What if oil is less substitutable?
A New-Keynesian Model with Oil, Price and Wage Stickiness including Capital Accumulation

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CES, PSE, University Paris I

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Outline

Goals

Model

Special Features
  Households
  Intermediate Good Firms
  GDP Definition

Estimation
  Setting
  Estimation Results

Impulse Response Functions
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• Construct a New-Keynesian model with labor, capital and oil in the production function; and domestic goods and oil in the consumption flow. Both using CES functions.

• Add price and wage stickiness.

• Estimate the model’s parameters with Bayesian techniques, using U.S data from 1984:Q1 to 2007:Q1.

• Study the impact of an oil shock in this economy.
Why a CES?

Figure: US Oil consumption per capita (Mtoe) and Spot Oil Prices
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Model Structure

Domestic Economy
Model Structure

- Domestic Economy
- Final Good Firm
- Households
Model Structure

Domestic Economy

Households

Final Good Firm

<table>
<thead>
<tr>
<th>l.s taxes</th>
<th>invest</th>
<th>work</th>
<th>consume</th>
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</thead>
</table>

Foreign exogenous price

Labor

Capital

Oil produces

Intermediate Firms

Oil

Final Goods

Taylor

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Model Structure

Domestic Economy

- l.s taxes
- invest
- work
- capital
- consume

Final Good Firm

Oil produces

Intermediate Firms

Labor, Capital

Foreign exo p., exogenous price

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Domestic Economy → Final Good Firm

- Households
  - investment
  - work
  - consumption

- Final Goods
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- Final Goods Firm
  - Produces
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Households

Final Good Firm

Final Goods

Oil

Households

- invest
- work
- consume

Domestic Economy

- l.s taxes
- bonds
- capital

Final Good Firm

- Oil

Intermediate Firms

- Oil
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- Capital

Foreign exo p.

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- **Domestic Economy**
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- **Final Good Firm**
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- **Households**
  - Final Goods
    - Oil

- **Intermediate Firms**
  - Oil
  - Labor
  - Capital
  - Profit

- **Final Goods**
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Final Goods

Oil

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  - l.s taxes
  - bonds
  - capital

- **Final Good Firm**

- **Intermediate Firms**
  - exo p.

- **Final Goods**
  - Oil
  - Labor
  - Capital

- **Oil**

- **Taylor**

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- **Goals**
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exo p.

exo p.
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Exo P.

Exo P.

Exo P.

Exogenous Price
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- Households
- Intermediate Good Firms
- GDP Definition

Estimation

Impulse Response Functions
Households’ CES Consumption Function

\[ C_t(j) := ((1 - x_c)^{1-\sigma} C_{q,t}(j) + x_c^{1-\sigma} C_{e,t}(j))^{\frac{1}{\sigma}} \]
Households’ CES Consumption Function

\[ C_t(j) := ((1 - x_c)^{1-\sigma} C^\sigma_{q,t}(j) + x_c^{1-\sigma} C^\sigma_{e,t}(j))^{\frac{1}{\sigma}} \]

\[ \sigma = \frac{\eta_c - 1}{\eta_c} \]
Households’ CES Consumption Function

\[ C_{q,t}(j) := \left( \int_0^1 C_{q,t}(i,j)^{1-\frac{1}{\epsilon_p}} \, di \right)^{\frac{\epsilon_p}{\epsilon_p-1}} \]

\[ C_t(j) := ((1 - \chi_c)^{1-\sigma} C_{q,t}(j) + \chi_c^{1-\sigma} C_{e,t}(j))^\frac{1}{\sigma} \]

\[ \sigma = \frac{\eta_c-1}{\eta_c} \]
Intermediate Good Firms’ CES Production Function
Intermediate Good Firms’ CES Production Function

\[ Q_t(i) := (x_p(A_{E,t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^\rho)^{1/\rho} \]

\[ \rho = \frac{\eta_p - 1}{\eta_p} \]
GDP (in value added)

\[ P_{c,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t \]
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Impulse Response Functions
### Calibrated Parameters

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\epsilon_p$</th>
<th>$\epsilon_w$</th>
<th>$\omega$</th>
<th>$\chi_c$</th>
<th>$\chi_p$</th>
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</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.025</td>
<td>8</td>
<td>8</td>
<td>0.18</td>
<td>0.03</td>
<td>$1.6635 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

**Table**: Calibrated Parameters
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Share” parameter on capital</td>
<td>$\alpha$ Normal(0.3,0.05)</td>
<td>0.3329 0.3399 0.3129 0.3711</td>
</tr>
<tr>
<td>Elast. substitution in production</td>
<td>$\eta_p$ Inv_Gamma(0.135,inf)</td>
<td>0.1215 0.1254 0.1053 0.1461</td>
</tr>
<tr>
<td>Elast. substitution in consumption</td>
<td>$\eta_c$ Inv_Gamma(0.135,inf)</td>
<td>0.0625 0.1112 0.0319 0.2085</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\phi$ Normal(1.17,0.5)</td>
<td>1.2732 1.2148 0.7667 1.6638</td>
</tr>
<tr>
<td>Taylor rule response to inflation</td>
<td>$\phi_{\pi}$ Normal(1.2,0.1)</td>
<td>1.0000 1.0226 1.0000 1.0499</td>
</tr>
<tr>
<td>Taylor rule response to output</td>
<td>$\phi_{y}$ Normal(0.5,0.1)</td>
<td>0.8744 0.9097 0.7994 1.0336</td>
</tr>
<tr>
<td>Calvo price parameter</td>
<td>$\theta_p$ Beta(0.5,0.2)</td>
<td>0.5000 0.5181 0.5001 0.5386</td>
</tr>
<tr>
<td>Calvo wage parameter</td>
<td>$\theta_w$ Beta(0.5,0.2)</td>
<td>0.8188 0.8075 0.7620 0.8535</td>
</tr>
</tbody>
</table>

**Table**: Prior and Posterior Distribution of Structural Parameters
## Goals Model

### Special Features

- **Estimation Impulse Response Functions**

### Parameter Prior and Posterior Distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mode</td>
</tr>
<tr>
<td>Autoregressive parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real oil price</td>
<td>$\rho_{se}$ Beta(0.5,0.2)</td>
<td>0.9976</td>
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<tr>
<td>Real capital price</td>
<td>$\rho_{sk}$ Beta(0.5,0.2)</td>
<td>0.9692</td>
</tr>
<tr>
<td>Government</td>
<td>$\rho_{g}$ Beta(0.5,0.2)</td>
<td>0.9364</td>
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<tr>
<td>Monetary</td>
<td>$\rho_{i}$ Beta(0.5,0.2)</td>
<td>0.9414</td>
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<tr>
<td>Oil productivity</td>
<td>$\rho_{ae}$ Beta(0.5,0.2)</td>
<td>0.6862</td>
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<tr>
<td>TFP</td>
<td>$\rho_{alk}$ Beta(0.5,0.2)</td>
<td>0.7317</td>
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<tr>
<td>Oil Prod. in Gov</td>
<td>$\rho_{ae,g}$ Beta(0.5,0.2)</td>
<td>0.2043</td>
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<tr>
<td>TFP in Gov.</td>
<td>$\rho_{alk,g}$ Beta(0.5,0.2)</td>
<td>0.7103</td>
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<tr>
<td>Price markup1</td>
<td>$\rho_{p}$ Beta(0.5,0.2)</td>
<td>0.7838</td>
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<tr>
<td>Wage markup1</td>
<td>$\rho_{w}$ Beta(0.5,0.2)</td>
<td>0.3469</td>
</tr>
<tr>
<td>Price markup2</td>
<td>$\nu_{p}$ Beta(0.5,0.2)</td>
<td>0.2849</td>
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<tr>
<td>Wage markup2</td>
<td>$\nu_{w}$ Beta(0.5,0.2)</td>
<td>0.4953</td>
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**Table:** Prior and Posterior Distribution of Shock Parameters
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Mode</td>
</tr>
<tr>
<td>Standard deviations</td>
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</tr>
<tr>
<td>Real oil price</td>
<td>$\sigma_{se}$</td>
<td>Inv_Gamma(1,2)</td>
</tr>
<tr>
<td>Real capital price</td>
<td>$\sigma_{sk}$</td>
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<td>TFP</td>
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<tr>
<td>Price markup</td>
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<tr>
<td>Wage markup</td>
<td>$\sigma_{w}$</td>
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Impulse Response Functions
IRF to a Real Oil Price Shock
Log-linearized real wage inflation equation

\[ \pi_{q,t} + \pi_{wr,t} = \beta \mathbb{E} [\pi_{q,t+1} + \pi_{wr,t+1}] + \frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w(1 + \phi \epsilon_w)} (mrs_{t+s_c,t - wr,t}) + \epsilon_{w,t} \]
Optimal Expenditure Allocation

Household’s Optimal Expenditure Allocation
Optimal Expenditure Allocation

Household’s Optimal Expenditure Allocation

$$\max_{C_q,t, C_e,t} P_{C,t}(j)$$

s. t

$$P_{C,t}(j) = P_{e,t} C_{e,t}(j) + P_{q,t} C_{q,t}(j)$$

$$C_t(j) := \left((1 - \chi_c)^{1-\sigma} C_{q,t}(j) + \chi_c^{1-\sigma} C_{e,t}(j)\right)^{\frac{1}{\sigma}}$$
Optimal Expenditure Allocation

Household’s Optimal Expenditure Allocation

\[ \max_{C_q,t, C_e,t} P_{c,t} C_t(j) \]

subject to

\[ P_{c,t} C_t(j) = P_{e,t} C_{e,t}(j) + P_{q,t} C_{q,t}(j) \]

\[ C_t(j) := \left( (1 - x_c)^{1 - \sigma} C_{q,t}(j) + x_c^{1 - \sigma} C_{e,t}(j) \right)^{\frac{1}{\sigma}} \]

\[ P_{c,t} = \left( (1 - x_c) P_{q,t}^{\frac{\sigma}{\sigma-1}} + x_c P_{e,t}^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma-1}{\sigma}} \]

\[ C_{q,t}(j) = (1 - x_c) \left( \frac{P_{q,t}}{P_{c,t}} \right)^{\frac{1}{\sigma-1}} C_t(j) \]

\[ C_{e,t}(j) = x_c \left( \frac{P_{e,t}}{P_{c,t}} \right)^{\frac{1}{\sigma-1}} C_t(j) \]
Household

\[
\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t(j), L_t(j)) \right], \quad 0 < \beta < 1
\]

subject to

\[
P_{c,t} C_t(j) + P_{k,t} I_t(j) + B_t(j) \leq (1 + i_{t-1}) B_{t-1}(j) + W_t(j) L_t(j) + D_t + r_t^k P_{k,t} K_t(j) + T_t
\]
Household

\[ \text{Problem} \]

\[
\max_{\mathbb{E}_0} \left[ \sum_{t=0}^{\infty} \beta^t U(C_t(j), L_t(j)) \right], \quad 0 < \beta < 1
\]

subject to

\[
P_{c,t} C_t(j) + P_{k,t} I_t(j) + B_t(j) \leq (1 + i_{t-1}) B_{t-1}(j) + W_t(j) L_t(j) + D_t + r^k_t P_{k,t} K_t(j) + T_t
\]

\[
U(C_t(j), L_t(j)) = \log(C_t(j)) - \frac{L_t(j)^{1+\phi}}{1+\phi}
\]
Household

Problem

\[ \max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t(j), L_t(j)) \right], \quad 0 < \beta < 1 \]

s. t

\[ P_{c,t}C_t(j) + P_{k,t}I_t(j) + B_t(j) \leq (1 + i_{t-1})B_{t-1}(j) + W_t(j)L_t(j) + D_t + r_t^k P_{k,t}K_t(j) + T_t \]

\[ U(C_t(j), L_t(j)) = \log(C_t(j)) - \frac{L_t(j)^{1+\phi}}{1+\phi} \]

\[ I_t := K_{t+1} - (1 - \delta)K_t \]
Household's Optimization

1 = \beta E_t \left[ (1 + i_t) \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \right]

Euler

First Order Conditions

Fisher

\frac{W_t}{P_{c,t}} = C_t L_{t}^{\phi}

1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{k,t+1}}{P_{k,t}} (r_{t+1}^{k} + 1 - \delta) \right]
The Labor "Packer"

Labor "Packer"
The Labor "Packer"

Household labor service $j \in [0, 1]$

Labor "Packer"
Household labor service $j \in [0, 1]$

\[ L_t^d := \left( \int_0^1 L_t(j) \frac{e_w - 1}{e_w} \, dj \right) \frac{e_w}{e_w - 1} \]
The Labor "Packer"

Household labor service $j \in [0, 1]$

$L^d_t := \left( \int_0^1 L_t(j) \frac{\epsilon_w - 1}{\epsilon_w} \, dj \right) \frac{\epsilon_w}{\epsilon_w - 1}$

$\epsilon_w$: the elasticity of substitution among tips of labor
Labor "Packer" Problem

Labor "Packer" Profit Optimization

\[
\max_{L_t(j)} W_t L_t^d - \int_0^1 W_t(j) L_t(j) \, dj
\]

\[L_t^d = \left( \int_0^1 L_t(j) \frac{1}{\epsilon_w} \, dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}
\]

\[L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} L_t^d
\]

\[W_t = \left( \int_0^1 W_t^{1-\epsilon_w}(j) \, dj \right)^{\frac{1}{1-\epsilon_w}}
\]
Wage Optimization

Wage Maximization (at each date $t$) (Calvo Setting)

\[ W_t(j) = W_{t-1}(j) \quad \text{and} \quad W_t(j) = W_t^o(j) \]

\[ W_t = (\theta_w W_{t+1}^{1-\epsilon_w} + (1 - \theta_w) W_t^{o1-\epsilon_w}) \frac{1}{1-\epsilon_w} \]
Final Good Producers

Intermediate Good: $i \in [0, 1]$

$$Q_t = \left( \int_0^1 Q_t(i) \epsilon_p - 1 \epsilon_p \right) \epsilon_p \epsilon_p - 1 \epsilon_p$$: the elasticity of substitution among intermediate goods
Final Good Producers

Intermediate Good $i \in [0, 1]$

Final Good Firm

$Q_t = \left( \int_1^0 Q_t(i) \epsilon_p - \epsilon_p \right) \epsilon_p - \epsilon_p$: the elasticity of substitution among intermediate goods
Final Good Producers

Intermediate Good $i \in [0, 1]$

Final Good Firm

\[
Q_t = \left( \int_0^1 Q_t(i) \frac{\epsilon_p - 1}{\epsilon_p} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}
\]
Final Good Producers

Intermediate Good $i \in [0, 1]$

Final Good Firm

\[ Q_t = \left( \int_0^1 Q_t(i) \frac{\epsilon_p - 1}{\epsilon_p} \, di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \]

$\epsilon_p$: the elasticity of substitution among intermediate goods
Final Good Firm Profit Optimization

\[
\max_{Q_t(i)} P_{q,t} Q_t - \int_0^1 P_{q,t}(i) Q_t(i) di \\
\text{s. t.} \\
Q_t = \left( \int_0^1 Q_t(i) \frac{\epsilon_p - 1}{\epsilon_p} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}
\]

\[
Q_t(i) = \left( \frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t
\]

\[
P_{q,t} = \left( \int_0^1 P_{q,t}(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}
\]
Intermediate Good Firms’ CES Production Function

Intermediate Firms
Intermediate Good Firms’ CES Production Function

\[ Q_t(i) := (x_p(A_{E_t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_{t(i)}^\alpha L_t^{d(i)}(1-\alpha))^\rho)^{1/\rho} \]

\[ \rho = \frac{\eta_p - 1}{\eta_p} \]
Intermediate Good Firms’ CES Production Function

\[ Q_t(i) := (x_p(A_{E,t} E_t(i))^\rho + (1-x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^\rho)^{1/\rho} \]

\[ \rho = \frac{n_p - 1}{n_p} \]
Intermediate Good Firms’ CES Production Function

\[ Q_t(i) := (\chi_P(A_{E,t}E_t(i))^\rho + (1 - \chi_P)(A_{L,K,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^{\rho})^{1/\rho} \]

Given: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)

Choses: \( E_t(i), L_t(i) \) and \( K_t(i) \)

Strategy of firm \( i \)
Intermediate Good Firms’ CES Production Function

\[ Q_t(i) := (x_p(A_{E,t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^\rho)^{1/\rho} \]

Given: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)
Choses: \( E_t(i), L_t(i) \) and \( K_t(i) \)

\( \rho = \frac{\eta_p - 1}{\eta_p} \)

Given: prices and quantities
Choses: \( P_{q,t}(i) \)
Intermediate Good Firms

Intermediate Firms
Intermediate Good Firms

\[ Q_t(i) := (x_p(A_{E,t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^\rho)^{1/\rho} \]

\[ \rho = \frac{\eta_p - 1}{\eta_p} \]
Intermediate Good Firms

\[ Q_t(i) := (x_p(A_{E,t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^\rho)^{1/\rho} \]

\[ \rho = \frac{\eta_p - 1}{\eta_p} \]

strategy of firm \(i\)
Goals

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Intermediate Good Firms

Intermediate Firms

\[ Q_t(i) := (x_p (A_{E,t} E_t(i))^\rho + (1 - x_p) (A_{L_K,t} (K_t(i)^\alpha L_t^d(i)^{1-\alpha}))^{\rho})^{1/\rho} \]

\[ \rho = \frac{\eta_p - 1}{\eta_p} \]

Given: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)

Choses: \( E_t(i), L_t(i) \) and \( K_t(i) \)

strategy of firm \( i \)

cost minimization
Intermediate Good Firms

\[ Q_t(i) := (x_p(A_{E,t}E_t(i))^\rho + (1 - x_p)(A_{LK,t}(K_t(i)^\alpha L_t^d(i)^{1-\alpha})))^{\frac{1}{\rho}} \]

*Given*: \( P_{e,t}, P_{k,t}, W_t \) and \( Q_t(i) \)

*Choses*: \( E_t(i), L_t(i) \) and \( K_t(i) \)

\( \rho = \frac{\eta_p - 1}{\eta_p} \)

*Given*: prices and quantities

*Choses*: \( P_{q,t}(i) \)
Cost Minimization

Cost minimization

\[ MC_t := P_{e,t} \cdot \frac{x_p A_{E,t}^\rho Q_t(i)^{1-\rho} E_t(i)^\rho-1}{1 - \rho} \]

\[ = \frac{W_t}{(1 - \alpha)Q_t(i)^{1-\rho}(1-x_p)A_{LK,t}^\rho K_t(i)^{\alpha \rho} L_t^d(i)(1-\alpha)^\rho-1} \]

\[ = \frac{r_t^k P_{k,t} A_{LK,t}^\rho K_t(i)^{\alpha \rho-1} L_t^d(i)(1-\alpha)^\rho}{1 - \rho} \]

\[ cost(Q_t(i)) = MC_t Q(i) \]
Price Optimization

Price Maximization (at each date $t$) (Calvo Price Setting)

\[ P_{q,t}(i) = P_{q,t-1}(i) \]

\[ P_{q,t}(i) = P_{q,t}(i) \]

\[ P_{q,t} = \left( \theta_p P_{q,t-1}^{1-\epsilon_p} + (1 - \theta_p) P_{q,t}^{1-\epsilon_p} \right) \]
Calvo Price Setting

\[ P_{q,t}(i) = P_{q,t-1}(i) \]

\[ P_{q,t}(i) = P_{q,t}^o(i) \]

\[ P_{q,t} = (\theta P_{q,t-1}^{1-\epsilon} + (1 - \theta)(P_{q,t}^o)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \]
Calvo Price Setting Problem

\[
\max_{P_{q,t}(i)} E_t \left[ \sum_{k=0}^{\infty} \theta^k d_{t,t+k} \left[ P_{q,t}(i) Q_{t,t+k}(i) - \text{cost}(Q_{t,t+k}(i)) \right] \right]
\]

s.t

\[
Q_{t,t+k}(i) = \left( \frac{P_{q,t}(i)}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0
\]
Calvo Price Setting

Calvo Price Setting Solution

\[ \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \theta_p^k d_{t+k} Q_{t+k}^o \left( P_{q,t}^o - M^p m c_{t+k}^o \right) \right] = 0 \]

\[ d_{t,t+k}(j) := \beta^k \frac{\lambda_{t+k}(j)}{\lambda_t(j)} \]

\[ M_{t+k}^{o} := M_{t+k} \]

\[ Q_{t+k}^o := \left( \frac{P_{q,t}^o}{P_{q,t+k}^o} \right)^{-\epsilon_p} Q_{t+k} \]
Government
Government

\[ 1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \]
1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_{\pi}} \left( \frac{Y_t}{Y} \right)^{\phi_{y}} \epsilon_{i,t}

\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}

ln(\epsilon_{i,t}) = \rho_i ln(\epsilon_{i,t-1}) + e_{i,t}
1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \\

\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}} \\

(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t \\

\ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t}
Government

\[ \ln(G_{r,t}) = (1 - \rho_g)(\ln(\omega Q)) + \rho_g \ln(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t} \]

\[ 1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_{\pi}} \left( \frac{Y_t}{Y} \right)^{\phi_y} \varepsilon_{i,t} \]

\[ \Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}} \]

budget constraint

\[ (1 + i_{t-1})B_{t-1} + G_t = B_t + T_t \]

spending function

\[ \ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t} \]
Shocks

Government Spending Shock

\[
\ln(G_{r,t}) = (1 - \rho_g)(\ln(\omega Q)) + \rho_g \ln(G_{r,t-1}) \\
+ \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}
\]
Shocks

**Government Spending Shock**

\[
\ln(G_{r,t}) = (1 - \rho g) \ln(\omega Q) + \rho g \ln(G_{r,t-1}) \\
+ \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}
\]

**Monetary Policy**

\[
1 + i_t = \frac{1}{\beta} (\Pi_q,t) \phi_\pi \left( \frac{Y_t}{Y} \right) \phi_y \varepsilon_{i,t}
\]

\[
\ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t}
\]
Shocks

\[ S_{e,t} := \frac{P_{e,t}}{P_{q,t}} \]

\[ \log(S_{e,t}) = \rho_{s,e} \log(S_{e,t-1}) + e_{se,t} \]
Shocks

Oil Price

\[ S_{e,t} := \frac{P_{e,t}}{P_{q,t}} \]

\[ \log(S_{e,t}) = \rho_{s,e} \log(S_{e,t-1}) + e_{se,t} \]

Capital Price

\[ S_{k,t} := \frac{P_{k,t}}{P_{q,t}} \]

\[ \log(S_{k,t}) = \rho_{s,k} \log(S_{k,t-1}) + e_{sk,t} \]
Shocks

\[ \ln(A_{LK,t}) = \rho_a \ln(A_{LK,t-1}) + e_{alk,t} \]
Shocks

TFP

\[ \ln(A_{LK,t}) = \rho_a \ln(A_{LK,t-1}) + e_{alk,t} \]

Oil Productivity

\[ \ln(A_{E,t}) = \rho_a \ln(A_{E,t-1}) + e_{ae,t} \]
Other Shocks

Price Markup

\[ \varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1} \]

Wage Markup

\[ \varepsilon_{w,t} = \rho_w \varepsilon_{w,t-1} + e_{w,t} - \nu_w e_{w,t-1} \]
Definition of Equilibrium

Equilibrium
Definition of Equilibrium

Agents maximize its problems

- All markets clear
- Equilibrium
- Government budget constraint fulfilled
## Data

<table>
<thead>
<tr>
<th>Observed Variable</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>cqobs</td>
<td>$detrend\left( ln\left( \frac{PCE - PCE_{energy}}{GDPDEF} \right) \right) * 100$</td>
</tr>
<tr>
<td>invobs</td>
<td>$detrend\left( ln\left( \frac{PFI}{GDPDEF} \right) \right) * 100$</td>
</tr>
<tr>
<td>yobs</td>
<td>$detrend\left( ln\left( \frac{GDPC09}{LNSIndex} \right) \right) * 100$</td>
</tr>
<tr>
<td>wrobs</td>
<td>$detrend\left( ln\left( \frac{Hourlycompensation}{LNSIndex} \right) \right) * 100$</td>
</tr>
<tr>
<td>labobs</td>
<td>$ln\left( \frac{Averagehours<em>CE16OVIndex}{LNSIndex} \right) * 100 - mean\left( ln\left( \frac{Averagehours</em>CE16OVIndex}{LNSIndex} \right) \right) * 100$</td>
</tr>
<tr>
<td>infobs</td>
<td>$ln\left( \frac{GDPDEF}{GDPDEF(-1)} \right) * 100 - mean\left( ln\left( \frac{GDPDEF}{GDPDEF(-1)} \right) \right) * 100$</td>
</tr>
<tr>
<td>iobs</td>
<td>$(ln(1 + \frac{FEDFUND}{400}) - mean(ln(1 + \frac{FEDFUND}{400}))) * 100$</td>
</tr>
<tr>
<td>eobs</td>
<td>$ln\left( \frac{TotalSAOil}{LNSIndex} \right) * 100 - mean\left( ln\left( \frac{TotalSAOil}{LNSIndex} \right) \right) * 100$</td>
</tr>
</tbody>
</table>
No Ponzi Scheme

Transversality condition (no Ponzi Scheme)

\[
\lim_{k \to \infty} \mathbb{E}_t \left( \frac{B_{t+k}}{t+k-1} \prod_{s=0}^{t+k-1} \left(1 + i_{s-1}\right) \right) \geq 0, \quad \forall t.
\]
Calibrated Parameters

<table>
<thead>
<tr>
<th>Elasticty</th>
<th>$\alpha_e$</th>
<th>0.013</th>
<th>0.07</th>
<th>0.1</th>
<th>0.3</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_p$</td>
<td>0.05</td>
<td>0.135</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table: Set of starting values