Price formation of exhaustible resources: An Experimental Investigation of the Hotelling Rule

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Outline

- Introduction
- Experimental Design
- Hypotheses
- Results & Discussion
- Conclusion
A central concept of resource economics

- Hotelling, 1931: The Economics of Exhaustible Resources
  - The Hotelling rule states that the prices, more specifically the scarcity rent, of non-renewable resources should rise at the rate of interest. Then a resource owner is indifferent between extracting now and extracting in the future.

- Foundation of our understanding of how exhaustible resource markets will respectively should evolve

- BUT: Most of the empirical studies reject the theory, see e.g. Livernois 2009
Price development in the long run: oil, coal and gas

Source: BP (Oil, CPI), IEA (Coal 1972-2010, Gas), NMA (Coal 1900-1971)
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Experimental Design, Theoretical model and solution

- Modeling a market by using a continuous double auction (CDA)
- Each buyer has a utility function with a diminishing marginal utility

\[ U = \sum_{i=1}^{n} \frac{ax_i - \frac{1}{2}bx_i^2 - p_i x_i}{(1 + r)^{i-1}} \text{ with } a, b > 0 \text{ and } \frac{\partial^2 U}{\partial x_i^2} < 0 \]

- Each seller has the objective function

\[ \Pi = \sum_{i=1}^{n} \frac{p_i x_i}{(1 + r)^{i-1}} \text{ s.t. } \sum_{i=1}^{n} x_i \leq S_0 \]

- To solve with a Lagrange approach
- Parameter
  - Interest rate \( r = 20\% \)
  - Stock \( S_0 = 50 \)
  - Periods \( n = 10 \)

<table>
<thead>
<tr>
<th>period</th>
<th>price</th>
<th>quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.852</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>4.622</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>5.548</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>6.656</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>7.988</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9.586</td>
<td>5</td>
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<tr>
<td>7</td>
<td>11.502</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>13.804</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>16.564</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \Sigma 50 \]
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Hypotheses

The behavior is close to the theoretical solution, so that:

1. Prices will increase over the 10 periods.
2. The price increase is in accordance with an exponential growth law.
3. The market performance in the second market of each session will be closer to the equilibrium path than the performance in the preceding market.
4. Consumption quantities decrease over time, so that (a) the entire stock will be depleted after 10 periods and (b) there will be no trades in the final period.
Procedures

- Subjects
  - 220 subjects (students of the Clausthal University of Technology) participated in 11 sessions
  - 20 participants per market (10 buyers and 10 sellers)

- Experiment
  - Computerized experiment was programmed in zTree (version 3.2.12)
  - Each participant earned on average 15.65 € plus a show-up fee of 3 €
  - Session lasted around 90 minutes
  - Instructions contained
    - Written set of regulations
    - A test of understanding
    - A test period
  - 2 x 10 periods
  - One period lasted 3 minutes
  - All interactions were anonymous
  - Inexperienced, untrained subjects!
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Results (1) - Price formation: Median prices from 10 markets

Result: An increasing price path can be observed.
Results (2) - Regressions: Prices

- The price path can be described with $p_{opt} = 3.852 \cdot (1.2)^{period-1}$
- Nonlinear least-squares estimation of the marginal prices, period 1-9
  \[ p = \beta_1 \cdot (1.2)^{period-1} + \varepsilon \]

<table>
<thead>
<tr>
<th></th>
<th>price level $\beta_1$</th>
<th>95% Conf. Interval</th>
<th>N</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st markets</td>
<td>3.710 (0.086)</td>
<td>3.540 3.880</td>
<td>265</td>
<td>0.875</td>
</tr>
<tr>
<td>2nd markets</td>
<td>3.911 (0.035)</td>
<td>3.842 3.979</td>
<td>263</td>
<td>0.980</td>
</tr>
<tr>
<td>All markets</td>
<td>3.826 (0.044)</td>
<td>3.739 3.912</td>
<td>528</td>
<td>0.935</td>
</tr>
<tr>
<td>Theory</td>
<td>3.852</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: We use a regression model without intercept. Therefore this modified $R^2$ cannot be interpreted as the common $R^2$. See Davidson and Mackinnon (2004) or Wooldridge (2009) for a detailed discussion.*
**Result:** A learning process can be observed. The data from the second run is closer to the equilibrium.
Result: Decreasing quantities in line with the theory, but final round effect
The quantity path can be described with $x_{opt} = 100 - 19.26 \cdot (1.2)^{period-1}$

Nonlinear least-squares estimation of the marginal prices, period 1-9

\[ x = 100 - \beta_2 \cdot (1.2)^{period-1} + \varepsilon \]

<table>
<thead>
<tr>
<th></th>
<th>quantity level $\beta_2$</th>
<th>95% Conf. Interval</th>
<th>N</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st markets</td>
<td>18.585 (0.569)</td>
<td>17.441</td>
<td>19.730</td>
<td>50</td>
</tr>
<tr>
<td>2nd markets</td>
<td>19.967 (0.667)</td>
<td>18.626</td>
<td>21.308</td>
<td>50</td>
</tr>
<tr>
<td>All markets</td>
<td>19.276 (0.442)</td>
<td>18.399</td>
<td>20.153</td>
<td>100</td>
</tr>
</tbody>
</table>

Theory: 19.26

*Note: We use a regression model without intercept. Therefore this modified R² cannot be interpreted as the common R². See Davidson and Mackinnon (2004) or Wooldridge (2009) for a detailed discussion.*
Results (6) - Efficiencies

- Efficiency of a market

\[ \eta_{eff} = \frac{\text{producer} + \text{consumer surplus}}{\text{total surplus}} \]

- Benchmark efficiency (each period the same consumption)

- Relative efficiency

\[ \eta^*_{eff} = \frac{\eta_{eff} - \eta_{benchmark}}{(100\% - \eta_{benchmark})} \]

<table>
<thead>
<tr>
<th></th>
<th>efficiency $\eta_{eff}$</th>
<th>efficiency benchmark $\eta_{benchmark}$</th>
<th>relative efficiency $\eta^*_{eff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>all markets</td>
<td>97.75%</td>
<td>92.81%</td>
<td>68.63%</td>
</tr>
<tr>
<td>1st markets</td>
<td>96.93%</td>
<td></td>
<td>57.32%</td>
</tr>
<tr>
<td>2nd markets</td>
<td>98.56%</td>
<td></td>
<td>79.95%</td>
</tr>
</tbody>
</table>
Interpretation: For the market outcome only the aggregated consideration is crucial.
Further treatments

- Overview: conducted treatments

<table>
<thead>
<tr>
<th></th>
<th>Basic-Treatment</th>
<th>Trader-Treatment</th>
<th>Interest-Treatment</th>
<th>BST-Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interest rate</strong></td>
<td>20%</td>
<td>20%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td><strong>Stock</strong></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><strong>Periods</strong></td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td><strong>Participants</strong></td>
<td>10 buyer, 10 seller</td>
<td>10 buyer, 10 trader</td>
<td>10 buyer, 10 seller</td>
<td>10 buyer, 10 seller</td>
</tr>
</tbody>
</table>
| **Utility function**      | \( U = a \cdot x - \frac{1}{2} \cdot b \cdot x^2 \)  \\
with \( a = 20, b = 2 \) | \( U = a \cdot x - \frac{1}{2} \cdot b \cdot x^2 \)  \\
with \( a = 20, b = 2 \) | \( U = a \cdot x - \frac{1}{2} \cdot b \cdot x^2 \)  \\
with \( a = 20, b = 2 \) | \( U = a \cdot x - \frac{1}{2} \cdot b \cdot x^2 \)  \\
with \( a = 20, b = 2 \) |
| **Seller costs**          | 0               | 0                | 0                  | 0             |
| **Backstop costs**        | -               | -                | -                  | 8             |
| **Markets**               | 10              | 4                | 4                  | 4             |
Insights from the BST-Treatment

- **Motivation:** A lot of energy policy instruments are aiming on the demand side. The behavior of the supply side is less or even not focused.

- **Treatment-Design**
  - Buyers can purchase an unlimited substitute technology for a fix relative high price
  - Counterintuitive effect of premature "exploration"

**Result:** Shift of consumption from later to earlier periods through introducing a substitute technology.
Agenda

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Conclusion

- The Basic-Treatment shows, by and large, a confirmation of the theory.
- The market institution (CDA) seems to be like a (partial) substitute for the individual rationality of the participants (in line with Gode/Sunder, 1993).
- High market efficiencies (Smith, 1982: „scientific mystery“) despite a sophisticated intertemporal dynamic allocation problem.
- Existence of learning effects.
- In real markets there are many determinants of resource prices.
  - A price path deviating from the Hotelling Rule can’t be directly evaluated as a non-optimal path.
- Extensions of the treatment were conducted, for example introducing a Backstop-Technology.
Thank you for your attention!

Comments? Questions?
Experimental Design

- Modeling a market by using a continuous double auction (CDA)
  - Sellers and Buyers can submit orders
  - Placing an order actively or passively
  - Continuously, because placing an order is always possible till the trading time is over
  - One can trade more than one unit of the good per period
  - The transaction list and the list of open bids and offers is public information

![Diagram](based on Smith, 2003)
Insights from the BST-Treatment

Result: No discrete jump from the exhaustible resource to the BST, but technology change in the „right“ period
Sources