Uncertainties in the Uranium and Enrichment Markets: a Stochastic Approach

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Overview

• **Uranium (U) mining and enrichment** are steps in the **front end** of the nuclear fuel cycle (NFC)

• Competitive markets exist for these services

• They provide enriched U fuel to **nuclear reactors**, whose demand for fuel is **nearly inelastic** in the short run
Overview

• We have devised a **market-clearing model** that evaluates the evolution of these markets through 2030:
  • **primary uranium** from mines,
  • **secondary uranium** from e.g. depleted uranium (DU) upgrading or highly enriched uranium (HEU) down blending
  • **conversion** to uranium hexafluoride (UF₆),
  • **enrichment** to 4-5% ²³⁵U, as required by most reactors.

• The model is **not** used to predict prices, but rather to **evaluate policy measures affecting the front end markets**, while taking a stochastic approach to account for uncertainties
Outline

- **Methodology**
  - Underlying databases
  - Depiction of supply and demand & assumptions
  - Coupling between markets
- **Reference case and one sensitivity study**
  - Constraints on primary U supply
- **Monte Carlo implementation**
- **Other applications**
Supply databases

- **Primary uranium (CSM):**
  - contains more than 350 uranium properties at various stages of development, representing reserves and resources of over 7 million tonnes U (tU).

- **Secondary uranium (UT-Austin):**
  - includes inventories of excess HEU, DU, other government- and utility-held U stocks, and excess weapons grade plutonium (WGPu)

- **Enrichment (UT-Austin)**
  - Existing capacity as well as announced and planned expansions through 2018; growth assumed thereafter
Market clearing model

- Constructs **annual** supply curves from the **primary U**, **secondary U** and **enrichment plant** databases assembled by CSM and UT-Austin

![Uranium supply and demand cartoon](image)

- **Demand**
  - Tails Assay = 0.1%
  - Tails Assay = 0.2%
  - Tails Assay = 0.3%
  - Tails Assay = 0.4%

- **Supply**
  - Price [$/kg U]
  - Quantity [kg U]

- **Price** $/kg U
- **Quantity** [kg U]

Intersection determines quantity of uranium produced and market-clearing price

‘Tails’ = leftover depleted uranium from enrichment
Interplay between markets

- It would take many years for a reactor (at large cost) to modify its demand for fuel
- But utilities can substitute uranium for enrichment services
  - so the uranium and enrichment demand curves exhibit considerable short run price elasticity

The substitution is achieved by adjusting the amount of U-235 left in the depleted uranium “tails”.

Enrichment services are measured in separative work units, SWU.
Market clearing model

- Simulation marches forward through time
  - Uranium and enrichment market clearing states are solved at each time step in coupled calculations
  - The model responds to shifts in supply and demand, for instance following closure of a major mine:

```
Price [$/kg U]
```

```
Supply
```

```
Demand
```

```
Tails Assay = 0.1%
```

```
0.2%
```

```
0.3%
```

```
Tails Assay = 0.4%
```

```
... if a mine closes, shifting supply...
```

```
... utilities consume less U (and more SWU)
```

Constrained optimization problem solved each time step

- **Demand curves** are calculated as the **locus of points** that minimize **cost**, $C \ [\$]$, to the consumer:

$$C = (P_U + P_C) \cdot U_d + P_{SWU} \cdot SWU_d$$

$P_U, P_C, P_{SWU} = \text{market prices of U, conversion, enrichment } [\$/kg \text{ or } \$/SWU]$  

$U_d, SWU_d, NU_d = \text{amount } [\text{kg or SWU}] \text{ of U or SWU}$  

**Each term is a function of the DU ‘tails’ U-235 enrichment, } x_w$ (and the reactor fuel and natural uranium enrichments $x_p$ and $x_f$).

- **Constraints:**
  - Demand for reactor fuel (fixed in the short run) must be satisfied
  - $P_U$ treated as free, $P_{SWU} = f(SWU_d)$ obtained from clearing enrichment market at each trial $x_w$
Choose an $x_w$, set $dC/dx_w = 0$, solve for $P_U$:

$$0 = (P_U + P_C) \frac{\partial}{\partial x_w} \left[ \frac{x_p - x_w}{x_f - x_w} \right] + P_{SWU} \left[ \frac{\partial}{\partial x_w} \left( \frac{x_p - x_w}{x_f - x_w} \right) \left( V(x_w) - V(x_f) \right) + \left( \frac{x_p - x_w}{x_f - x_w} - 1 \right) \frac{\partial V(x_w)}{\partial x_w} \right]$$

where

$$\frac{\partial}{\partial x_w} \left[ \frac{x_p - x_w}{x_f - x_w} \right] = \frac{1}{x_f - x_w} \left( \frac{x_p - x_w}{x_f - x_w} - 1 \right), \quad \frac{\partial V(x_w)}{\partial x_w} = -2 \ln \left( \frac{1 - x_w}{x_w} \right) - \left( \frac{1 - 2x_w}{1 - x_w} \right) \left( 1 + \frac{1 - x_w}{x_w} \right)$$

Use constraint to solve for $U_d(x_w)$, put one point on $U$ demand curve

Solve for $P_U$ at which this $x_w$ minimizes $C$

Repeat for all feasible $x_w$ to derive $U$ demand curve
Supply and demand curves

• Supply steps are color-coded according to deposit type
  • Supply curve changes with each time step as deposits are mined out and other deposits become available
  • NU demand curve can shift over time due to exogenous fuel demand growth rate and change shape as enrichment cost structure evolves
Reference Case

- Demand growth rate derived from average of OECD Redbook ‘low demand’ and ‘high demand’ scenarios
- Initial demand reflects world values in 2010

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>World LEU reactor fuel demand in 2010</td>
<td>6,790 tonnes / yr</td>
</tr>
<tr>
<td>World U-as-fuel demand in 2010*</td>
<td>3,390 tonnes / yr</td>
</tr>
<tr>
<td>Average U-235 content of LEU fuel</td>
<td>4.3 w/o</td>
</tr>
<tr>
<td>World LEU &amp; NU fuel demand growth rate</td>
<td>2.6 % / year</td>
</tr>
<tr>
<td>Year enrichment facility capacity growth commences</td>
<td>2018</td>
</tr>
<tr>
<td>Enrichment facility capacity growth rate</td>
<td>2.6% / year</td>
</tr>
<tr>
<td>Cost of yellowcake-to-fluoride conversion</td>
<td>$10 / kg U</td>
</tr>
<tr>
<td>WGPu fraction in MOX</td>
<td>5 w/o of IHM</td>
</tr>
<tr>
<td>U-235 content of HEU downblend stock</td>
<td>0.25 w/o</td>
</tr>
</tbody>
</table>

* Certain reactors, e.g. CANDU, consume fuel fabricated from unenriched natural uranium.
World Nuclear Association (WNA) case, similar demand growth rate, provided for comparison: WNA doesn’t clear markets, only add up potential supply sources (figure source: WNA)
Each supplier assumed to expand at 2.6% p.a. after 2018.

WNA does not assume supply growth except for Russian sources, seeks to identify shortfalls.

(figure source: WNA)
The reference case predicts

- Uranium and SWU prices to remain high (in historical terms) through ~2015;
- Both prices to decline, thereafter in the case of SWU to below-historical levels, and remain low through 2020-25;
- Sympathetic increases in both U and SWU prices beginning in the early 2020s.
  - these increases may be spurious as we do not yet take undiscovered deposits or reserve growth into account.
Constrained primary supply scenario

- ‘Worst case’ primary supply scenario where:
  - Olympic Dam mine expansion doesn’t take place;
  - Queensland adopts U mining ban;
  - A worldwide U supply slowdown develops (mine capacity factors decreased).

Figures. U production by country, base scenario (left) and constrained supply (right)
Effects of constrained primary supply scenario

- U price rises to $260/kg U by 2030
- Substitution of SWU and secondary U for primary U is seen, but these ‘safety valves’ have only a limited mitigating effect:

<table>
<thead>
<tr>
<th>Item</th>
<th>Reference Case</th>
<th>Constrained Primary Supply (CPS)</th>
<th>Difference, (CPS – Reference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary U extracted [tU]</td>
<td>1,464,700</td>
<td>1,445,700</td>
<td>-19,000</td>
</tr>
<tr>
<td>NUE of reenriched DU [tU]</td>
<td>125,500</td>
<td>129,600</td>
<td>+4,100</td>
</tr>
<tr>
<td>NUE of HEU and WGPU [tU]</td>
<td>71,400</td>
<td>70,600</td>
<td>-800</td>
</tr>
<tr>
<td>SWU consumed [kSWU]</td>
<td>1,332,100</td>
<td>1,348,300</td>
<td>+16,200</td>
</tr>
<tr>
<td>NU/NUE purchase expend. [M$]*</td>
<td>161,100</td>
<td>184,900</td>
<td>+23,800</td>
</tr>
<tr>
<td>SWU purchase expenditures [M$]*</td>
<td>141,500</td>
<td>149,300</td>
<td>+7,800</td>
</tr>
<tr>
<td>Total Expenditures [M$]**</td>
<td>317,200</td>
<td>348,700</td>
<td>+31,500</td>
</tr>
</tbody>
</table>

*Undiscounted.
** Includes oxide-to-fluoride conversion for primary U at $10/kg U.
Monte Carlo implementation

- **Objective**: incorporate and propagate uncertainties in supply factors & reactor fuel demand
- **Method**: sample inputs from distributions, run independent realizations, aggregate

**Table. Supply and demand uncertainties for sample case**

<table>
<thead>
<tr>
<th>Description</th>
<th>Mean Value</th>
<th>Distribution &amp; range or standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactor fuel demand growth rate</td>
<td>2.6 %/yr</td>
<td>Uniform, 1.7% - 3.5%</td>
</tr>
<tr>
<td>Enrichment capacity growth rate</td>
<td>2.6 %/yr</td>
<td>Uniform, 1.7% - 3.5%</td>
</tr>
<tr>
<td>Uranium Production Cost Multiplier</td>
<td>1.0</td>
<td>Normal, $\sigma = 0.3$</td>
</tr>
<tr>
<td>SWU Production Cost Multiplier</td>
<td>1.0</td>
<td>Normal, $\sigma = 0.3$</td>
</tr>
<tr>
<td>Mine Capacity Factor Multiplier</td>
<td>1.0</td>
<td>Normal, $\sigma = 0.3$</td>
</tr>
</tbody>
</table>
Results: U price

Median shown in black, 90% confidence interval in red
Results: SWU consumption (top), Market clearing SWU price (bottom)

Bottleneck in 2015:
supply constraints amidst transition from diffusion to centrifuge technology
Conclusions: limitations, applications

**Limitations:**
- Assumes a single, frictionless world market for uranium and enrichment services at equilibrium
- Uses production rather than operating costs to dispatch supply
- Used as policy tool, not as a predictor of uranium prices

**Policy relevance:**
- Evaluate market impact of inventory sales strategies
- Assess buying into / disbursing from a uranium bank
- **Support R&D decision making** by identifying surplus associated with advanced technologies, e.g. SILEX enrichment