Robust Optimal Taxation and Environmental Externalities

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Outline

1. Introduction
2. One-Energy-Sector Model
3. Complete Model
4. Numerical Results
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2. One-Energy-Sector Model
3. Complete Model
4. Numerical Results
Environmental Externality: $CO_2$ Emission $\rightarrow$ Global Temperature $\rightarrow$ output.

Model Uncertainty: Our knowledge is limited regarding how $CO_2$ emission affects future global mean temperature and output.

Robust Control: The fear of catastrophic events alters the “optimal” path of energy extraction and thereby production and consumption.

We want to find out:

- Robust version of optimal energy extraction
- How to achieve it in a decentralized economy
Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2012):

1. Assume that the mapping from carbon concentration to output damages is subject to a risk $\gamma$: $y = e^{-\gamma S \tilde{y}}$ (the true distribution of $\gamma$ is known).

2. Obtain analytical expressions for carbon externality.

3. Assess optimal use of energy based on average performance (w.r.t. the know distribution)

4. Optimal taxation: A Pigouvian tax on carbon emission with a rate equal to the marginal externality of carbon.
Hansen and Sargent (2002, 2008) — Robust Control:

1. Decision maker does not know the true distribution of $\gamma$, but has an approximating one in his mind.

2. Only concerns about social (individual) welfare in the worst case scenario.

3. Chooses the worst case distribution of $\gamma$ around a neighborhood centered at the approximating one.

4. Uses “entropy” as a measure of the deviation from the approximating model.

5. Linear-Quadratic-Gaussian world.
Our Contribution

To the Literature of Environmental Externality:

1. Incorporate model uncertainty into GHKT (2012) and characterize optimal allocation and tax as functions of the size of model uncertainty.

2. The concern of robustness matters both qualitatively and quantitatively.

To the Literature of Robust Control:

1. Study a dynamic model outside the LQG world.

2. Develop a simple method which divide the model into two parts and solve them separately.
Outline

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2 One-Energy-Sector Model

3 Complete Model

4 Numerical Results
Basic Setup

- **Preference:**
  \[ E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \]

- **Final Good Production:**
  \[ F(K, E) = K^\theta E^\nu, \quad \theta + \nu \leq 1 \]
  where \( E \) denotes energy input measured in its carbon content. In addition, the extraction cost of \( E \) is zero.

- **Capital Accumulation (with 100 percent depreciation):**
  \[ \tilde{K}' = F(K, E) + (1 - \delta)K - C \]

- **Law of motion of atmospheric carbon concentration:**
  \[ S' = S + \phi_0 E \]
Introducing a stochastic variable $\gamma$, which reduces the end-of-period capital stock $\tilde{K}'$ by a factor of $h(S', \gamma) = e^{-S'\gamma}$ to $K'$. That is,

$$K' = h(S', \gamma)\tilde{K}',$$

The approximating distribution of $\gamma$ is $\pi(\gamma) = \lambda e^{-\lambda \gamma}$. Let $\hat{\pi}(\gamma)$ be an alternative distribution, and $m(\gamma) = \frac{\hat{\pi}(\gamma)}{\pi(\gamma)}$ be the likelihood ratio.

The “distance” between $\hat{\pi}(\gamma)$ and $\pi(\gamma)$ is measured by relative entropy $\rho(\hat{\pi}(\gamma), \pi(\gamma)) \equiv \int [m(\gamma) \log m(\gamma)] \pi(\gamma) d\gamma$. 
We first focus on a robust version of social planner’s problem and then show that the robust social optimal allocation is implemented via a Pigouvian tax in a decentralized economy.

Two-person zero-sum dynamic game: In any period,
- The beginning-of-the-period capital $K$ is given. The planner chooses $E$, production takes place and the planner chooses $C$.
- $\tilde{K}'$ and $S'$ evolve according to $\tilde{K}' = F(K, E) + (1 - \delta)K - C$ and $S' = S + \phi_0 E$, respectively.
- The malevolent player chooses an alternative distribution $\hat{\pi}(\gamma)$ or, equivalently, $m(\gamma)$, to minimize welfare. Accordingly, the next period $K' = h(S', \gamma)\tilde{K}'$.
A Zero-Sum Dynamic Game (Continued)

(Elaborate)

\[
V(K, S) = \max_{\{C, E, \tilde{K}', S'\}} \min_{m(\gamma)} \{ u(C) \} \\
+ \beta \int [m(\gamma)V(K', S') + \alpha m(\gamma) \log m(\gamma)] \pi(\gamma) d\gamma \\
\text{s.t.} \\
\tilde{K}' = F(K, E) - C \\
K' = h(S', \gamma)\tilde{K}' \\
S' = S + \phi_0 E \\
1 = \int m(\gamma)\pi(\gamma) d\gamma
\]
A Zero-Sum Dynamic Game (Continued)

- \( \int \alpha m(\gamma) \log m(\gamma) \pi(\gamma) d\gamma = \alpha \rho(\hat{\pi}(\gamma), \pi(\gamma)) \): Deviation from the approximating distribution will be penalized by adding \( \alpha \rho(\hat{\pi}(\gamma), \pi(\gamma)) \) to the objective function.

- A greater \( \alpha \) means a greater penalty associated with the deviation of \( \gamma \) from its approximating distribution, thus, a less concern about robustness.
Robustness (the inner minimization problem)

Define the robustness part of the problem by

\[ R(V)(K', S') = \min_{m(\gamma)} \int \left[ m(\gamma) V(K', S') + \alpha m(\gamma) \log m(\gamma) \right] \pi(\gamma) d\gamma \]

s.t.

\[ K' = e^{-S' \gamma} \tilde{K}' \]

\[ 1 = \int m(\gamma) \pi(\gamma) d\gamma \]
Suppose \( V(\cdot) \) takes the form \( V(K', S') = f(S') + \bar{A} \log(K') + \bar{D} \), and substitute it into the above problem, we obtain:

\[
m^*(\gamma) = (1 - \Delta S') e^{\Delta \lambda S' \gamma}
\]

and

\[
R(V)(\tilde{K}', S') = f(S') + \bar{A} \log(\tilde{K}') + \bar{D} + H(S'; \alpha, \bar{A})
\]

where \( \Delta = \frac{\bar{A}}{\alpha \lambda} \) and \( H(S'; \alpha, \bar{A}) = \alpha \log(1 - \Delta S') \) is the robust version externality of carbon.
Optimal Choice (the outer maximization problem)

With the above results, the maximization problem can be written as

$$V(K, S') = \max_{\{C,E,\tilde{K}',S'\}} \{ \log(C) + \beta R(V)(\tilde{K}', S') \}$$

or equivalently,

$$f(S') + \bar{A} \log(K) + \bar{D} = \max_{C,E} \{ \log(C) + \beta [f(S') + \bar{A} \log(\tilde{K}') + \bar{D} + H(S'; \alpha, \bar{A})] \}$$

subject to

$$\tilde{K}' = F(K, E) - C$$

$$S' = S + \phi_0 E$$
Proposition 1:
The two-person zero-sum dynamic game described by admits a unique feedback (Markov perfect) equilibrium, in which the equilibrium strategies are given by:

\[ C^* = (1 - \beta \theta) K^\theta E^* \nu = (1 - \beta \theta) K^\theta [c_E(1 - \Delta S)]^\nu \]
\[ E^* = c_E(1 - \Delta S) \]
\[ S'^* = S + \phi_0 c_E(1 - \Delta S) \]
\[ \hat{\pi}^*(\gamma) = m^*(\gamma) \pi(\gamma) = \lambda^* e^{-\lambda^* \gamma} \]

where \( \lambda^* = \lambda(1 - \Delta S'^*) = \lambda(1 - \Delta \phi_0 c_E)(1 - \Delta S) \) and \( c_E = \frac{\nu(1-\beta)}{[\beta \alpha(1-\beta) + \nu] \phi_0 \Delta} \) and \( \Delta = \frac{\bar{A}}{\alpha \lambda} \).
Remarks

- $V(K, S')$ is increasing in $K$, decreasing in $S$, and jointly concave in $K$ and $S$.

- A greater concern of robustness ($\alpha \downarrow$) is going to lower $E^*$, $C^*$ and $S'^*$.

- The worst case distribution is also exponential, 
  \[ \hat{\pi}^*(\gamma) = \lambda^* e^{-\lambda^* \gamma}. \]
  Since $\lambda^* < \lambda$, the worst case mean of $\gamma$, 
  \[ (\lambda^*)^{-1}, \]
  is strictly greatly than the approximating mean, $\lambda^{-1}$. 
The marginal externality of Carbon emission, measured in the unit of utility and evaluated at $E^*$, is given by

$$\lambda_s = -\beta \frac{\partial V(K', S')}{\partial E} \bigg|_{K', S'} = \frac{\nu}{c_E(1 - \beta \theta)(1 - \Delta S)} = \frac{\nu}{(1 - \beta \theta)E^*}.$$ 

The marginal externality of Carbon emission, measured in the unit of consumption goods and evaluated at $E^*$, is given by

$$\Lambda_s = \frac{\lambda_s u'(C^*_t)}{u'(C^*_t)} = \nu K^\theta E^* \nu^{-1}.$$ 

The marginal externality of Carbon emission, measured as a percentage of output and evaluated at $E^*$, is given by

$$\hat{\Lambda}^s = \frac{\Lambda_s}{F(K, E)} = \frac{\nu}{E^*}.$$
We now turn to the decentralized problem. Suppose the government imposes a percentage tax $\tau_t$ on emissions, $E_t$.

\[
V(k, K, S) = \max_{c, k'} \min_{\pi(\gamma)} \left\{ u(c) + \beta \tilde{E}_\gamma \left[ V(k', K', S') + \alpha \log \left( \frac{\tilde{\pi}(\gamma)}{\pi(\gamma)} \right) \right] \right\}
\]

s.t.
\[
c + \tilde{k}' = r(K, S)k + \tau(K, S)E(\tau) + \pi^{profit} \\
\tilde{K}' = G(K, S) \\
k' = k' - \gamma S' \tilde{k}' \\
K' = K' - \gamma S' \tilde{K}' \\
S' = S + \phi_0 E(S)
\]
Proposition 2

Suppose the government sets $E = c_E (1 - \Delta S)$ or, equivalently, $\tau_t = \Lambda^S$, and the tax proceeds are rebated lump-sum to the representative consumer. Then the competitive equilibrium allocation coincides with the solution to the robust planner’s problem. That is, $c^* = C^* = (1 - \beta \theta) K^\theta [c_E (1 - \Delta S)]^\nu$. 
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In this section, we extend the analytical one-sector model as follows:

- **Three Energy Sectors:**

  1. The oil sector produces oil \( E_1 \) at zero cost but is subject to a resource feasibility constraint, \( R_0 > 0 \).

  2. The coal and the green energy sector use linear technologies \( E_i = A_i N_i; \ i = 2, 3 \). The stock of coal is assumed to be infinity.

  3. \( A_2 N_2 \) and \( A_3 N_3 \) grow at a rate of two percent per year.
The composite energy is produced by

\[ E = (\kappa_1 E_1^\rho + \kappa_2 E_2^\rho + \kappa_3 E_3^\rho)^{1/\rho}. \]

As in GHTK, suppose \( S = P + T \) where \( P \) and \( T \) are the permanent and temporary components of \( S \), respectively.

\[ P' = P + \phi_L (E_1 + E_2) \text{ and } \]
\[ T' = (1 - \phi)T + (1 - \phi_L)\phi_0 (E_1 + E_2). \]
Now assume that the approximating distribution of $\gamma$ is normal with mean $\bar{\gamma}$ and variance $\sigma^2$; i.e., $\pi \sim \mathcal{N}(\bar{\gamma}, \sigma^2)$. It follows from the inner-minimization problem that

$$
\hat{\pi}^*(\gamma) \sim \mathcal{N}(\bar{\gamma} + \frac{\bar{A}\sigma^2}{\alpha} S'^2, \sigma^2)
$$

$$
H(S'; \alpha, \bar{A}) = - (\bar{\gamma} + \frac{\bar{A}\sigma^2}{2\alpha} S') \bar{A} S'
$$
Social Planner

$$f(N, P, T, R) = \max_{E_1, E_2, E_3, E, P', T', S', R'} \left\{ \frac{1}{1 - \beta\theta} \log\left[ \left( 1 - \frac{E_2}{A_2 N} - \frac{E_3}{A_3 N} \right)^{1 - \theta - \nu} E^\nu \right] + \beta[f(N', P', T', R') + H(S'; \alpha, \bar{A})] \right\}$$

s.t.

$$E = (\kappa_1 E_1^\rho + \kappa_2 E_2^\rho + \kappa_3 E_3^\rho)^{1/\rho}$$

$$N' = (1 + g) N$$

$$R' = R - E_1 \geq 0$$

$$P' = P + \phi_L (E_1 + E_2)$$

$$T' = (1 - \phi) T + (1 - \phi_L) \phi_0 (E_1 + E_2)$$

$$S' = P' + T'$$
We calculate the marginal externalities caused by $P$ and $T$ respectively, and show that the externality caused by Carbon emission is a weighted sum of the two.

\[
\hat{\Lambda}^P = -(1 - \beta \theta) \frac{\partial f}{\partial P} = \theta \bar{\gamma} \sum_{j=1}^{+\infty} \frac{\beta^j}{1 - \Delta S_{t+j}}
\]

\[
\hat{\Lambda}^T = -(1 - \beta \theta) \frac{\partial f}{\partial T} = \theta \bar{\gamma} \sum_{j=1}^{+\infty} \frac{[\beta(1 - \phi)]^j}{1 - \Delta S_{t+j}}
\]

\[
\hat{\Lambda}^S = \phi_L \hat{\Lambda}^P + \frac{(1 - \phi_L) \phi_0}{1 - \phi} \hat{\Lambda}^T
\]

\[
\lim_{\alpha \to +\infty} \hat{\Lambda}^S_t = \theta \bar{\gamma} \left[ \frac{\phi_L \beta}{1 - \beta} + \frac{(1 - \phi_L) \phi_0 \beta}{1 - (1 - \phi) \beta} \right]
\]
FONC and Externalities

(skip) FONC’s:

\[
\frac{\nu \kappa_1}{E_1^{1-\rho} E^{\rho}} - \hat{\Lambda}^S = \beta \left[ \frac{\nu \kappa_1}{(E_1')^{1-\rho}(E')^\rho} - (\hat{\Lambda}^S)' \right], \quad (\partial E_1)
\]

\[
\frac{\nu \kappa_2}{E_2^{1-\rho} E^{\rho}} - \hat{\Lambda}^S = \frac{1 - \theta - \nu}{A_2 N_0}, \quad (\partial E_2)
\]

\[
\frac{\nu \kappa_3}{E_3^{1-\rho} E^{\rho}} = \frac{1 - \theta - \nu}{A_3 N_0}. \quad (\partial E_3)
\]
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3 Complete Model

4 Numerical Results
## Calibration

### Table: Calibration Summary

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Optimal Energy Path when $R_0 = 253.8GtC$ (The same as in GHKT)

**Figure:** Optimal Use of Energy when $R_0 = 253.8$
Optimal Energy Path when $R_0 = 8000\, GtC$ (Including Methane)

**Figure:** Optimal Use of Energy when $R_0 = 8000$
Optimal Energy Path when \( R_0 = \infty \)

**Figure:** Optimal Use of Energy when \( R_0 = \infty \)
More Graphs for $R_0 = \infty$

Figure: Global Average Temperature when $R_0 = \infty$
More Graphs for $R_0 = \infty$ (Continued)

Figure: The Path of Output and Capital when $R_0 = \infty$
Thank You!