Optimal Dynamic Carbon Taxation in the Presence of Business Cycles and Market Frictions

By Travis Roach

Though carbon dioxide emissions have implications for both national and international policy, they are by and large the result of decisions made at a more micro level. This paper begins from this realization and models the United States economy in a dynamic stochastic general equilibrium framework while including market imperfections in the form of monopolistic competition, labor income taxation, and nominal frictions in optimal price setting on the part of firms. It is within this second-best setting that the question of optimal carbon taxation is considered alongside optimal revenue recycling. The central findings from this analysis are that using environmental revenues to reduce distortionary taxes does not yield the same benefit as returning revenues in a lump-sum fashion, and that following the implementation of an environmental tax average utility increases by as much as 24%.

Keywords: Carbon Dioxide, Environmental Tax, Double Dividend, DSGE Model

JEL Codes: H23, Q43, Q53, Q58
Optimal environmental taxation has long been a focus of study in the economic literature, but has recently received more popular attention due to growing concerns over climate change. Indeed, the once bright burning fire of literature concerning the topic of environmental taxation, and the double dividend hypothesis in particular, seemed to reduce to the mere glow of embers at the turn of the century. Since then, a few prominent economists have rearticulated their calls for environmental taxation (Mankiw 2009). Lawyers too have joined the call for carbon taxation as Hsu (2011) demonstrates. Despite the recent fervor surrounding major economic policies to reduce carbon dioxide there is still a lack of consensus among politicians and economists alike on the optimal path to take with regards to pollution taxation – and even less consensus on whether or not the revenues gained by such a tax could be used to reduce pre-existing distortionary taxes, the so-called double dividend (Starrett, 1999; Fernandez, et al. 2010, 2011). In fact, the majority of research to date on environmental taxation has focused on estimating a single tax rate that is time-invariant, for instance a “second-best” tax that is lower than the implied Pigouvian tax, to correct for activities that result in an externality. Such a tax may not be an appropriate approach in practice as there will be differing welfare impacts as the economy fluctuates between boom and bust, or the price of the underlying asset, which in the case of carbon dioxide emissions is energy, is volatile. This paper re-addresses this problem of optimal carbon taxation and redistribution in the context of a dynamic stochastic general equilibrium (DSGE) model. Environmental macroeconomics, as this type of framework has been referred to by previous authors in this limited field of research (Heutel and Fischer, 2013), is somewhat of a misnomer. Because carbon dioxide (CO₂) emissions are a trans-boundary

---

1 Calculating optimal carbon tax rates, and interactions with other pre-existing taxes, has most commonly been pursued in large-scale computable general equilibrium type models (CGE). Examples of CGE models that have been used include: GTAP, AIM, and GEM-E3
pollutant, its regulation and control tends naturally to be that of a macroeconomic problem. CO\textsubscript{2} emissions, however, are the result of individual firm and consumer actions and are thusly a macro-effect that is the result of micro-motives. This is one reason that DSGE modeling is a particularly robust setting to analyze optimal taxation because it allows for the most advantageous policy to be determined in the context of an economy in which forward-looking consumers maximize utility, firms have market power, there are labor market frictions due to taxation, there are nominal frictions in price adjustment, and the economy fluctuates along the real business cycle. Further, by simulating shocks to productivity and the price of energy, an optimal tax policy that minimizes welfare loss can be estimated and easily implemented in the real economy.

To date, there have been two published works which have considered emissions control in the context of a DSGE model, one paper that addresses energy use as an externality to consumers as an extension to their model, and one paper that addresses the double dividend hypothesis with a revenue-neutral green tax. Heutel (2012) uses a real business cycle (RBC) style DSGE model to infer the relationship between persistent productivity shocks and emissions. In a RBC type model there are no market imperfections, and markets always clear under the standard classical assumptions, e.g., laborers are paid the value of their marginal product. If markets were truly perfect, though, then there would not be an externality associated with CO\textsubscript{2} emissions because the socially efficient level would occur through the price mechanism. Clearly this is not the scenario that policymakers face today. A key finding in Heutel (2012) is that the optimal tax is pro-cyclical, and should decrease as the economy enters into a recessionary state. Though this paper comes to a similar conclusion, there are a few ways in
which the modeled economy in this paper differs from the model of Heutel (2012): first, Heutel
does not include labor in the production process, and second there are no frictions – nominal or
real. Similarly, Manzano (2006) finds the optimal tax rate on gasoline, when its consumption
produces a negative externality, to be pro-cyclical. Fischer and Springborn (2009) also study
optimal environmental policy in the context of an RBC model, though their model includes labor
as a production input. These authors propose emissions-intensity targets as the optimal policy
against other popular mechanisms like quotas or taxes. Fischer and Springborn mention at the
end of their paper that the interaction between revenue generating instruments and pre-existing
distortions within the economy is an avenue which merits future research. The present work
furthers this line of research by using the DSGE framework and incorporating real frictions in
the form of imperfect competition and labor taxation, and nominal frictions in the form of price
rigidities. Too, the model presented here addresses the question of tax interaction by allowing for
the revenue generated by carbon taxation to reduce distortionary labor taxes. The only other
paper to model an environmental tax reform and the double dividend hypothesis in a dynamic
general equilibrium context focuses on capital taxation as the distorting tax. Glomm et al. (2007)
finds that using a green tax to reduce capital taxation results in an increase in consumption of
market goods, an “efficiency dividend” in their words, and an improvement in environmental
quality, a green dividend in their words. The authors make note, however, that environmental
quality may decrease in the steady state.

The double dividend hypothesis began to be vigorously discussed beginning with Oates
(1995) and Goulder (1995), though it was already instantiated in the environmental and public
finance literature. Oates and Goulder both set a useful platform from which many empirical and
numerical studies were later based upon; Oates by noticing that a “tax interaction effect: could potentially counteract any double dividend effects,” and Goulder by distinguishing between “weak” and “strong” dividends. The weak double dividend is a scenario in which the introduction of an environmental tax to reduce distortionary taxes, instead of being transferred in a lump-sum fashion, leads to costs savings. The strong dividend is a scenario in which overall welfare, in terms of output or employment, is improved without regard to environmental improvements. In response to Goulder, Bovenberg (1999) provides an “updated reader’s guide” and sheds doubt on the possibility of a strong double dividend existing. Other authors have joined the debate on the double dividend hypothesis, many of which cast doubt on its existence. Overwhelmingly, though, authors note that setting a price on carbon emissions will be beneficial in bolstering the development of energy-efficient technology (Parry, 2005; Helm, et al., 2005) Parry (2005) also argues that not only should a price instrument be used in abating carbon dioxide emissions, but that the price instrument should be a harmonized international tax scheme. An internationally unified tax-scheme is a large charge, no doubt, but one that is much more likely to succeed than a permit system, the author notes.

The rest of the paper continues as follows: section 2 is a short empirical analysis provided to compare the impulse responses found in the simulation with real world data; section 3 develops the DSGE model; section 4 describes the parameters used in calibrating the simulation model; section 5 compares the outcomes of the baseline model with two different environmental policy rules and discusses the double dividend hypothesis; and section 6 concludes.
II. Empirical Analysis

The primary goal of this section is to motivate and provide a reference point for the simulation that follows. In order to do so, a vector error correction model is presented that uses quarterly data from 1973:1 to 2012:4 on carbon dioxide emissions from energy (CO₂), real GDP, and the energy CPI as variables. Data on CO₂ comes from the Energy Information Administration, and data on GDP and the energy CPI come from the St. Louis Federal Reserve Economic Database. Seasonality has been removed from CO₂ using a simple moving average. Figures 1 and 2 below show the progression of CO₂ emissions and GDP (fig. 1), and the energy CPI (fig. 2), over the sample period studied here with NBER recessions indicated by bars.
At first glance, figures 1 and 2 suggest that the variables under consideration are likely non-stationary. Augmented Dickey-Fuller and KPSS tests indicate that this is in fact the case. In order to properly specify the empirical model, then, a Johansen cointegration test is conducted under numerous lag lengths. The Johansen cointegration test indicates that the variables are cointegrated of order 2. In accordance with the cointegrating relationship found, the following vector error correction model is estimated:

\[
\Delta y_t = \alpha + \sum_{i=1}^{2} C y_{t-i} + \sum_{i=1}^{2} \beta_i \Delta y_{t-i} + \epsilon_t
\]

Where: \( y_t \) is a vector of the aforementioned variables, \( C \) denotes the impact matrix of long-term dynamics, and \( \beta_i \) can be manipulated to interpret reduced-form innovations for each variable to structural shocks. Further, a lag length of two quarters is chosen based on Akaike information
criterion. Using the standard Cholesky decomposition, the recursive structural interpretation is as follows,

\[
\begin{pmatrix}
    e_t^{GDP} \\
    e_t^{CPI} \\
    e_t^{CO_2}
\end{pmatrix}
= \begin{bmatrix}
    a_{11} & 0 & 0 \\
    a_{21} & a_{22} & 0 \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{pmatrix}
    \epsilon_t^{Real \ GDP \ shock} \\
    \epsilon_t^{Energy \ price \ shock} \\
    \epsilon_t^{CO_2 \ demand \ shock}
\end{pmatrix}
\]

Here it is assumed that shocks to GDP affect all other variables contemporaneously, but the converse is not true. Also, it is assumed that energy price shocks affect CO₂ contemporaneously. Though the model would have a different structural interpretation if the variables were re-ordered, it is worth noting that the general shapes of the impulse response functions do not differ.

Figure 3, below, displays the long run effect of shocks to GDP and the energy CPI on CO₂ emissions.
From the first row of impulse response functions presented in figure 3 it is plainly evident that innovations in both GDP and the energy CPI have lasting effects on CO$_2$ emissions. In the case of a GDP shock, CO$_2$ emissions are expected to increase on impact, and remain above zero for the following 20 periods. When there has been a shock to the price of energy, it is expected that CO$_2$ emissions will decrease for roughly 12 periods before beginning to rise again to pre-shock levels.

The second row of figure 3 shows how innovations in GDP and the energy CPI affect each other. As one might expect, the energy CPI responds in a positive manner to a positive shock in GDP. Positive energy price shocks will cause GDP to decrease for a short time but eventually lead to GDP returning to pre-shock levels. This finding echoes what many others have found with respect to the price of oil and overall economic activity since Hamilton (1983, 1995) including Rotemberg and Woodford (1996).

III. Theoretical Model

The theoretical model presented below incorporates several frictions, both nominal and real, in order to portray the United States economy in a simple yet tractable manner. Specifically,
the model presented here is a closed economy model in which firms have market power, but face adjustment costs when optimally choosing their price level. In addition, emissions (energy that pollutes upon combustion) are used as an input in the production process, much like the models of Copeland and Taylor (2004) in the environmental economics literature, and the papers that follow in the vein of Kim and Loungani (1992) and Rotemberg and Woodford (1996) in the DSGE literature. Finally, the externality effect of increased emissions negatively affect consumers, and are included in the individual’s utility function.

A. Households

The representative infinitely-lived consumer wishes to maximize expected lifetime utility which is represented by the separable utility function,

\[
\text{max } E_t \sum_{t=0}^{\infty} \xi^t \left( \frac{c_t^{1-\theta}}{1-\theta} - \chi \frac{N_t^{1-\varphi}}{1-\varphi} - \ln M_t \right)
\]

Where \( \xi \) denotes the discount rate, \( N_t \) labor supplied, \( M_t \) emissions, and \( C_t \) represents consumption of the Dixit-Stiglitz aggregate over a continuum of goods indexed by \( j \),

\[
C_t \equiv \left[ \int_0^1 c_{jt} \frac{v-1}{\nu} \, dj \right]^{\frac{\nu}{v-1}}
\]

The representative consumer is subject to the budget constraint,

\[
P_t C_t + K_t \leq \tau_t K_{t-1} + (1 - \delta) K_{t-1} + (1 - \tau_t) w_t N_t + T_t
\]
which includes an income tax of \( \tau_t \) and transfer payments \( T_t \) from the government in the form of a lump-sum rebated environmental tax; wages \( w_t \) and capital \( K_t \).

The first-order conditions from the household’s problem yield the following equilibrium conditions:

\[
C_t^{-\theta} = \xi (r_{t+1} + 1 - \delta) E_t(C_{t+1}^{-\theta})
\]

\[
\chi C_t^{\theta} N_t^{\phi} = (1 - \tau_t) w_t
\]

**B. Firms**

There is a continuum of firms who produce a differentiated final-good \( y_f \) and sell in a monopolistically-competitive market using capital, labor, and emissions (or energy that pollutes on combustion) as inputs according to the following Cobb-Douglas production function:

\[
F_t(K_t N_t M_t) = z_t K_t^\alpha N_t^\beta M_t^\gamma
\]

where \( z_t \) is an exogenous productivity shock that develops according to the autoregressive process,

\[
\ln z_t = \rho \ln z_{t-1} + \varepsilon_{zt}
\]

with \( \varepsilon_{zt} \sim N(0, \sigma_z^2) \) drawn once each period, and \( \alpha + \beta + \gamma = 1 \). This specification of the dynamic total factor productivity process follows the macroeconomic literature on DSGE models.
The firm minimizes total costs,

\[ w_t N_t + r_t K_t + (p_t^E + \tau_t^E) M_t \]  

where \( w_t \) and \( r_t \) are the real wage and the rental rate of capital, respectively; and the price of energy, \( p_t^E \), is exogenously determined and subject to shocks of the form,

\[ \ln p_t^E = \rho_p \ln p_{t-1}^E + \epsilon_{pt} \]  

with \( \epsilon_{pt} \sim N(0, \sigma_p^2) \), and the per-unit emissions tax, \( \tau_t^E \), is endogenously determined according to various policy rules discussed below. Exogenous energy pricing is an assumption that is common to RBC models that include energy use in the production function, and is also used in Leduc and Sill’s (2004) paper that allows for adjustment costs faced by the firm.

Cost minimization implies the following labor, capital, and emissions demand functions

\[ r_t = mc_t z_t \alpha K_t^{\alpha-1} N_t^\beta M_t^\gamma \]  

\[ w_t = mc_t z_t \beta K_t^\alpha N_t^{\beta-1} M_t^\gamma \]  

\[ (p_t^E + \tau_t^E) = mc_t z_t \gamma K_t^\alpha N_t^\beta M_t^{\gamma-1} \]

Where \( mc_t \) is the Lagrange multiplier from the cost minimization problem, and can be otherwise thought of as the real marginal cost of producing an additional unit of output. Given this
interpretation, and after substituting (12-14) into (10), the total cost for the firm can be re-stated as,

\begin{equation}
mc_t y_t
\end{equation}

In order to incorporate price stickiness, each firm faces quadratic nominal price-adjustment costs, or menu costs, as in Rotemberg (1982).

\begin{equation}
\frac{\psi}{2} \left( \frac{p_{jt}}{\pi p_{jt-1}} - 1 \right)^2 Y_t
\end{equation}

Where \( \psi \geq 0 \) denotes the extent of price adjustment costs. Each firm, then, seeks to maximize profits by choosing their price level

\begin{equation}
\max_{p_{jt}} E_t \sum_{t=0}^{\infty} \xi^t \lambda_t \left[ \frac{p_{jt} y_{jt}}{p_t} - mc_t y_{jt} - \frac{\psi}{2} \left( \frac{p_{jt}}{\pi p_{jt-1}} - 1 \right)^2 Y_t \right]
\end{equation}

subject to the firms production function,

\begin{equation}
y_{jt} = z_t K_{jt}^{\alpha} N_{jt}^{\beta} M_{jt}^{\gamma}
\end{equation}

and the series of demand constraints for the \( j^{th} \) firm

\begin{equation}
y_{jt} = \left( \frac{p_{jt}}{p_t} \right)^{-\nu} Y_t
\end{equation}
Note that if $\psi = 0$ the problem reduces to the (nominal) frictionless problem in which prices are fully flexible, and firms set a mark-up over marginal cost in accordance with the Lerner index. Finally, there is no entry or exit in the final-goods sector, and capital and labor markets are perfectly competitive.

By assumption the firm is not able to exercise market power in input markets. Thus, after equating labor supply and demand conditions (equations 7 and 13) the static “labor wedge” can be found; the difference between the consumers marginal rate of substitution between consumption and labor and the after-tax wage rate, and the firms demand for labor based on equating the marginal product of labor with the wage rate.

$$\chi C^\beta_t N^\psi_t = (1 - \tau_t)mc_t z_t \beta K^\alpha_t N^\beta_t - 1 M^\gamma_t$$

After noting that in a symmetric equilibrium all firms will set the same price, the efficient pricing scheme can be derived. This is also known in the macroeconomics literature as the new-Keynesian Phillips curve (NKPC).

$$0 = (1 - \nu)\lambda_t + \nu\lambda_t mc_t - \psi\lambda_t \left(\frac{\pi_t}{\pi} - 1\right) \frac{\pi_t}{\pi}$$

$$+ \xi \psi E_t \left[\lambda_{t+1} \left(\frac{\pi_{t+1}}{\pi} - 1\right) \left(\frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t}\right)\right]$$

which can more easily be seen and interpreted as a linear approximation around its steady state,

$$\pi_t = \frac{v - 1}{\psi} mc_t + \xi E_t \pi_{t+1}$$
Here, $\pi_t = \frac{p_t}{p_{t-1}}$, and it is assumed that there is no inflation in the steady state ($\pi = 1$). Also, note that in the absence of adjustment costs ($\psi = 0$) the NKPC reduces to the usual Lerner index mark-up rule.

One assumption that has been made in the analysis so far is that there is not a market for intermediate goods used in the production of final goods. This has been done because the primary intermediate good of interest, emissions or energy, can take many forms; e.g., fuel expenditures for a fleet of vehicles, electricity demand, or on site combustion of fossil fuels for production. The local market structure will have large effects on the price of energy that each firm faces when making purchasing decisions; e.g., Edgeworth price-cycling in gasoline markets, municipal monopolies for electricity production, and wholesale prices for combustible commodities that are contingent on demand from the rest of the world. Too, it is reasonable to assume that firms are price-takers for energy, and thus accept the price of energy as given. Leduc and Sill (2004) make a similar assumption in their new-Keynesian DSGE model that incorporates energy as an input in the production process citing the role that OPEC plays in affecting world prices for energy. For the sake of generality, then, each different type of energy source is included in the variable, $\mathcal{M}_t$. Because this paper seeks to find an optimal tax on the carbon output of an economy the fuel-type will not be of concern at present because it would be straightforward to weight a tax on a certain energy source based on its carbon content.\footnote{The EIA provides a summary of all fuel-type carbon content factors (EIA, 2013)}
C. Ramsey Taxation

Due to the distortions in the modeled economy, the labor wedge and pricing frictions, any tax on CO₂ emissions will naturally be a “second-best” type policy. In order to derive the second-best policy, then, a basic Ramsey planner problem must be solved. The problem of the Ramsey planner is to maximize the representative consumer’s utility subject to the aggregate resource constraint and private-sector equilibrium conditions. The “primal” approach used here is similar to that of Chari and Kehoe (1999). The Ramsey problem can formally be written as,

\[
\text{(23)} \quad \max E_0 \sum_{t=0}^{\infty} \xi^t u(C_t, N_t, M_t)
\]

subject to

\[
\text{(24)} \quad E_0 \sum_{t=0}^{\infty} \xi^t [u_{c,t} C_t + u_{N,t} N_t + u_{M,t} M_t] = A_0
\]

\[
\text{(25)} \quad C_t + K_{t+1} = z_t K^z_{t} N^r_{t} M^r_{t} + (1 - \delta) K_t
\]

Here, the left-hand-side of equation (24) is the present-value implementability constraint (PVIC) and it is equal to the constant, \( A_0 \). Lagrange multipliers for the constraints are \( \mu_1 \) and \( \mu_{2,t} \), respectively. Using the first order conditions with respect to consumption and labor it can be shown\(^4\) that the labor income tax rate is constant and equal to

\[
\text{(26)} \quad \tau_t = \frac{\mu_1 (\theta - \varphi)}{1 + \mu_3 (1 - \varphi)}
\]

\(^3\) A collection of present value utilities, all of which are constants and are not influential or interesting in the derivation that follows.

\(^4\) This result, as well as the optimal carbon tax, is derived in a mathematical appendix that is available upon request.
An important feature that is of utmost importance to describing the nature of the double dividend is the way in which firms hire from input markets. If firms are able to exercise some amount of market power, then the optimal labor tax will vary over time. If firms are not able to exercise market power, though, then the solution to the Ramsey problem implies that the labor-income tax rate should be constant over time. This result is known as tax smoothing, and can be found as early as Barro (1979). If we are to take Goulder’s definition of strong and weak double dividends, then, a weak double dividend will always be found due to tax smoothing. Due to this finding, and the assumption that capital and labor markets are competitive, the revenue neutral emissions tax will be refunded to consumers in the form of a transfer payment, the double dividend will be examined in terms of whether or not there is an increase in consumption - an efficiency dividend, and whether or not there has been a decrease in the amount of emissions – a green dividend as in Glomm, et al. (2007).

Unlike the optimal labor tax, the tax on emissions does in fact vary over time. Using the first order conditions for consumption and emissions the optimal tax on emissions is given by\(^5\)

\[
\tau_t^E = \frac{c_t^p (1 + \mu_1 - \delta \mu_2)}{\bar{M}_t} - p_t^E
\]  

From equation (27) it is apparent that the optimal emissions tax is increasing in the amount of consumption, and decreasing in both the level of emissions and the price of energy.

\(5\) Derivation of the optimal emissions tax is shown in an appendix that is available from the author on request.
IV. Parameters

Although DSGE models have rarely been used in the environmental literature, the history of DSGE modeling is rich. Thus, many of the parameters used in this study rely on findings and assumptions from the macroeconomic and monetary theory literature. There are, however, a few variables of interest that will be derived here.

First, the persistence and standard deviation for the energy price level is calculated by estimating an ordinary least squares regression of the equation governing the price shock using the aforementioned energy CPI data. From this regression it is found that the energy price at time $t$ is highly persistent. The persistence parameter, $\rho_p$, is calculated to be .9817, and the standard deviation of innovations in the energy price is .0513. The standard deviation of the productivity shock is calibrated in such a way that the standard deviation of output is similar to that of the United States, a common practice in the DSGE literature.

The share of output devoted to energy is also calculated. This study uses the same method and data source as Fischer and Springborn (2011), namely calculating the mean ratio of total energy expenditures to GDP from information gathered from the Energy Information Administration (EIA). The value used in this paper differs from the aforementioned paper due to a longer time horizon of data. The calculated value of the energy share in output is found to be .0818, a slightly lower value than Fischer and Springborn.

Another variable of interest in the calibration of the model presented below is the menu-cost parameter, which has been set to a value of 20. This parameter corresponds with the structural estimates of Lubik and Schorfheide (2004). Further, while the presence of adjustment costs causes there to be a wedge between consumption available to consumers, which is made
greater when increasing the menu-cost parameter, the qualitative results that follow are not
impeded by perturbations to this parameter. This result is shown in the sensitivity analysis
section below for output, consumption, and emissions, though it applies to the other variables in
the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Energy share of output</td>
<td>.0818</td>
<td>Mean ratio of total energy expenditure to GDP (1973-2010), EIA (2013)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of output</td>
<td>.33</td>
<td>Standard assumption from DSGE literature</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Labor share of output</td>
<td>.5882</td>
<td>Calculated from former two parameters as $1 - \alpha - \gamma$, consistent with Fischer and Springborn (2011)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>.025</td>
<td>Standard assumption from DSGE literature</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Rotemberg menu-cost parameter</td>
<td>20</td>
<td>Lubbick and Schorfheide (2004)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Coefficient of relative risk aversion</td>
<td>2</td>
<td>Standard assumption from DSGE literature</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Frisch labor supply elasticity</td>
<td>.25</td>
<td>Standard assumption from DSGE literature</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Quarterly discount rate</td>
<td>.98267</td>
<td>Heutel (2012)</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>Marginal labor tax rate</td>
<td>.10</td>
<td>Average marginal income tax rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Price-elasticity of demand</td>
<td>11</td>
<td>Consistent with a mark-up of 10%</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Energy price autocorrelation</td>
<td>.981742</td>
<td>Calculated using energy CPI data</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>Standard deviation of energy price innovation</td>
<td>.0513</td>
<td>Calculated using energy CPI data</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Productivity autocorrelation</td>
<td>.95</td>
<td>Standard assumption from DSGE literature</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of TFP innovation</td>
<td>.007</td>
<td>Standard assumption from DSGE literature</td>
</tr>
</tbody>
</table>
V. Simulation

In this section the results from the baseline model are presented followed by a discussion of two different policy rules. The first policy rule discussed is a time-invariant tax rule that is only concerned with reaching a target level of steady-state emissions, and is thus agnostic to changes in consumer welfare. The second policy rule, however, is a time varying tax rule that takes consumer welfare as a central goal. This rule is developed along the lines of Miguel and Manzano (2006), and Heutel (2012). Implications for revenue recycling are discussed alongside each policy rule as well.

The model is solved by taking a first order approximation about the stochastic steady state as in Sims (2002). Further, the model moments and impulse response functions are based on 1000 replications of 500 periods. Steady-state values as well as the mean of simulation moments for the baseline model are presented below in table 2. The baseline model considered here is one in which there are no environmental taxes, but all other frictions remain.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady-state</th>
<th>Simulated Mean</th>
<th>Simulated Standard Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.13905</td>
<td>1.146992</td>
<td>0.006099</td>
</tr>
<tr>
<td>c</td>
<td>0.607612</td>
<td>0.614811</td>
<td>0.002787</td>
</tr>
<tr>
<td>k</td>
<td>20.7425</td>
<td>20.787134</td>
<td>0.010745</td>
</tr>
<tr>
<td>n</td>
<td>0.33911</td>
<td>0.334578</td>
<td>0.001205</td>
</tr>
<tr>
<td>m</td>
<td>0.057</td>
<td>0.069141</td>
<td>0.114158</td>
</tr>
</tbody>
</table>

From the impulse response functions, which are shown below in figures 4, it is evident that emissions, output, and consumption respond somewhat similarly to shocks in GDP and the energy price as the impulse response functions from the structural-VEC model presented above. Though the model is not able to accurately portray the hump-shaped response of emissions to a
shock in GDP, the same dynamics are at work. Specifically, a shock to TFP (left) will increase emissions for a time but the effect is not permanent. The same general caveat can be applied to the response of emissions to a shock in the energy price (right).

In order to check the sensitivity of the model to perturbations of the underlying parameters many different potential scenarios were tested including, but not limited to: sensitivity to changes in the menu cost parameter, and the degree of market power each firm yields. The model performs well under each of these different parameter specifications with no changes to any of the qualitative results, and only minor changes to the actual numerical values that are simulated. Figure 5, below, displays the sensitivity of the impulse response functions for output, consumption, and emissions to changes in the menu cost parameter\(^6\). Each subfigure shows the response for 40 periods after a shock in total factor productivity depending on the

\(^6\) Market power sensitivity is not displayed because it is very similar to the menu-cost sensitivity. Available upon request.
menu-cost parameter, $\psi$, which ranges from a very flexible price environment ($\psi = 5$) to an inflexible environment ($\psi = 50$). Note, that the baseline model assumes a menu-cost value of 20 which corresponds to prices changing every 2-3 quarters (Lubok and Schorfheide, 2004).

**Figure 5A – Output Sensitivity Analysis – TFP Shock**

![Output Sensitivity Analysis – TFP Shock](image)

**Figure 5B – Consumption Sensitivity Analysis – TFP Shock**

![Consumption Sensitivity Analysis – TFP Shock](image)

**Figure 5C – Emissions Sensitivity Analysis – TFP Shock**

![Emissions Sensitivity Analysis – TFP Shock](image)
For each of the impulse response functions we can see that varying the menu-cost parameter only has a moderate effect on the values of the response, but no real changes qualitatively. Specifically, as the price becomes less flexible ($\psi \to 50$) output and emissions decrease from their initial shock values slightly quicker, and consumption is less hump-shaped and slightly flatter in the first 15 periods.

A. “Environment-First” Tax Rule

The policy rule described in this section seeks only to reach a specified level of emissions without regard to welfare or consumption of the individuals in the modeled economy. The rule here is modeled after the 2020 climate initiative in which the United States set a goal to reach 17% reductions in CO$_2$ emissions from 2005 levels by the year 2020. In order to reach this level of CO$_2$ reductions in the steady-state, compared to the baseline model with no environmental taxation, a tax of 20.3% is required.

A question that has received considerable attention in the public finance and environmental literature is whether an environmental tax should be used to reduce distortionary labor taxation, or whether environmental taxes should be returned to individuals in a lump-sum manner. The results from this paper indicate that redistributing the revenues from an environmental tax in the form of a lump-sum transfer is the preferred mechanism. As table 3 indicates, when the revenues from emissions taxation are returned to consumers in a lump-sum fashion steady-state consumption decreases only slightly while emissions decrease by more than the required 17%.
Table 3 – Environment-First Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline steady-state</th>
<th>Without recycling</th>
<th>Decrease labor tax</th>
<th>Lump-sum transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.146992</td>
<td>1.061956</td>
<td>1.067331</td>
<td>1.073608</td>
</tr>
<tr>
<td>c</td>
<td>0.614811</td>
<td>0.612986</td>
<td>0.613469</td>
<td>0.613579</td>
</tr>
<tr>
<td>k</td>
<td>20.787134</td>
<td>17.541301</td>
<td>17.729437</td>
<td>18.458814</td>
</tr>
<tr>
<td>n</td>
<td>0.334578</td>
<td>0.334335</td>
<td>0.335557</td>
<td>0.331397</td>
</tr>
<tr>
<td>m</td>
<td>0.069141</td>
<td>0.054125</td>
<td>0.053718</td>
<td>0.053639</td>
</tr>
</tbody>
</table>

Goulder (1995) provides a useful starting point for any discussion on the double dividend hypothesis. Goulder determines two types of double dividends that may occur, strong and weak. The former classification being a situation in which welfare increases following an environmental tax regardless of the degree of environmental improvement, and the latter a situation in which welfare increases by a greater amount when tax revenues are used to reduce distortionary taxes than when revenues are returned in a lump-sum fashion.

There are many possible reasons why the “weak” double dividend hypothesis does not hold for the modeled economy here. First, as Deaton (1979) and Babiker, Metcalf, and Reilly (2003) show, a lump-sum transfer may be preferred because any efforts to reduce distortionary taxes causes a larger gap between the after tax prices of energy and labor. Second, a finding in the macroeconomics literature on tax-smoothing suggests that when the degree of market power is unchanging the optimal labor tax rate is constant over time. Thus, any effort to reduce distortionary taxes is suboptimal in the first place and the planner should instead solve for an optimal labor tax with the knowledge that environmental revenues will be refunded in a lump-sum fashion. Glomm, et al. (2007) distinguishes their double dividend in terms of an efficiency dividend, an increase in consumption, and a green dividend, an increase in environmental
quality. When the flat tax is used on emissions a double dividend is not found because consumption decreases by .3%. It is worth noting, however, that the average utility of the consumer actually increases compared to the baseline scenario of no environmental taxes. Thus, the decrease in consumption is offset in utility terms by an increase in environmental quality.

B. Dynamic Tax Rule

Unlike the former tax rule, the tax rule proposed here is developed with an eye toward consumer welfare. As both Miguel and Manzano (2006), and Heutel (2012) show, the optimal environmental tax will increase as the economy expands, and decrease in response to energy price shocks. That is to say, in the presence of positive productivity shocks the optimal tax should increase to dissuade the heightened use of energy, and should decrease following spikes in energy prices because energy demand will naturally be lower. Using the results from the planner’s problem presented above, an optimal tax rule is instead estimated here. Specifically, an environmental Taylor-rule of sorts is estimated that depends on deviations from the steady-state level of output, and the steady-state energy price with the estimated optimal tax increasing as output increases, and decreasing as energy price increases. This rule is developed to offer policymakers a viable policy-rule that is determined by readily available macroeconomic aggregates, such as those used in the empirical analysis of section two. The tax rule is expressed below in equation (28),

\[
\tau_t^M = \mu_t + \beta_y \ln(y_t - \bar{y}) - \beta_p \ln(p_t - \bar{p})
\]
where \( \mu_t \) denotes the preferred tax when the economy is in a steady-state, bars above output and the price of energy indicate steady-state levels, and the variables, \( \beta_y \) and \( \beta_p \) are estimated weights that maximize average utility of the consumer. Table 4, below, displays the results of using the dynamic tax rule and returning the revenues in various fashions compared to the baseline steady-state of no environmental taxation.

**Table 4 – Dynamic Tax Rule**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline steady-state</th>
<th>Without recycling</th>
<th>Decrease labor tax</th>
<th>Lump-sum transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.146992</td>
<td>1.061447</td>
<td>1.066927</td>
<td>1.072750</td>
</tr>
<tr>
<td>c</td>
<td>0.614811</td>
<td>0.612500</td>
<td>0.613114</td>
<td>0.613215</td>
</tr>
<tr>
<td>k</td>
<td>20.787134</td>
<td>17.539623</td>
<td>17.726865</td>
<td>18.459009</td>
</tr>
<tr>
<td>n</td>
<td>0.334578</td>
<td>0.334636</td>
<td>0.335927</td>
<td>0.331503</td>
</tr>
<tr>
<td>m</td>
<td>0.069141</td>
<td>0.053574</td>
<td>0.053172</td>
<td>0.053085</td>
</tr>
</tbody>
</table>

Notes: This table presents results when the environment-first tax rule is used for \( \mu_t \), and weights are .5 for both the output gap and energy price gap.

We can again see that the double dividend hypothesis does not hold in terms of a weak/strong dividend, or as an efficiency dividend. The major benefit to a dynamic tax-rule can be seen, though, by the fact that average utility of the consumer increases by roughly 24%. This is a major benefit to the policy-maker because as Hsu (2011) notes, the major impediment to a carbon tax is simply its political infeasibility. Given these results, introducing a tax on emissions is a policy that can be justified as helping consumers in the long-run, though average output does fall by roughly 6.7%.
VI. Conclusions

This paper has furthered a new line of research that examines the problem of carbon dioxide mitigation policy in a dynamic and stochastic environment by incorporating important elements, such as monopolistic competition and price-adjustment frictions, in order to better represent the United States economy. Too, this paper finds that rebating tax revenues to consumers in the form of a lump-sum subsidy is preferred to reducing distortionary labor taxes. For both policy scenarios presented above, a flat tax rate and a dynamic tax rate, a double dividend fails to exist in terms that previous authors have considered, but under both policy scenarios average utility increases compared to the baseline scenario of no environmental taxation when revenues are returned as a lump-sum. This is a boon to policy-makers that wish to address the pressing problem of global climate change because the present analysis indicates that the implementation of an environmental tax is beneficial in utility terms. It should be noted, however, that average output declines following an environmental tax.

While a theoretical analysis of the problem of environmental taxation is certainly necessary for the ongoing discussion and research on confronting climate change, a shortcoming of this paper lies in the fact that a representative consumer is used to express the interests of a population that is actually quite varied. For instance, the distributional effects of returning revenues in a lump-sum are of interest because lower income individuals typically have a much higher propensity to consume than their more wealthy counterparts. Thus, returning revenues to these individuals may spur economic activity by more than this paper is able to analyze. Similarly, in the modeled economy presented here consumers are forced to consume an aggregate good that represents a basket of goods. If revenues were instead used to incentivize
cleaner consumption, from an emissions standpoint, then a larger green dividend may actually be realized, a triple dividend of sorts. Evidence of this type of policy having success can readily be seen by the many accomplishments of regional greenhouse gas initiative (RGGI) states’ policies toward using revenues from permit auctions to further environmental goals; for instance: funding energy audits for the poor, installing solar panels on universities, and replacing government fleet vehicles with vehicles that emit less (RGGI, 2013). Further research that considers this “third” dividend would certainly be of merit. Too, a model that includes wage setting frictions and habit formation in consumption may prove useful in matching the hump-shaped impulse responses found in the empirical analysis

REFERENCES


