]Optimal in the sense of the level that would be desirable if no supply-side adaptation would be made.

[6]In this case, the result could indeed be improved even further when the tax rate is not exactly fixed in this way. Beyond the scope of this article, a discussion of the optimal tax accounting for the strategic consumer-owner interaction on the resource market can be found in Liski and Tahvonen (2004) who examine a first best climate taxation in general, and Dullieux et al. (2010) who examine the optimal tax given a 2° C warming equivalent emission constraint. Both studies find that under certain conditions the optimal tax may contain an import tariff component, i.e. be larger than the pure Pigou tax.

the Green Paradox, both based on the fact that even if we were to abstain from introducing a carbon tax today, other future climate related developments may influence the resource market some time in the future and therewith the carbon emission path. Such possible developments do not only encompass technological innovations driven by rising fossil fuel extraction costs, but also political movements alimented by ever rising emissions and temperature, severely affecting many densely populated regions all over the world. The potential measures include, among others, backstop technologies, demand cartels, carbon capture and storage systems or prohibitively high future carbon taxes. Both our reservations suggest that given the possibility of such future measures, a currently introduced carbon tax may be more favorable for the evolution of our climate than predicted according to the Green Paradox:

First, if some of the mentioned future climate regime switches were to materialize at the specified time in the medium-term, then the cumulative emissions may be more relevant than the detailed evolution of the emission path, and the analytical analysis of the optimal behavior of the resource owners suggests that these cumulative emissions up to the time of the regime switch may be reduced for any tax path with positive tax rates, independently of the rate of increase of the tax level.

Second, if a future regime switch such as the introduction of a backstop technology is stochastic, our model suggests that even the weak version of the Green Paradox does not hold anymore: not only cumulative emissions in future periods, but also current emissions can be reduced by carbon taxes whose levels increase more rapidly than at the real interest rate. More precisely this is the case for any tax whose rate of increase is below the sum of the real interest rate plus the perceived conditional probability of the introduction of the backstop.

In addition to the impact of the taxes on the climate, the anticipation effects can even be beneficial for the consumer countries in the sense that the tax allows these countries to extract part of the suppliers' resource rent, which may increase the carbon tax related welfare gains for the demand countries.

Some caveats regarding the findings presented in the paper are in order. First, in the framework of the stochastic regime switch, our result gives only a clear indication for a tax whose maximal rate of increase is still limited, even, if due to the possibility of the backstop this limit may be substantially higher than the one originally suggested by the Green Paradox. It is clear, however, that, along the line of our argumentation brought forward in the deterministic case, the examination of the stochastic case should not stop here: even a tax that may rise faster than our elevated threshold rate of increase identified in the stochastic analysis, may overall be beneficial: it may slightly rise the initial periods' emissions, but lead to substantial emission reductions later on. In the case where the probability distribution for the occurrence of the regime switch may indicate that the latter is likely to occur in the medium-term, our argumentation for the primary relevance of the *cumulative* emissions should be considered as well: if the tax leads to substantial cuts of future emissions, these reductions may more than compensate for the smaller increases in earlier emissions.

Second, we ignored the potential endogeneity of the future climate scheme change. This is a severe limitation, as it is clear that the eventual carbon tax affects virtually all variables influencing the potential future regime switch, e.g. the temperature path, the consumer price, the general economic development, or the technical progress with alternative energies.

Finally, we explained that especially for the here relevant medium-term future the cumulative emissions may prime in importance over the detailed emission path. This is only a simplified view. Ideally, one would more properly weight increases of current emissions against reductions of cumulative medium or long-term emissions. For this, a more realistic model for total net present damage would be desirable: some limited discounting of future damages, coupled with a non-linear mapping of cumulative emissions (resp. concentrations) to damages would ideally be considered.

An encompassing *analytical* examination of all these issues seems infeasible. In order to address them, it would thus be interesting to explore the case for the Green Paradox about carbon taxes by means of numerical simulations. Even if many of the relevant parameters for such an undertaking - especially the ones about the future climate regime switch - may be subject to large uncertainties, it should allow at least some approximate quantitative assessment of the qualitative claims brought forward by the Green Paradox resp. by our analysis.

Broadening the perspective, we would like to conclude by stressing the implications of this analysis for climate policy evaluation beyond the question of the Green Paradox. While we have shown here how future independent climate-relevant developments may dramatically influence how a carbon tax qualifies regarding the Green Paradox, the potential future climate developments may be crucial for the net impact of any currently debated climate measure. These potential future developments should therefore be taken into account when assessing current measures' desirability and impacts in general, as is hardly being done so far. Predictions about future climate-relevant developments, be they policy measures or technological developments, are intrinsically linked to large uncertainty and complicating reflections. Yet, the uncertainty of predictions is not truly reduced by simply ignoring its sources, rather the latter introduces some potentially large bias which, as shown here, may crucially affect the conclusions about possible policies.

9 Annex

(A) Single-crossing property for monopolist's revenue

In order to rule out some theoretically possible multiple local maxima that would be difficult to deal with analytically, we assume that the demand functions $r(p)$, resp. their inverses $p(r)$, exhibit the property that the marginal revenue of a monopolist's resource sales at a specific period is falling in the current rate of extraction, i.e. that $\frac{\partial[p(r)+p'(r)r]}{\partial r} < 0$, over the full range of considerable extraction rates. This condition guarantees that $P_t(r_t)$ is a strictly decreasing function not only in the competitive but also in the monopolistic case. It notably implies that, should the value of $P_t(r_t)$ decrease, its argument r_t increases, and vice versa. Note that the property represents only an absolutely mild assumption: typically considered demand functions, be they linear, quadratic, isoelastic, or exponential, all meet this assumption in any case. For the case of the world with a monopolist and a tax in a region covering only a fraction of the worldwide demand, stringency of the analytically derived conclusions will require an extension of this assumption: in this case we will assume that for any considered regional tax level τ, the worldwide demand $r(p, \tau)$, which is the sum of the demand $r_1(p+\tau)$ in Region 1 that levies the tax and the demand in the second, non-taxing region, $r_2(p)$, is such that $\frac{\partial[p(r,\tau)+p'(r,\tau)r]}{\partial r} < 0$. This condition is rather likely to hold as well in most cases. It can analytically be shown that it notably holds for all linear, exponential and quadratic demand forms for which the corresponding condition from the worldwide tax case holds - for the quadratic at least for limited tax levels. Exceptions are, however, possible for a limited subset of situations with isoelastic demand in the case of the regional tax.

(B) Proof of Proposition 1

Consider two situations in the same model but with notably differing final multipliers, λ_T. The difference between the two models' variables be called $\Delta\lambda_t$, ΔA_t, Δr_t and Δc_t, respectively. The claim can then be stated as

$$\Delta\lambda_T > 0 \Rightarrow \Delta A_T < 0, \tag{A.1}$$

with the considered time span being $t = [0, T]$. We will show by contradiction that the claim in Eq. (A.1) holds unambiguously.

Assume thus the contrary,

$$\Delta\lambda_T > 0 \wedge \Delta A_T > 0, \tag{A.2}$$

which we will proove to be inconsistent.

All considered variables, A_t, λ_t, r_t and c_t, exhibit continuous time paths.

This implies that $\lim_{t\to T}\lambda_t = \lambda_T$ and $\lim_{t\to T} A_t = A_T$, i.e. $\lim_{t\to T}\Delta\lambda_t = \Delta\lambda_T$ and $\lim_{t\to T}\Delta A_t = \Delta A_T$. Assuming Eq. (A.2) to hold, we thus know that the RHS in Eq. (3) (with $\tau = 0$) will be larger for $t \to T$ in the case of the increased final multiplier, i.e. $\Delta\text{RHS} > 0$. Therefore Property 5 implies that the chosen extraction rates become lower in the region where t is close to T:

$$\lim_{t\to T}\ \Delta r_t < 0$$

Argument 1: If λ_t of both situations coincide, then knowing that either of the remaining variables r_t or A_t coincide as well would imply that the other of the two latter variables must coincide as well (from Eq. (3)), and that thus the whole model paths as well, because the similarity at t of all variables implies a similar evolution of all variables. Demand- and extraction cost-curves are the same in both situations. Thus, it is thus easy to verify the following rule:

$$\Delta A_t = 0 \wedge \Delta\dot{A}_t \neq 0 \Rightarrow \text{sign}\Delta r_t = \text{sign}\Delta\dot{A}_t \wedge \text{sign}\Delta\lambda_t = -\text{sign}\Delta\dot{A}_t$$

Argument 2: $\Delta\lambda_t > 0 \underset{t\epsilon[0,T]}{\forall}$ would imply lower rather than higher cumulative emissions, i.e. violate Eq. (A.2). $\lim_{t\to 0}\Delta\lambda_t > 0$ would imply $\lim_{t\to 0}\Delta r_t < 0$. This would imply decreasing Δc_t for low t, which would tend to alleviate the impact of the positive $\Delta\lambda_t$ on Δr_t. However, the negative value of Δc_t could never fully compensate for the strictly positive value of $\Delta\lambda_t$ in a way that could allow non-negative ΔA_t-values in this subcase: as soon as ΔA_t would approach zero, it would again be the positive $\Delta\lambda_t$ value that would dominate, reducing current Δr_t and therefore prevent ΔA_t to achieve zero or even a positive value at any $t > 0$. This argument extends to any sub-period $[\underline{s}, \bar{s}]$, and analogously to the case of an inverted sign of $\Delta\lambda_t$. Thus we state:

$$\Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t > 0 \underset{\underline{s}\leq t\leq\bar{s}}{\forall} \quad\Rightarrow\quad \Delta A_{\bar{s}} < 0 \underset{\underline{s}\neq\bar{s}}{\forall} \tag{A.3}$$

$$\Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t < 0 \underset{\underline{s}\leq t\leq\bar{s}}{\forall} \quad\Rightarrow\quad \Delta A_{\bar{s}} > 0 \underset{\underline{s}\neq\bar{s}}{\forall} \tag{A.4}$$

Argument 3: Consider the case of Argument 2, adapted in the way that the multiplier difference converges to zero as time approaches $\bar{s}$, i.e. $\lim_{t\to\bar{s}}\Delta\lambda_t = 0$ and $\Delta\lambda_{\bar{s}} = 0$. The implied strict inequalities for $\Delta A_{\bar{s}}$ do not become weak in that case: if we had $\Delta A_{\bar{s}} = 0$ simultaneously with $\Delta\lambda_{\bar{s}} = 0$, there would be no possibility how the extraction rates, multipliers or cumulative extractions could have differed in the pre-$t_{\bar{s}}$ periods. As they did differ, however, the results from Argument 2 extend to

$$\begin{aligned}\Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t > 0 \underset{\underline{s}\leq t<\bar{s}}{\forall} \wedge \Delta\lambda_{\bar{s}} = 0 \quad &\Rightarrow \quad \Delta A_{\bar{s}} < 0 \underset{\underline{s}\neq\bar{s}}{\forall} \\ \Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t < 0 \underset{\underline{s}\leq t<\bar{s}}{\forall} \wedge \Delta\lambda_{\bar{s}} = 0 \quad &\Rightarrow \quad \Delta A_{\bar{s}} > 0 \underset{\underline{s}\neq\bar{s}}{\forall}.\end{aligned}$$

Argument 4: Consider the case of Argument 3, adapted in the way that the multiplier difference becomes zero already at some time s before $\bar{s}$, and remains so up to time $\bar{s}$, i.e. $\Delta\lambda_t = 0 \underset{s\leq t\leq\bar{s}}{\forall}$. During the time where the two λ_t-values coincide, the difference between the cumulative emissions cannot become zero at any time before $\bar{s}$: Argument 3, if applied to the interval $[\underline{s}, s]$, implies that $\Delta A_s \neq 0$. Moreover, whenever $\Delta A_t = 0$ for a specific $s < t \leq \bar{s}$, i.e. for a time when $\Delta\lambda_t = 0$, extractions and therewith the corresponding cost curve evolved along the exact same path in both models within the whole time interval $[s, t]$. (See also Argument 1 for a similar argument.) This would, however, require that $\Delta A_s = 0$, which is impossible because $\Delta A_s \neq 0$. Thus, the results from Argument 2 and 3 extend to

$$\Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t > 0 \underset{\underline{s}\leq t<s}{\forall} \wedge \Delta\lambda_t = 0 \underset{s\leq t\leq\bar{s}}{\forall} \quad \Rightarrow \quad \Delta A_{\bar{s}} < 0 \underset{\underline{s}<s<\bar{s}}{\forall} \tag{A.5}$$

$$\Delta A_{\underline{s}} = 0 \wedge \Delta\lambda_t < 0 \underset{\underline{s}\leq t<s}{\forall} \wedge \Delta\lambda_t = 0 \underset{s\leq t\leq\bar{s}}{\forall} \quad \Rightarrow \quad \Delta A_{\bar{s}} > 0 \underset{\underline{s}<s<\bar{s}}{\forall}, \tag{A.6}$$

where the signs of the inequalities for $\Delta A_{\bar{s}}$ on the RHS do not switch, as in order to do so ΔA_t would have to cross the value 0, which, as just shown, is impossible.

Argument 5: Closely related to what is shown in Argument 1, if $\Delta A_0 = 0$ then $\Delta\lambda_0 \neq 0$; otherwise we would be in the case where from time 0 onwards the two FOCs would necessarily imply that the future evolution is the same in both cases, which notably would not allow for the existence of any difference in the final λ_T-values. Thus we get

$$\Delta\lambda_T \neq 0 \Rightarrow \Delta\lambda_0 \neq 0. \tag{A.7}$$

Eq. (A.2), together with Arguments 2 (Eq. (A.3)), 4 (Eq. (A.5)) and 5 (Eq. (A.7)), imply that there must exist some time t for which the multiplier-difference is strictly negative. As $\lim_{t\to T}\Delta\lambda_t > 0$, this implies that there is a time t^* in the inner of the interval for which $\Delta\lambda_{t^*} = 0$. Moreover, if final cumulative emissions should grow larger as the final multiplier increases, there would necessarily have a time interval $[\underline{t}, \bar{t}]$, $0 \leq \underline{t} < \bar{t} < T$, to exist for which $\Delta A_{\underline{t}} = 0$, $\Delta\lambda_t < 0 \underset{\underline{t}\leq t<\bar{t}}{\forall}$, $\Delta\lambda_{\bar{t}} = 0$ and $\Delta A_t > 0 \underset{t\epsilon(\underline{t},\bar{t}]}{\forall}$. Together with Eq. (5), these relations imply

$$\left.\begin{aligned}\Delta\lambda_{\underline{t}} = \int_{t=\underline{t}}^{\bar{t}} \mathrm{e}^{\rho(\underline{t}-t)}\Delta\dot{c}_t dt < 0 \\ \Delta A_t = \int_{t=\underline{t}}^{t} \Delta\dot{c}_t dt > 0 \underset{t\epsilon(\underline{t},\bar{t}]}{\forall}\end{aligned}\right| ,$$

but this is ruled out by Lemma 2 (Part C of the Annex). Therefore, an increase of the final multiplier λ_T is necessarily associated with a decrease in cumulative emissions up to time T, A_T. ■

(C) Lemma 2

Lemma 2. *For any two continuous and differentiable functions $G(t)$ and $F(t)$ and their finite derivatives $g(t)$ and $f(t)$, and any $T > 0$ and $\rho > 0$,*

$$\left.\begin{array}{r} G(0) = F(0) \\ G(t) > F(t) \underset{t\epsilon(0,T)}{\forall} \end{array}\right\} \Rightarrow \int_0^T e^{-\rho t} \left[g(t) - f(t)\right] dt > 0 \ .$$

Proof. Define $H(t) \equiv G(t) - F(t)$ and $h(t) \equiv g(t) - f(t)$. We thus have $H(0) = 0$ and $H(t) > 0 \underset{t\epsilon(0,T)}{\forall}$. Use further $\eta \equiv \int_0^T \mathrm{e}^{-\rho t} h(t) \mathrm{d}t$. Then, define h^m as the path that minimizes the discounted integral while respecting the imposed condition:

$$\begin{array}{rl} & \min_{h^m} \eta \\ s.t. & \int_0^t h^m(s) \mathrm{d}s > 0 \underset{t\epsilon(0,T)}{\forall} \end{array} \tag{A.8}$$

By the following reasoning h^m cannot contain any periods with negative values:

- If $h^m(t)$ were to contain any negative values without that they were preceded (in terms of lower values of t) by some positive values, the condition in Eq. (A.8) would be violated: the integral over consequentially negative values with at least some of them being strictly negative is necessarily negative.
- If $h^m(t)$ were to contain some strictly negative values that are preceded only by positive values, simultaneously reducing some of the preceding positive values and increasing some of the mentioned negative values, will on one hand leave unaffected the condition (A.8) and on the other hand reduce the value of η, as the reduction of the earlier occurring positive values is discounted less than the increase of the later occurring negative values, leaving a net reduction in η and therefore contradicting that the initial h^m minimized η.

As the path $h^m(t)$ can thus not contain any negative values, and in order for $H(t)$ to take on *strictly* positive values on the integral $(0, T)$, it is clear that η must be positive as it is an integral of weighted positive values with some of them being strictly positive, as well as with strictly positive weights $e^{-\rho t}$. Thus $\int_0^T \mathrm{e}^{-\rho t} \left[g(t) - f(t)\right] dt > 0$. ■

(D) Proof of Proposition 2

Assume an exogenously given, fix λ_T.

From Eq. (5) we know for the monopolistic supplier

$$\lambda_t = \lambda_T e^{\rho(t-T)} + \int_{s=t}^{T} e^{(t-s)\rho}\dot{c}_s ds. \tag{A.9}$$

(It will be intuitive that our analysis holds for the competitive case as well.) Inserting Eq. (A.9) in Eq. (3) yields

$$p_t(r_t) + r_t p_t'(r_t) \quad = \quad c_t + \tau_t + \lambda_T e^{\rho(t-T)} + \int_{s=t}^{T} e^{(t-s)\rho}\dot{c}_s ds. \tag{A.10}$$

In the following we are going to proove by contradiction that the tax necessarily reduces cumulative extractions up to T.

Suppose thus hypothetically that the contrary would be the case, i.e. that

$$A_{T,\text{tax}} > A_{T,\text{no}}, \tag{A.11}$$

where we introduced the indexes *tax* and *no* to designate the variable, here A_T, in the case with the tax resp. in the case without any tax.

Eq. (A.11) implies

$$c_{T,\text{tax}} > c_{T,\text{no}}.$$

We have $\lim_{t\to T} \int_{s=t}^{T} e^{(t-s)\rho}\dot{c}_s \mathrm{d}s = 0$ and, from Eq. (A.11), $\lim_{t\to T} c_{T,\text{tax}} > \lim_{t\to T} c_{T,\text{no}}$. Therefore, the RHS of Eq. (A.10) is strictly larger in the tax case (note that $\lim_{t\to T} \lambda_t = \lambda_T$ in both, the tax as well as the no-tax case), and thus Property 5 (see Section 2) implies

$$\lim_{t\to T} r_{t,\text{tax}} < \lim_{t\to T} r_{t,\text{no}}. \tag{A.12}$$

Because all our variables evolve smoothly over time Eqs. (A.11) and (A.12) imply that there exists a t^* that meets the definition that *the two variants' extraction rates equate each other for the last time* in the pre-T period, i.e. such that

$$r_{t^*,\text{tax}} = r_{t^*,\text{no}}, \tag{A.13}$$

and

$$r_{t,\text{tax}} < r_{t,\text{no}} \ \forall_{t^*<t\leq T}. \tag{A.14}$$

Relation Eq. (A.14) implies that the difference $A_{t,\text{tax}} - A_{t,\text{no}}$ is strictly decreasing during the time between t^* and T, which, considering Eq. (A.11) can only hold if

$$c_{t,\text{tax}} > c_{t,\text{no}} \, \forall_{t^* \leq t \leq T}. \tag{A.15}$$

Eq. (A.13) and Eq. (A.10), as well as the fact that $\tau_t \geq 0$ imply

$$c_{t^*,\text{tax}} + \int_{t=t^*}^{T} \mathrm{e}^{\rho(t^*-t)} \dot{c}_{t,\text{tax}} dt \leq c_{t^*,\text{no}} + \int_{t=t^*}^{T} \mathrm{e}^{\rho(t^*-t)} \dot{c}_{t,\text{no}} dt, \tag{A.16}$$

and thus

$$\int_{t=t^*}^{T} \mathrm{e}^{\rho(t^*-t)} (\dot{c}_{t,\text{no}} - \dot{c}_{t,\text{tax}}) dt \geq c_{t^*,\text{tax}} - c_{t^*,\text{no}}. \tag{A.17}$$

As according to Eq. (A.15) the RHS of Eq. (A.17) is strictly positive, it is easy to see that Lemma 2 (Part C of the Annex) implies that Eqs. (A.15) and (A.17) cannot be reconciled, which concludes our proof by contradiction. ■

(E) Proof of Lemma 1

First, note that from Property 5 (see Section 2) we know that, for a fixed tax, an increase in the value of the RHS of the first order condition yields a decrease of the momentary extraction rate.

Suppose that the value of the LHS expression in the FOC decreases when r_t is fixed and the tax τ_t is increased from zero to a positive value. Consider further a no-tax case, where the RHS has an initial value, called RHS_0, yielding an initial extraction rate $r_{t,0}$ at which the RHS and the LHS of the FOC are equalized. As we suppose, adding a tax τ_t decreases the value on the LHS of the FOC when $r_{t,0}$ is hypothetically held constant in a first step. We thus would need to have a lower hypothetical RHS-value, RHS_1 in order for the FOC to be equalized in the new situation with the tax. Now, the RHS-value is however given and will not really be reduced to RHS_1 but remain at RHS_0. In order to see what this implies for the instantaneous extraction rate, we then consider in a second step a hypothetical re-increase of the RHS-value from RHS_1 to RHS_0. Along with this hypothetical re-increase of the RHS, we, however, will have to decrease the instantaneous extraction rate in order for both sides of the FOC to still be equalized. This shows that *if* adding an instantaneous tax τ_t decreases the LHS-value of the FOC, then the extraction rate at that time will have to decrease, given that the value on the RHS remains unchanged. We are now proceeding to show that the tax τ_t will indeed decrease the LHS-value at time t, which therefore implies that it will decrease the extraction rate r_t. This will conclude our proof. Note that showing this property is not as obvious as it may seem at first sight, as adding a tax in region 1 and leaving *worldwide* demand unchanged, does not simply mean to decrease a demand, but to eventually decrease demand in Region 1 and simultaneously increase the demand in Region 2.

While $p_R(r_t, \tau_t)$ unambiguously decreases with an increasing tax for a given r_t, this cannot be claimed to necessarily be the case for the second term of the LHS of the corresponding FOC, $\frac{\partial p_R(r_t,\tau_t)}{\partial r}$, without any further assumptions about the demand function. Here, we show that the reduction of p_R induced by a tax, i.e. - $[p_R(r_t, \tau_t) - p_R(r_t, 0)]$, unambiguously dominates the potential increase of the second term, i.e. $\left[r_t \frac{\partial p_R(r_t,\tau_t)}{\partial r} - r_t \frac{\partial p_R(r_t,0)}{\partial r}\right]$, at least for not too large tax levels and demand curves with finite derivatives, wherewith the direct effect of the tax at time t unambiguously reduces the extraction rate in the current period, r_t.

Be $r(p)$, the worldwide demand curve for the resource, a continuous, strictly decreasing function with a third derivative that is finite for any $p > 0$. The worldwide demand is split into the regional demands r_1 and r_2, such that for a worldwide equal price, demand in Region 2 corresponds to x times the demand in Region 1:

$$\begin{aligned} r_1 + r_2 &= r \\ r_2(p) &= x \cdot r_1(p) \end{aligned} \tag{A.18}$$

Eq. (A.18) implies that all derivatives of the regional demand function differ by a factor x as well:

$$r_2^{(i)}(p) = x \cdot r_1^{(i)}(p), \tag{A.19}$$

where the indice $(\cdot)^{(i)}$ denotes the i^{th} derivative.

When Region 1 introduces a tax, the *consumer price* for the resource in that region, p_1, exceeds the *consumer price* in the tax free Region 2, p_2, as well as the *sales price* for the resource owners, p_R, by the tax rate τ:

$$p_1 = p_2 + \tau = p_R + \tau$$

The aggregate demand for a given sales price and a specific tax rate is

$$r(p_R, \tau) = r_1(p_1) + r_2(p_2) = r_1(p_R + \tau) + r_2(p_R). \tag{A.20}$$

As the demand curves in the two regions are continuous and strictly decreasing, Eq. (A.20) directly implies that the worldwide demand is strictly decreasing as well in p_R as in τ. It is therefore clear that the inverse demand curve, here the sales price which for a given tax yields a specific aggregate demand, $p_R(r, \tau)$, is strictly decreasing in r.

In the following, we will use the syntax Δvar in order to express the discrete change of the value of the variable var resulting from the introduction of the tax:

$$\Delta \text{var} \equiv \text{var}_{\text{tax}} - \text{var}_{\text{no tax}}$$

Consider the hypothetical case where a consumer tax is introduced in Region 1 and the sales price demanded by the resource owners is adapted accordingly in a way that overall the introduction of the tax does imply an unchanged global consumption. In this case, demand in Region 1 would have to decline by exactly the same amount as the demand in Region 2 would increase, and the corresponding changes in the regions' sales, denoted Δr, would have to exactly have the size that implies that the price difference between the two regions amounts to the level of the tax,

$$\Delta p_1 + \Delta p_2 = \tau. \tag{A.21}$$

Consider the illustration in Fig. A.1.

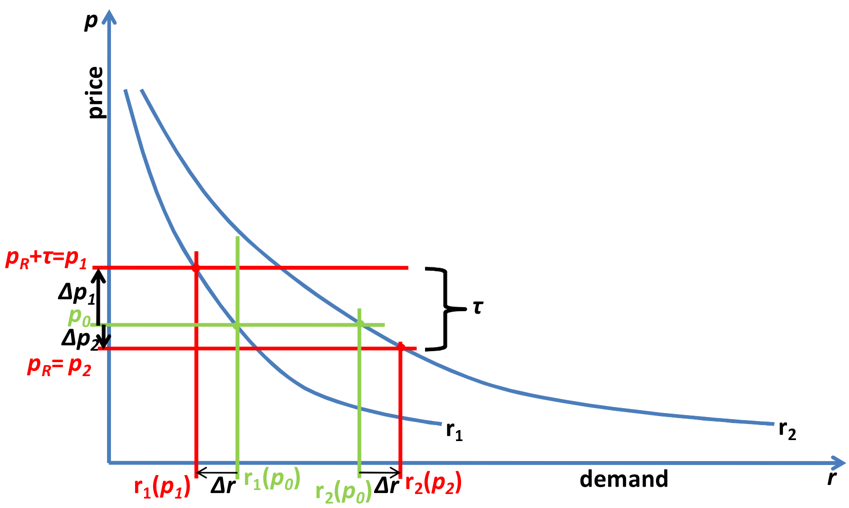

Figure A.1: Hypothetical situation of regional tax which is neutral for global emissions

In a first approximation we have:

$$\Delta r \approx \Delta p_1 \cdot r_1'(p_0) \tag{A.22}$$

$$\Delta r \approx \Delta p_2 \cdot r_2'(p_0) \tag{A.23}$$

With the inclusion of Eq. (A.19), using Eqs. (A.22) and (A.23) in Eq. (A.21) implies

$$\Delta p_2 \approx \frac{\tau}{1+x}, \tag{A.24}$$

wherewith

$$p_b \approx p_0 - \frac{\tau}{1+x}. \tag{A.25}$$

Eq. (A.25) expresses that, in order to keep aggregate demand constant, the sales price for the resource owner must decease by a value that is approximately proportional to the tax rate.

From Eqs. (A.24) and (A.21) follows

$$\Delta p_1 \approx x \cdot \Delta p_2. \tag{A.26}$$

In order to be able to make a statement about the corresponding change of the global demand, $\frac{\partial p_R(r,\tau)}{\partial r}$, we again develop two Taylor approximations:

$$\begin{aligned} r_1'(p_1) &\approx r_1'(p_0) + \Delta p_1 \cdot r_1''(p_0) + \frac{(\Delta p_1)^2}{2} r_1'''(p_0) && \text{(A.27)} \\ r_2'(p_2) &\approx r_2'(p_0) - \Delta p_2 \cdot r_2''(p_0) + \frac{(\Delta p_2)^2}{2} r_2'''(p_0) && \text{(A.28)} \\ &\approx x \cdot r_1'(p_0) - \Delta p_1 \cdot r_1''(p_0) + \frac{(\Delta p_1)^2}{2x} r_1'''(p_0), && \text{(A.29)} \end{aligned}$$

where the minus sign for the second term on the right hand side in Eq. (A.28) is due to the fact that Δp_2 is defined in absolute terms, and where Eq. (A.29) follows from Eq. (A.28) using Eq. (A.19) as well as Eq. (A.26).

As $\frac{\partial r}{\partial p} = \frac{\partial r_1}{\partial p} + \frac{\partial r_2}{\partial p}$, relying on continuity of all relevant functions we know that $\frac{\partial p}{\partial r} = \left[\frac{\partial r_1}{\partial p} + \frac{\partial r_2}{\partial p}\right]^{-1}$ and we can therefore write

$$\Delta\left[\frac{\partial p_R}{\partial r}\right] = \frac{1}{r_1'(p_R+\tau) + r_2'(p_R)} - \frac{1}{r_1'(p_0) + r_2'(p_0)}.$$

By using Eqs. (A.27) and (A.29), as well as Eq. (A.19), we can thus approximate this response of the first derivative of the selling-price, $\frac{\partial p_R}{\partial r}$, to the introduction of the tax as

$$\Delta\left[\frac{\partial p_R}{\partial r}\right] \approx \frac{1}{(1+x)r_1'(p_0) + (\Delta p_1)^2 \left(1+\frac{1}{x}\right) r_1'''(p_0)/2} - \frac{1}{(1+x)r_1'(p_0)}.$$

For relatively small $(\Delta p_1)^2$ this approximates to

$$\Delta\left[\frac{\partial p_R}{\partial r}\right] \approx \text{-}\,(\Delta p_1)^2 \frac{r_1'''(p_0)}{2x(1+x)r_1'(p_0)^2},$$

which is proportional to the square of the tax induced price change.

The response of the *seller price which leaves the global demand unchanged* to the introduction of the tax, $\Delta p = p_R - p_0$, can be approximated using Eq. (A.25):

$$\Delta p \approx -\frac{\tau}{1+x}$$

Using Eqs. (A.24) and (A.26) we therefore have the following ratio between the direct effect of the tax on the *seller price which leaves global demand unchanged* and the corresponding change of the price's derivative with respect to r:

$$\frac{\Delta\left[\frac{\partial p_R}{\partial r}\right]}{\Delta p} \approx \frac{\tau \cdot x \cdot r_1'''(p_0)}{2\left(1+x\right)^2 r_1'(p_0)^2}, \tag{A.30}$$

whose sign depends on the not specified sign of $r'''(p_0)$.

As the ratio in Eq. (A.30) is proportional to the tax rate, and the factor by which this tax rate is multiplied cannot be infinite due to the boundedness of our derivatives of the demand function, we thus know that Δp is larger in absolute terms than any finite multiple of $\Delta\left[\frac{\partial p_R}{\partial r}\right]$ for taxes that are not too large, which proves our claim. ■

(F) Proof of Proposition 5

Having a constant periodic probability (ψ) that a backstop technology may arise, we know from Dasgupta and Heal (1974) that the resource owners' maximization problem differs from the deterministic case without backstop solely by a corresponding increase of the discount factor. The first order conditions can thus be written as

$$\begin{aligned} P_t(r_t) &= c(A_t) + \tau e^{\theta t} + \lambda_t \\ \dot{\lambda}_t &= \lambda_t(\rho+\psi) - \dot{c}_t. \end{aligned} \tag{A.31}$$

Defining $\delta \equiv (\rho+\psi)$, we get

$$\lambda_t \quad = \quad \lambda_T e^{\delta(t-T)} + \int_{s=t}^{T} e^{\delta(t-s)} \dot{c}_s ds. \tag{A.32}$$

We are considering an exponentially increasing tax, $\tau_t = \tau_0 e^{\theta t}$, where θ may exceed ρ, as long as $\theta < \delta$.

We use the same syntax as in the proof for Lemma 1: $\Delta\text{var} \equiv \text{var}_{\text{tax}} - \text{var}_{\text{no tax}}$, where var can be a single variable or a combined mathematical term.

Note that, as the no-tax case corresponds to simply setting $\tau_{t,\text{no tax}} = 0 \underset{t\geq 0}{\forall}$ and in the tax case we have $\tau_{t,\text{tax}} > 0 \underset{t\geq 0}{\forall}$, we know that $\Delta\tau_t > 0 \underset{t\geq 0}{\forall}$.

In the next step we are going to show by contradiction that the described tax path cannot lead to increased cumulative emissions for any point in time:

Assume thus, hypothetically, that the contrary holds, i.e. $\Delta A_t > 0$ for some t.

We treat two possible subcases separately:

-*Subcase 1:* Suppose, $\exists t_0$ s.t.

$$\Delta A_{t_0} = 0 \tag{A.33}$$

$$\text{and} \qquad \Delta A_t > 0 \underset{t_0<t<\infty}{\forall}. \tag{A.34}$$

This requires $\Delta r_{t_0} \geq 0$, and therefore, due to Eq. (A.31) and Property 5, that $\Delta[\lambda_{t_0} + \tau_{t_0}] \leq 0$, wherewith we have

$$\Delta\lambda_{t_0} < 0. \tag{A.35}$$

However, from Eq. (A.32) (and the transversality condition), we know that $\lambda_{t_0} = \int_{t=t_0}^{\infty} \mathrm{e}^{\delta(t_0-t)}\dot{c}_t dt$, which can be rewritten as

$$\lambda_{t_0} = \mathrm{e}^{\delta t_0} \int_{t=t_0}^{\infty} \mathrm{e}^{-\delta t}\dot{c}_t dt. \tag{A.36}$$

It is, however, straightforward to see that Eqs. (A.33) through (A.36) are not reconcilable with Lemma 2 (Part C of the Annex). Therewith it is shown by contradiction that subcase 1 is impossible. ■*Subcase* 1.

-*Subcase 2:* Suppose $\exists t_1$, t_2, $t_1 < t_2$, s.t.

$$\Delta A_{t_1} = 0 \wedge \Delta r_{t_1} \geq 0, \tag{A.37}$$

$$\Delta A_{t_2} = 0 \wedge \Delta r_{t_2} \leq 0, \tag{A.38}$$

$$\text{and} \qquad \Delta A_t \geq 0 \underset{t_1<t<t_2}{\forall}. \tag{A.39}$$

Eqs. (A.37) and (A.38) imply

$$\Delta c_{t_1} = 0 \wedge \Delta c_{t_2} = 0, \tag{A.40}$$

and therewith also

$$\Delta[\lambda_{t_1} + \tau_{t_1}] \leq 0, \tag{A.41}$$

$$\text{and} \qquad \Delta[\lambda_{t_2} + \tau_{t_2}] \geq 0.$$

Eq. (A.39) indicates that

$$\Delta c_t > 0 \underset{t_1<t<t_2}{\forall}. \tag{A.42}$$

From Eq. (A.32) we know

$$\lambda_{t_1} = \lambda_{t_2} \mathrm{e}^{\delta(t_1-t_2)} + \int_{t_1}^{t_2} \mathrm{e}^{\delta(t_1-t)} \dot{c}_t dt.$$

Defining $\mu_t \equiv \lambda_t \mathrm{e}^{-\theta(t-t_1)}$, which yields $\mu_{t_1} = \lambda_{t_1}$ and $\lambda_{t_2} = \mu_{t_2} \mathrm{e}^{\theta(t_2-t_1)}$, we can write

$$\mu_0 = \lambda_0 = \mu_{t_2} \mathrm{e}^{[\delta-\theta](t_1-t_2)} + \int_{t_1}^{t_2} \mathrm{e}^{\delta(t_1-t)} \dot{c}_t dt.$$

Consider

$$\Delta[\lambda_{t_2} + \tau_{t_2}] \geq 0 \Rightarrow \mathrm{e}^{-\theta(t_2-t_1)} \Delta[\lambda_{t_2} + \tau_{t_2}] \geq 0 \Rightarrow \Delta[\mu_{t_2} + \tau_{t_1}] \geq 0. \tag{A.43}$$

As $\Delta\tau_{t_1} > 0$ the last expression in Eq. (A.43) implies

$$\Delta[a\mu_{t_2} + \tau_{t_1}] > 0 \underset{0 \leq a < 1}{\forall}. \tag{A.44}$$

From Eq. (A.41) we know $\Delta[\mu_{t_1} + \tau_{t_1}] \leq 0$, which we can rewrite as

$$\Delta[\mu_{t_2} \underbrace{\mathrm{e}^{[\delta-\theta](t_1-t_2)}}_{<1} + \int_{t_1}^{t_2} \mathrm{e}^{\delta(t_1-t)} \dot{c}_t dt + \tau_{t_1}] \leq 0. \tag{A.45}$$

Eqs. (A.44) and (A.45) imply

$$\Delta[\int_{t_1}^{t_2} \mathrm{e}^{\delta(t_1-t)} \dot{c}_t dt] \leq 0. \tag{A.46}$$

However, Eqs. (A.46), (A.40) and (A.42) violate Lemma 2 (Part C of the Annex), a contradiction. ■*Subcase* 2.

If the tax were to increase cumulative emissions for some period, either subcase 1 or subcase 2 would have to hold: we have $A_0 = 0$ in any case, and for any t^* where $\Delta A_{t^*} > 0$ there must exist a latest preceding period, $\underline{t}$, $\underline{t} < t^*$, for which the tax does not impact the cumulative emissions, $\Delta A_{\underline{t}} = 0$ ($\underline{t}$ may be 0). Then, there exist two possibilities: either the tax will increase cumulative emissions for all periods after time $\underline{t}$ - this is subcase 1 -, or there exists some future period for which the cumulative emissions are not affected by the tax - this is subcase 2. Therefore, the shown inconsistency of both subcases prooves that the considered taxes cannot increase the cumulative emissions, A_t, for any period t.

In addition, it is impossible that the tax does not change any periods' emissions: if this were the case, then λ_t would be unchanged as well, but in this case the tax τ_t would affect the extraction rate r_t in Eq. (A.31). Thus, the considered tax necessarily reduces the emissions, at least in some periods.

We conclude that the considered tax (i) does not increase any period's cumulative emissions, (ii) reduces cumulative emissions at least for some periods, and (iii) thus unambiguously reduces the expectancy of the cumulative emissions, QED. ■

References

P. Dasgupta and G. Heal (1974), The Optimal Depletion of Exhaustible Resources, *Review of Economic Studies* 41, pp. 3 – 28.

R. Dullieux, L. Ragot and K. Schubert (2010), Carbon Tax and OPEC's Rents under a Ceiling Constraint, mimeo, Université Paris 1 et al., June.

R. Gerlagh (2011), Too Much Oil, *CESifo Economic Studies* 57, pp. 79 – 102.

O.C. Herfindahl (1967), Depletion and Economic Theory, in M.H. Gaffney (ed.), *Extractive Resources and Taxation*, pp. 63 – 69.

M. Hoel (2010), Is There A Green Paradox?, *CESifo Working Paper* No. 3168, Munich, September.

D. Pearce (1991), The Role of Carbon Taxes in Adjusting to Global Warming, *Economic Journal* 101, pp. 938 – 948.

H.-W. Sinn (2008), Public Policies Against Global Warming: A Supply Side Approach, *International Tax and Public Finance* 15, pp. 360 – 394.

M. Liski and O. Tahvonen (2004), Can Carbon Tax Eat OPEC's Rents?, *Journal of Environmental Economics and Management* 47, pp. 1 – 12.

S. Polborn (2011), The Green Paradox and Increasing World Energy Demand, mimeo, University of Aarhus, January.

S. Solomon, G.K. Plattner, R. Knutti and P. Friedlingstein (2009), Irreversible Climate Change due to Carbon Dioxide Emissions, *Proceedings of the National Academy of Sciences* 106, pp. 1704–1709.

R. Van der Ploeg and C. Withagen (2010), Is There Really A Green Paradox?, *CESifo Working Paper* No. 2963, Munich, February.

R. Van der Ploeg and C. Withagen (2011), Optimal Carbon Tax with a Dirty Backstop: Oil, Coal, or Renewables?, *CESifo Working Paper* No. 3334, Munich, January.