

# Two-sided and Non-Convex Markets for Electricity Markets

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## I. OVERVIEW

Several markets present non-convexities due to their intrinsic characteristics, including start-up and shut-down costs, indivisibilities, economies of scale and minimum supply requirements, amongst others. These non-convexities create challenges for finding prices that support a market equilibrium. In this research, we focus on the effect of these non-convexities and potential quadratic costs that affect the market prices. We focus on deregulated electricity markets, with an independent system operator (ISO) receiving offers about the marginal generation costs and technical information including fixed commitment costs and minimum dispatch conditions. We incorporate the possible ramping costs, solving the mixed integer linear program (MILP) associated to the ISO's unit commitment problem and obtaining a set of market clearing prices that satisfies incentive compatibility.

This Walrasian market equilibrium conditions assure that suppliers would not want to change their energy dispatch at the solved prices. With these results in hand, we perform numerical experiments to show the impact of ramping costs on the clearing prices of a market with non-convexities.

## II. METHODS

We start with a classical formulation of the economic dispatch problem with unit commitment for the ISO. We use this original MILP formulation and reformulate it as a mixed integer second order cone program (MISOCP). This transformed problem has a linear objective and linear constraints, in addition to integer variables. One advantage of this reformulation is that commercial packages can be efficiently solved. By finding the optimal values of the MISOCP, we can relax the integer variables and obtain a second order conic program (SOCP). The key characteristic of the SOCPs we propose is that these are convex problems, which allows us to get the duals of the problem and therefore the shadow prices that constitute a Walrasian market equilibrium.

## III. RESULTS

Our results are presented as a consecutive sequence of models, starting with a classical problem (Scarf, 1994) that has startup and shutdown of generators. We compute and analyze the clearing prices associated to this classical Scarf problem, with two types of plants in the electricity market. We show that the presence of ramping affects the market clearing prices, and the optimal commitment of units to account for the flexibility provided and its tradeoff with the fuel cost. We measure the social welfare in each case and present the effects on the dispatch and operation of the electricity system

## IV. CONCLUSIONS

Our results show that using convex optimization techniques we can find a set of market clearing prices that more efficiently allocates resources in the case of instances characterized by deliverability costs and integer variables. This optimal allocation problem is particularly important for electricity markets due to the increased penetration of renewable energy sources, and to engage the demand side of the problem. The variability and uncertainty in the availability of these renewable sources require conventional generators to ramp up and down more frequently. In the

context of electricity markets, demand resources also present a number of characteristics that include deliverability and non-convexity. Hence, better prices can improve the efficiency of the market.

#### NOTES

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#### REFERENCES

- Scarf, H. 1994. "The Allocation of Resources in the Presence of Indivisibilities." *Journal of Economic Perspectives* 8(4):111–128.