Energy Real Option Valuation

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Direct Energy
Simple. Friendly. Direct.

Introduction

- Flexibility has value
- Operational and Strategic Flexibility
- How valuable is flexibility?
- How can this value be quantified?
- Different types of flexibility and how they interact

Introduction (cont.)

- Financial Options Research and framework is used for valuation of Flexibility
  - Comprehensive Approach, that transfers knowledge from Finance to resource allocation and planning decisions
  - Complexity
- The term “real option valuation” is used to describe a whole range of mathematical techniques used to assign value to managerial flexibility or the valuation of physical assets
- The valuation of real option deals requires combination of financial derivatives pricing and operations research methodologies

Agenda

- Financial Options (Black 76)
- Real Options Basics
- Risk Free Rate, CAPM & Hurdle Rate
- Real Options in the Energy Industry
  - Wildcatter Inc. (Decision Analysis)
  - Storage Valuation
  - Power Plant Valuation
    - Spark Spread: Margrabe
    - Jump Diffusion
    - Valuation
- Conclusions

FINANCIAL OPTIONS

Options Definition

- An option is “the right but not the obligation” to do something
  - Opt: to choose
- A financial option is “the right but not the obligation to buy (or sell) an asset”
  - The option buyer receives the right and pays premium
  - The option seller transfers the right and receives premium
  - Call option
    - right but not obligation to buy a certain quantity (of stock, commodity etc.) at a fixed price in the future
  - Put option
    - right but not obligation to sell a certain quantity (of stock, commodity etc.) at a fixed price in the future

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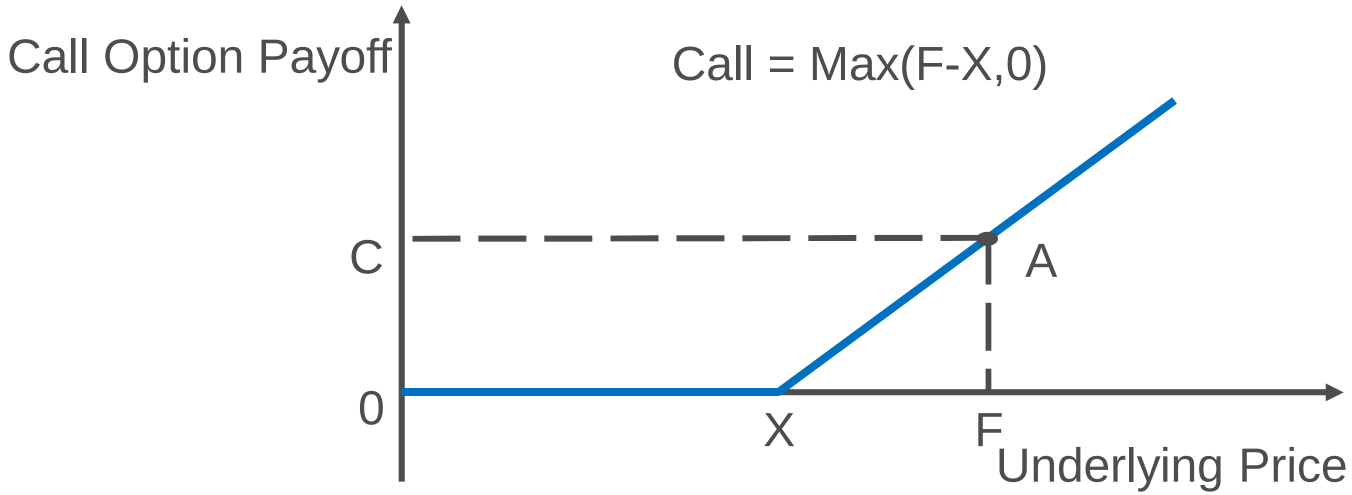
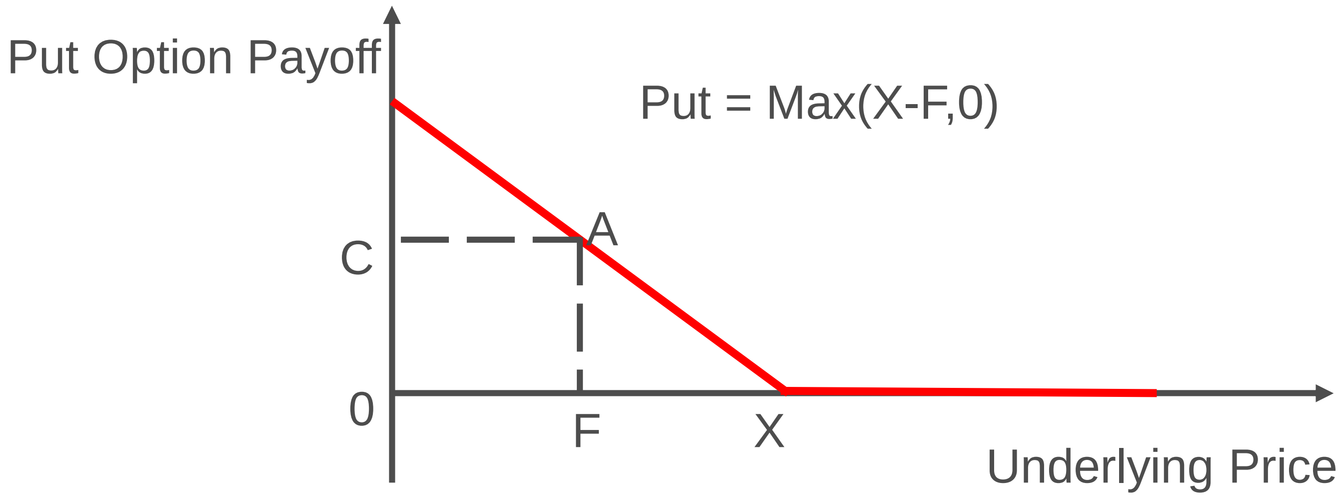
Call & Put Options Payoff





Black76 assumptions

- European, no Dividends
- Efficient markets (i.e., market movements cannot be predicted)
- Liquid markets
- No transaction costs
- Risk-free interest rate
- Constant Volatility
  - Annualized Standard Deviation
- Log-Normal Property of Commodities Prices
  - The natural logarithm of the price is Normally Distributed
  - The log-returns on the prices are Normally Distributed
- Expected Return of Underlying is zero (Martingale)

Black76 Model

$$\text{Call} = e^{-rT} \cdot \left[F \cdot N(d_1) - X \cdot N(d_2)\right]$$

$$\text{Put} = e^{-rT} \cdot \left[X \cdot N(-d_2) - F \cdot N(-d_1)\right]$$

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + 0.5 \cdot S^2}{S}$$

$$d_2 = \frac{\ln\left(\frac{F}{X}\right) - 0.5 \cdot S^2}{S} = d_1 - S$$

$$S^2 = \sigma^2 \cdot T$$

$$\text{Black}76 = f(F, X, T, \sigma, r)$$

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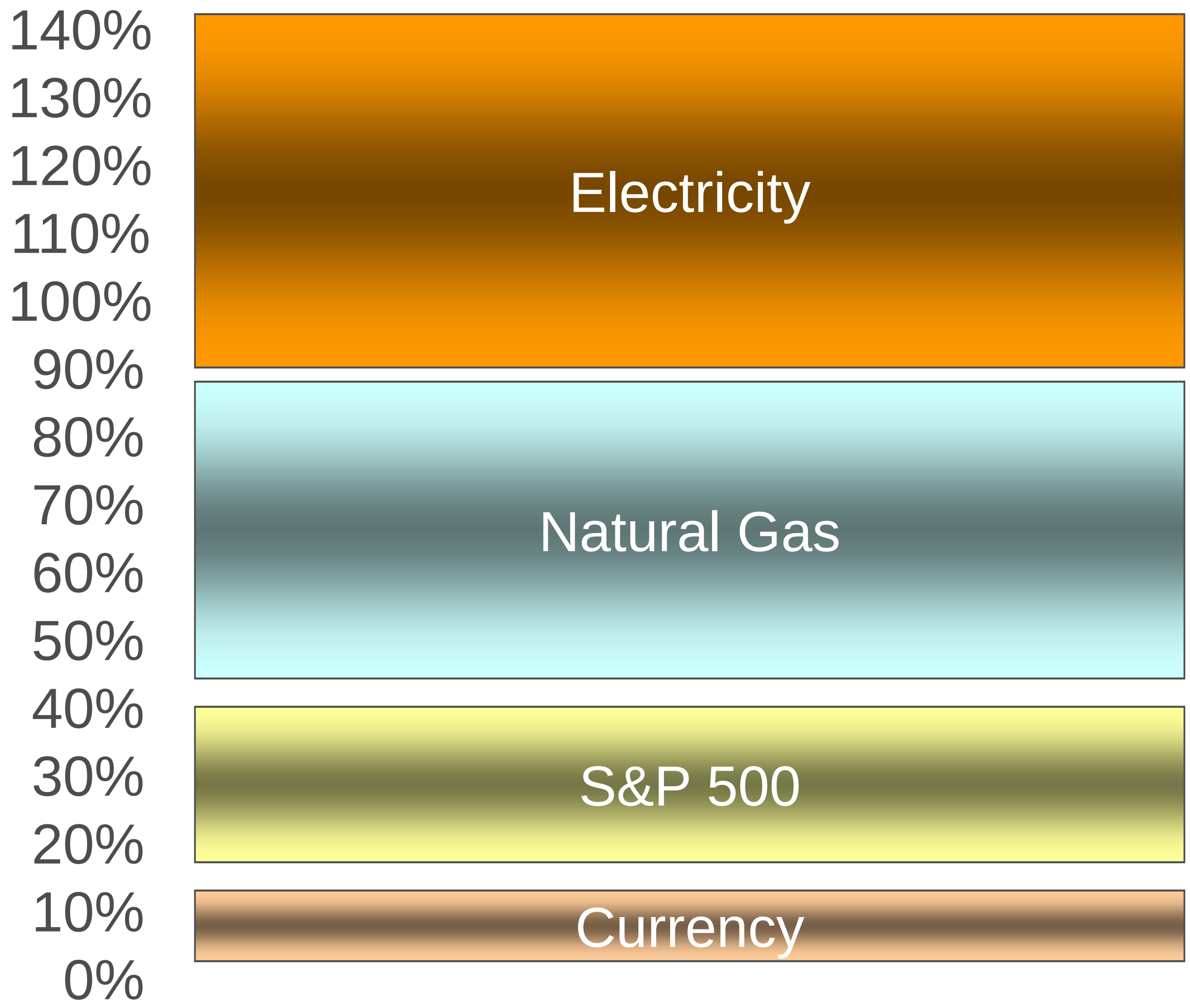
Volatility

- Moments of probability Distributions
  - Mean, Variance/Volatility, Skewness, Kurtosis etc.
- Volatility: Proxy for probability distribution of price/price returns
  - Normal Distribution
  - Log-Normal Distribution

Sources of Risk: Average Historical Price Volatility



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Black76 Model (cont.)

$$\text{Call} = e^{-rT} \cdot [F \cdot N(d_1) - X \cdot N(d_2)]$$

$$\text{Call} = e^{-rT} \cdot \left[F \cdot \frac{N(d_1)}{N(d_2)} - X \right] \cdot N(d_2)$$

$$N(d_2) = \Pr(F_T > X) =$$ Probability that the option will expire in the money

$$F \cdot \frac{N(d_1)}{N(d_2)} =$$ Conditional Expected Value of the underlying at expiration, on condition that the underlying expires in the money

$$F \cdot \frac{N(d_1)}{N(d_2)} = E(F_T / F_T > X)$$

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REAL OPTIONS BASICS

Real Options: Valuing Flexibility

- Managers have the right to revise decisions and adapt them in response to new market conditions and information or unexpected developments.
- Such flexibility is clearly valuable and should be accounted for in the valuation of a project or firm.
- The “Real Options Approach” focuses on finding the value of managerial flexibility and attaching a number to it.
- For Example:
  - Managers can expand or contract production in response to changes in demand.
  - Real Estate: Develop, or redevelop owned property, lease vs. sell
  - Power Markets: Demand Response
  - Airlines: Exit Row, Upgrades

Two Steps in Real Options Analysis:

- Identification
  - Are there real options imbedded in a given project?
  - What type of options?
  - Are they important?
- Valuation
  - How do we value the (important) options?
  - How do we value different types of options?
  - What methodology do we use?

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Identifying Real Options

- It is important to identify the options imbedded in a project, business process or service
- There are options imbedded in all but the most trivial projects.
  - Complex Projects
  - Projects over a long period of time
- The most crucial skill consists of:
  - Identifying those options that are "significant"
  - Ignoring those that are not.
- Identifying real options takes:
  - Practice
  - Vision
  - Management participation and buy-in

Is this a Real Option?

- Two conditions:
  - New Information will arrive in the future as uncertainty about market conditions is gradually resolved
  - This new information will affect or trigger further decisions.
- Identify the uncertainty that managers face:
  - What is the main thing that managers will learn over time?
  - What are the uncertainties that will be resolved?
  - How will they best use the new information?
  - Does new information affects future decisions?
  - What decisions will change as a result of the new information?
- “Phases”, “Review”, “Strategic investment”, “Scenarios”, …
- Pattern of cash flows and expenditures over time. For instance, large expenditures are likely to be discretionary.

Summary of Real Option Categories & Examples

Category	Description	Important in:
Option to Defer	Management holds a lease on (or an option to buy) land or resources. Can wait x years has opportunity to wait to invest, build, develop if/when market is favorable.	Natural resources extraction, real estate, farming, technology.
Time-to-build Option (Staged Investment)	Arrange investment as a series of smaller self-contained projects. Option to reevaluate and/or abandon at each stage if new information is unfavorable. Each stage is an option on the value of the subsequent stages.	R&D intensive industries, Large Scale projects, energy generation, start-up ventures.
Option to alter operating scale (i.e. to expand, contract, shut down, restart)	If market conditions change, the firm can expand/contract, accelerate production/extraction or temporarily shut down. i.e. Add a third shift at a factory. Slow steaming a tanker. Slow down rate of mineral extraction from a mine.	Natural resources (i.e. mining), real estate, fashion, consumer goods, cyclical industries
Option to abandon	If market conditions decline, management can abandon project and sell off assets in secondary markets. Cut your losses. Do not throw good money after bad.	Capital-intensive industries (i.e. airlines), financial services, new product introductions in uncertain markets.
Option to switch (i.e. inputs or outputs)	If prices or demand change, management can change product mix (product flexibility) or switch inputs (process flexibility)	Companies in volatile markets with shifting preferences, electric power companies, chemicals etc.
Growth options	Follow-up Investment. An early investment (R&D, lease on undeveloped land etc.) opens up future growth opportunities in the form of new products or processes, access to markets, or strengthening of core capabilities. Growth options can be “nested”. They are perceived to have “Strategic Value”	Movie Sequels, High tech; industries with multiple product generations (drug companies, computers, strategic acquisitions).
Multiple Interacting Options	Projects involve a collection of various options upward potential enhancing and downward protection options. Values can differ from the sum of individual option values because they interact.	Real life projects in many of the industries discussed above

Detail vs. Simplification

- Real projects, especially long-horizon ones, are complex:
  - They often combine existing assets and future options.
  - Options are often nested.
- Simplifying assumptions are needed:
  - To keep the model flexible.
  - To keep the model easy to communicate to decision makers

Detail vs. Simplification, Valuation Upper & Lower Bounds

- What should you do?
  - Divide the project into phases corresponding to simple options.
  - Search for the primary uncertainty that managers face.
  - A simplified model that dominates the project gives an upper bound for the project's value.
    - i.e. ignore transaction costs
- Begin by valuing the project as if there was no option involved, i.e., as if all decisions had to be taken immediately.
  - This benchmark is a lower bound for the project's value.
  - Examples:
    - Use European rather than American options.
    - Ignore some of the least important options.

Valuation of Real Options

- The tools developed to value financial options (i.e. calls and puts on stocks and commodities) can be used to estimate the value of real options embedded in some projects.
  - Black-Scholes is often used to value real options.
- Challenge: Real options are much more complex than financial options.
- Solution: Simplify real options to fit them into the valuation models for financial options.
- The aim is to develop numerical techniques to facilitate the decision-making process, not to replace sound business sense.

Valuation Methodologies

- DCF
- Analytic methods, i.e. Black-Scholes
- Monte Carlo simulation
- Tree Analysis (or forest analysis, as a more computationally intensive version)
- Numerical Analysis methods, i.e. numerical solutions for PDEs
- Decision Analysis
- Dynamic Programming and other optimization methods
- Heuristic methods
- Combination of the above

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RISK FREE RATE, CAPM & HURDLE RATE

Risk-Neutral Valuation & Hurdle Rate

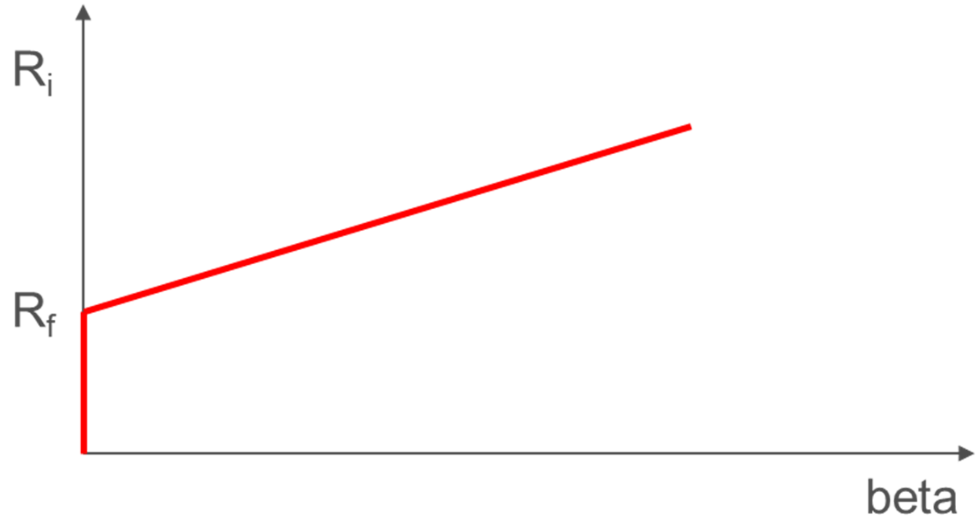
- Publicly traded Securities
  - Replicating Portfolio and Market Liquidity
  - Risk Neutrality
- One point of View:
  - Real Options "should" be valued using Risk-Neutrality as we are interested in their contribution to the "Market" Value of the Company
- But wait ..
- Corporate Practice
  - "Hurdle Rate"

CAPM

- Capital Asset Pricing Model
- Investors seek compensation for:
  – Time value of money (R_f)
  – Risk Exposure (beta)
- Investors seek compensation for asset not-diversifiable risk in the form of risk premium, above and beyond the risk free rate
- Beta is statistically derived from public data
  – bias towards public companies
  – riskiness of companies, vs. riskiness of projects

CAPM and the cost of capital

- Adjusts discount rate to risk exposure proxy (beta)
- $R_i = R_f + beta * (R_m - R_f)$
- $beta = Corr_{i,m} * vol_i / vol_m$
- Example:
  - R_f =2.5%, R_m =8%, beta =1.5
  - R_i = 10.75%
  - Rate of return below 10.75% under-compensates for the risk assumed



REAL OPTIONS IN THE ENERGY INDUSTRY

Real Options Popularity in Energy

- Homer City coal power plant
  - 1,884 megawatt coal-fired plant, located in PA
  - Around 2000 it sold for a price of 3X per kilowatt of capacity, than similar plants.
- Reason?
  - Location: into NY market
  - Location: into PA market
  - Location: into OH market
- Benefit from price discrepancies in the three regions
- This extrinsic/real option value was about two thirds of the value of the plant.

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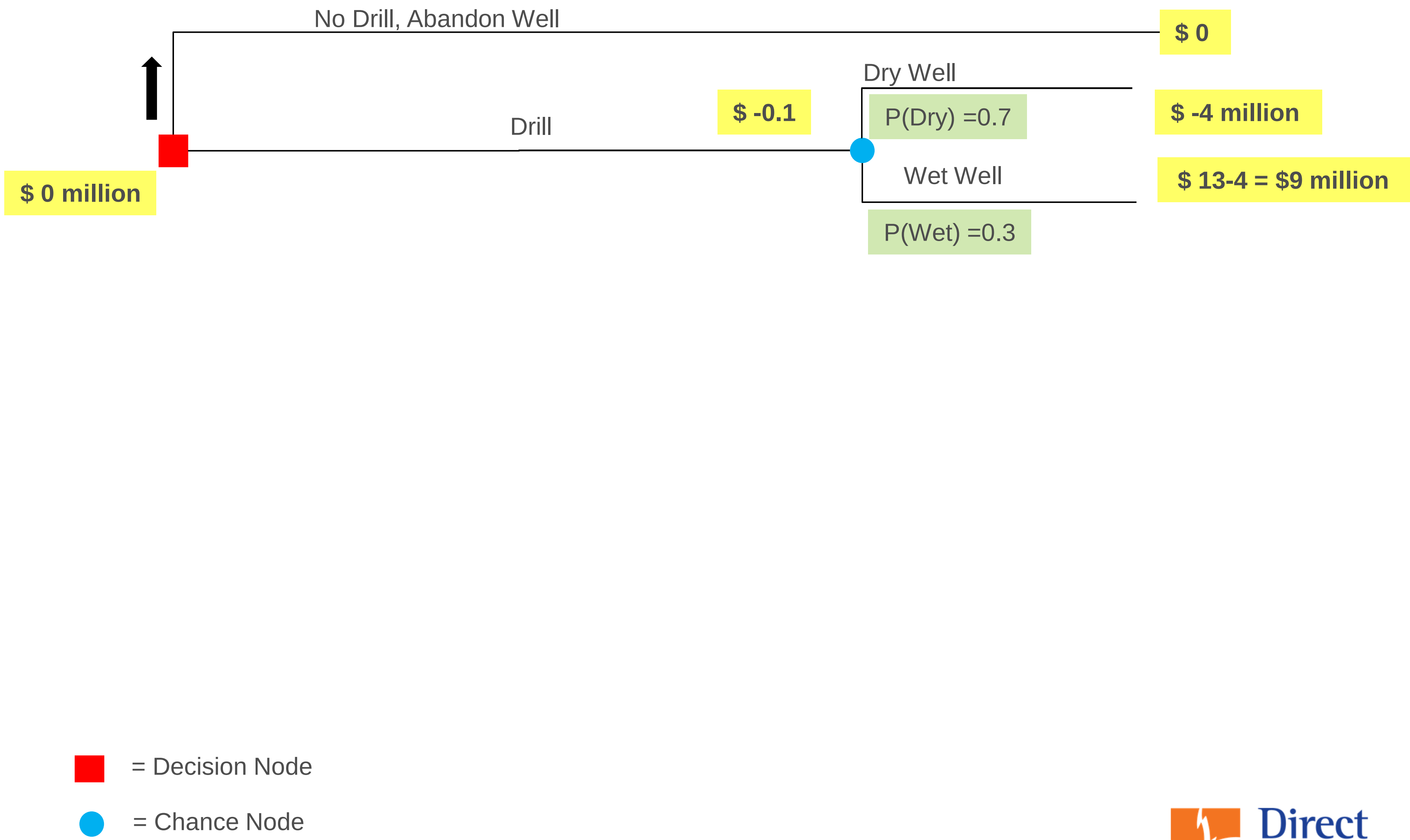
WILDCATTER INC.

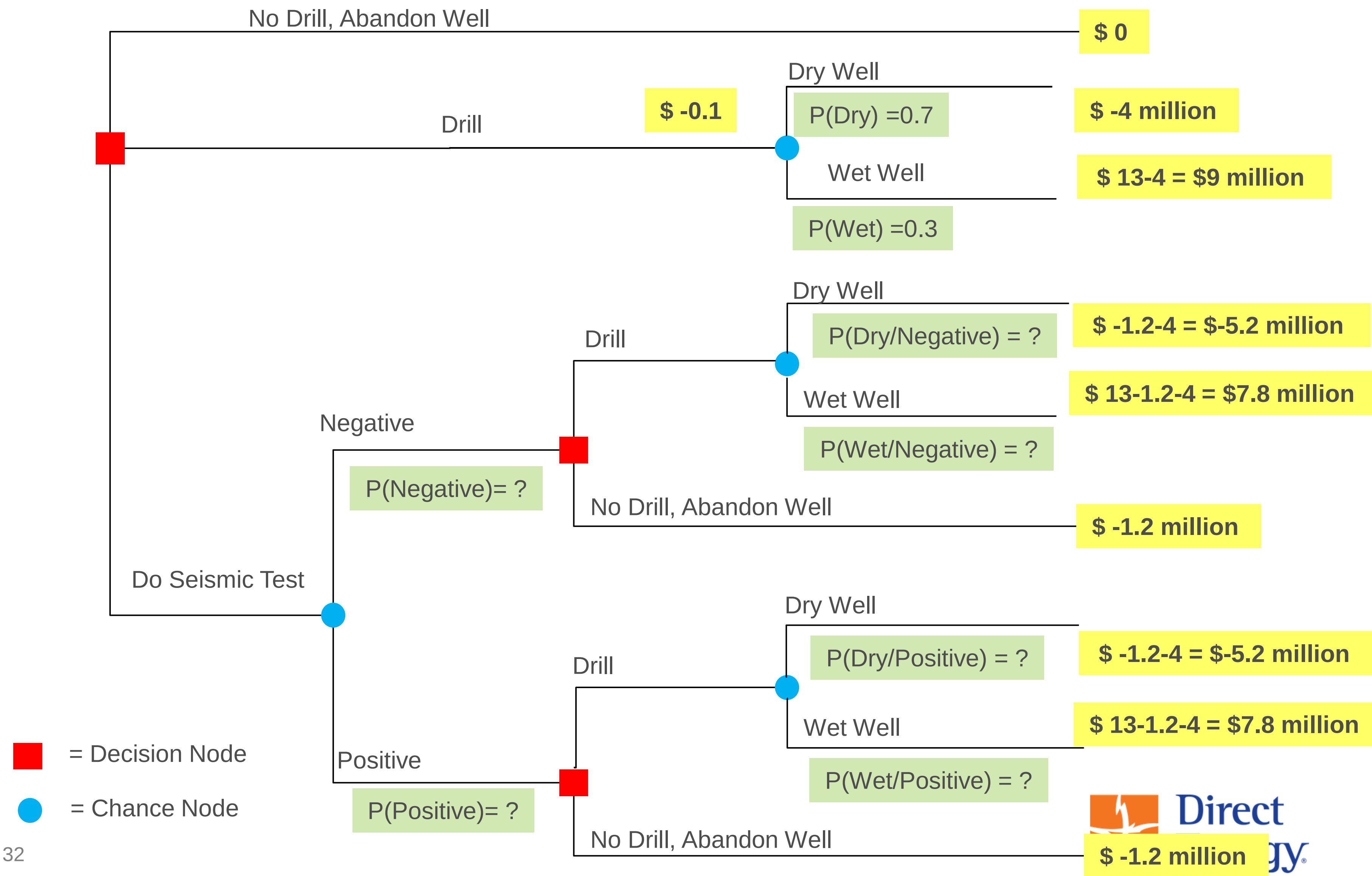
Wildcatter Inc.

- Drilling Rights
  - Drill and Find oil value = $13 million
  - Drill and does not find oil = $0 million
  - Cost of Drilling = $4 million
- Management believes that there is 30% chance to find oil
- Option #2:
  - Seismic test costs $1.2 million

Wildcatter Inc. (cont.)



Wildcatter Inc. with Seismic Test



Probabilities …

Probabilities		Source
Prob (Dry)	70%	Management
Prob (Wet)	30%	Management
Prob (Positive/Dry)	10%	Seismic Test Vendor
Prob (Negative/Dry)	90%	Seismic Test Vendor
Prob (Positive/Wet)	80%	Seismic Test Vendor
Prob (Negative/Wet)	20%	Seismic Test Vendor

…. and Bayesian Analysis

$$Prob(Dry/Positive) = \frac{Prob(Positive/Dry) \cdot Prob(Dry)}{Prob(Positive)}$$

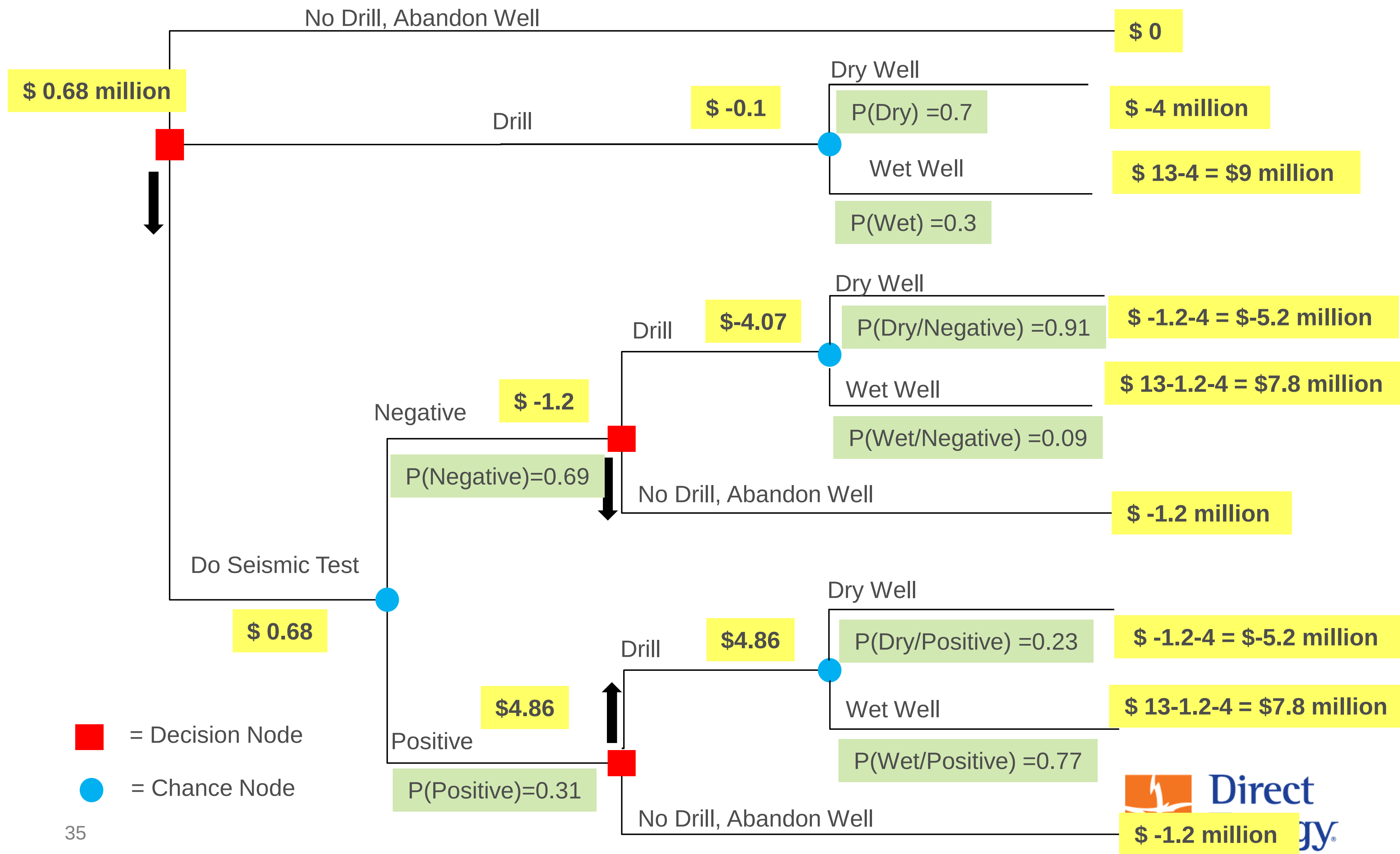
$$Prob(Positive) = Prob(Positive/Dry) \cdot Prob(Dry) + Prob(Positive/Wet) \cdot Prob(Wet)$$

$$Prob(Positive) = 0.1 \cdot 0.7 + 0.8 \cdot 0.3 = 31\%$$

$$Prob(Dry/Positive) = \frac{0.1 \cdot 0.7}{0.31} = 23\%, \quad Prob(Wet/Positive) = 77\%$$

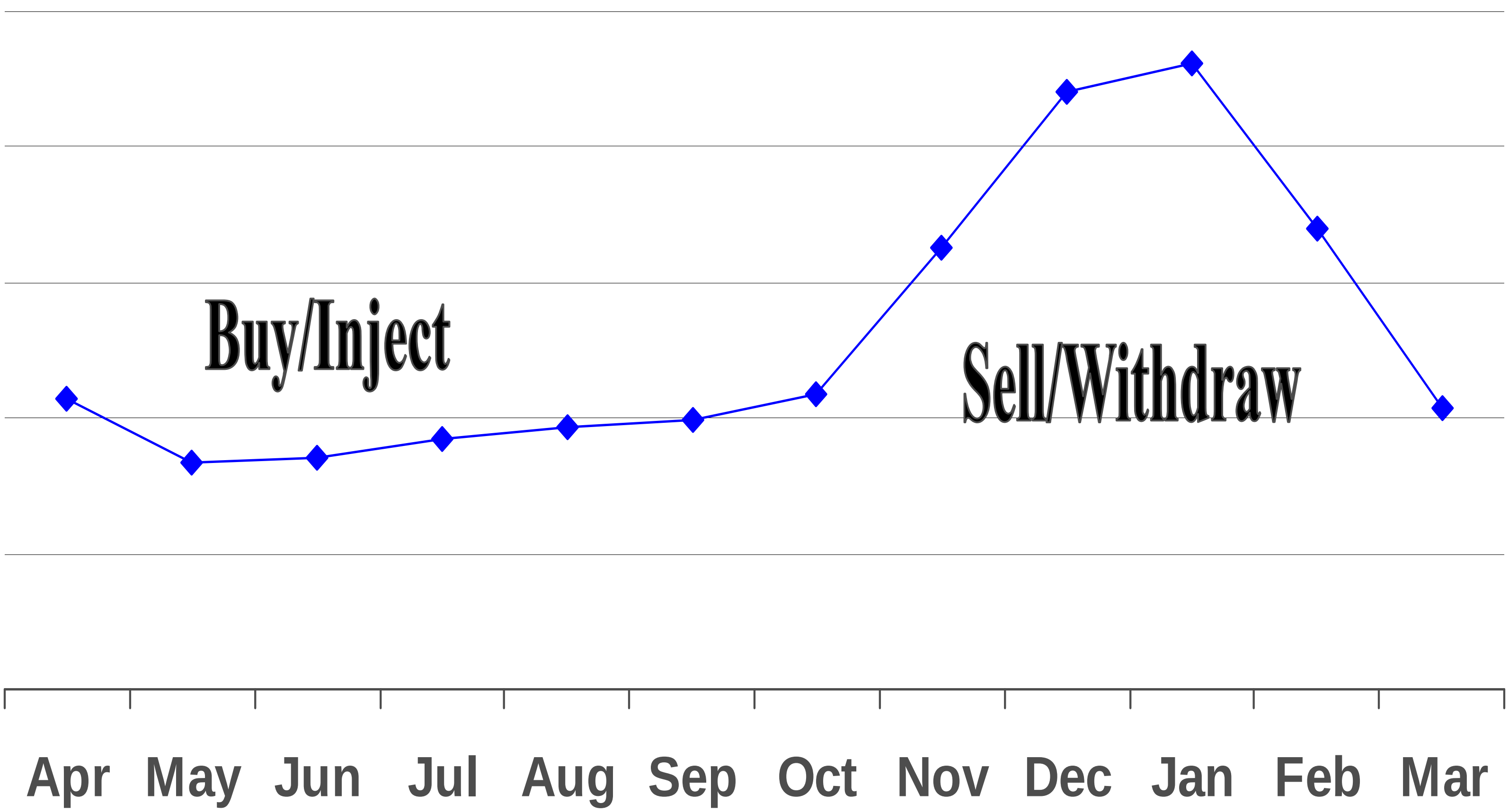
$$Prob(Dry/Negative) = 91\%, \qquad Prob(Wet/Negative) = 9\%$$

Wildcatter Inc. with Seismic Test (cont.)



STORAGE VALUATION

Gas Storage: Optimization of Future Revenues



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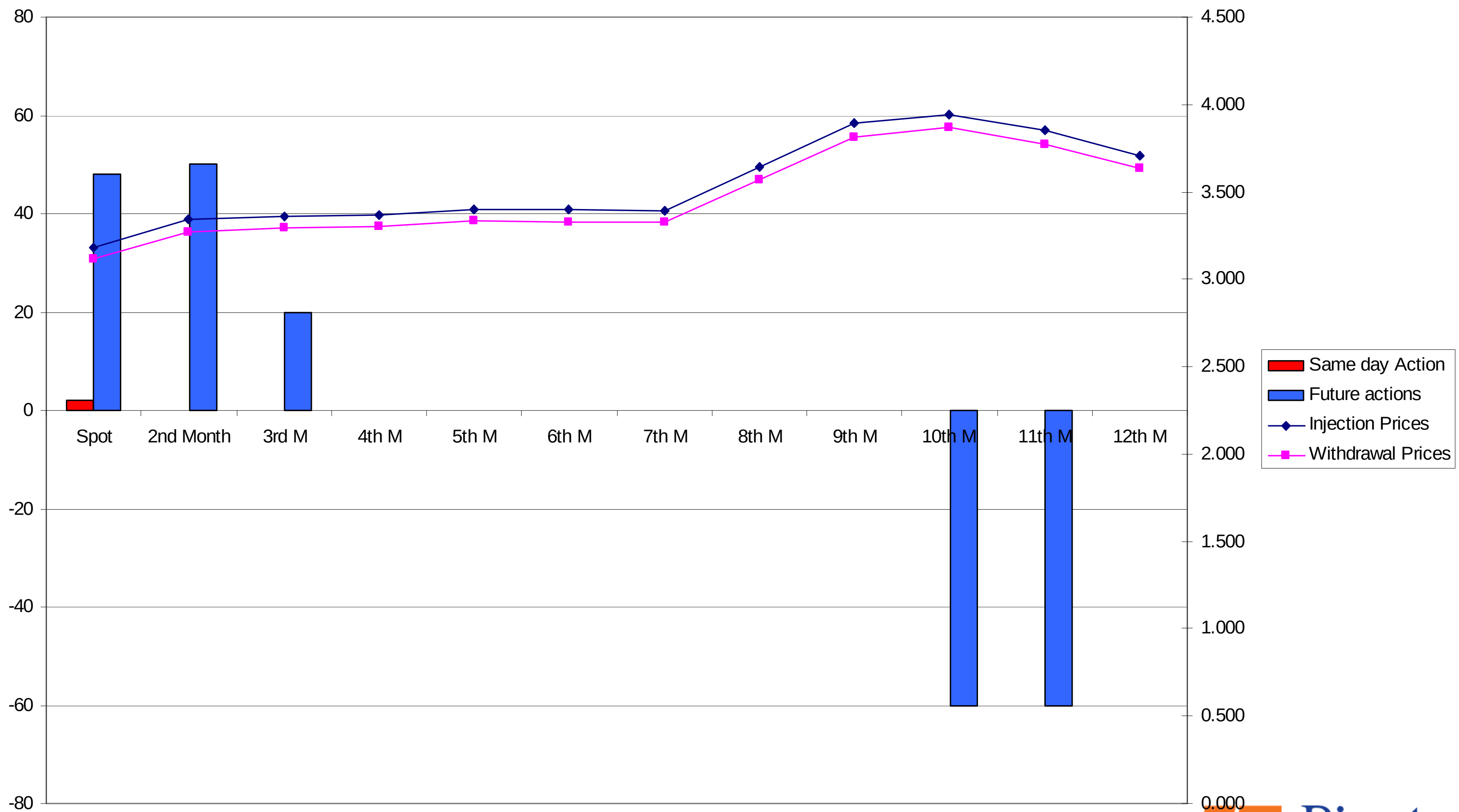
First Day : Prices and Injection-Withdrawal Schedule



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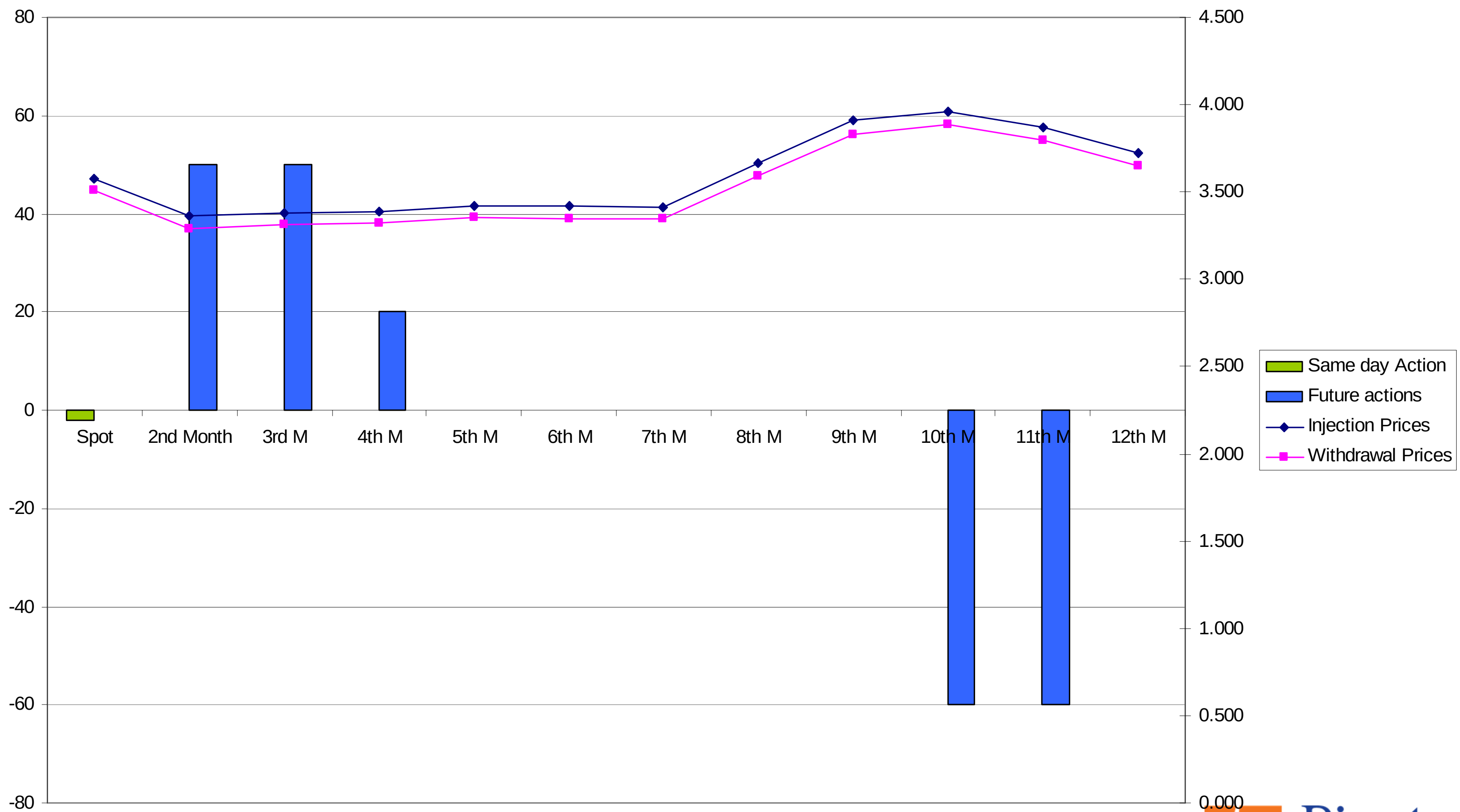
Second Day : Prices and Injection-Withdrawal Schedule



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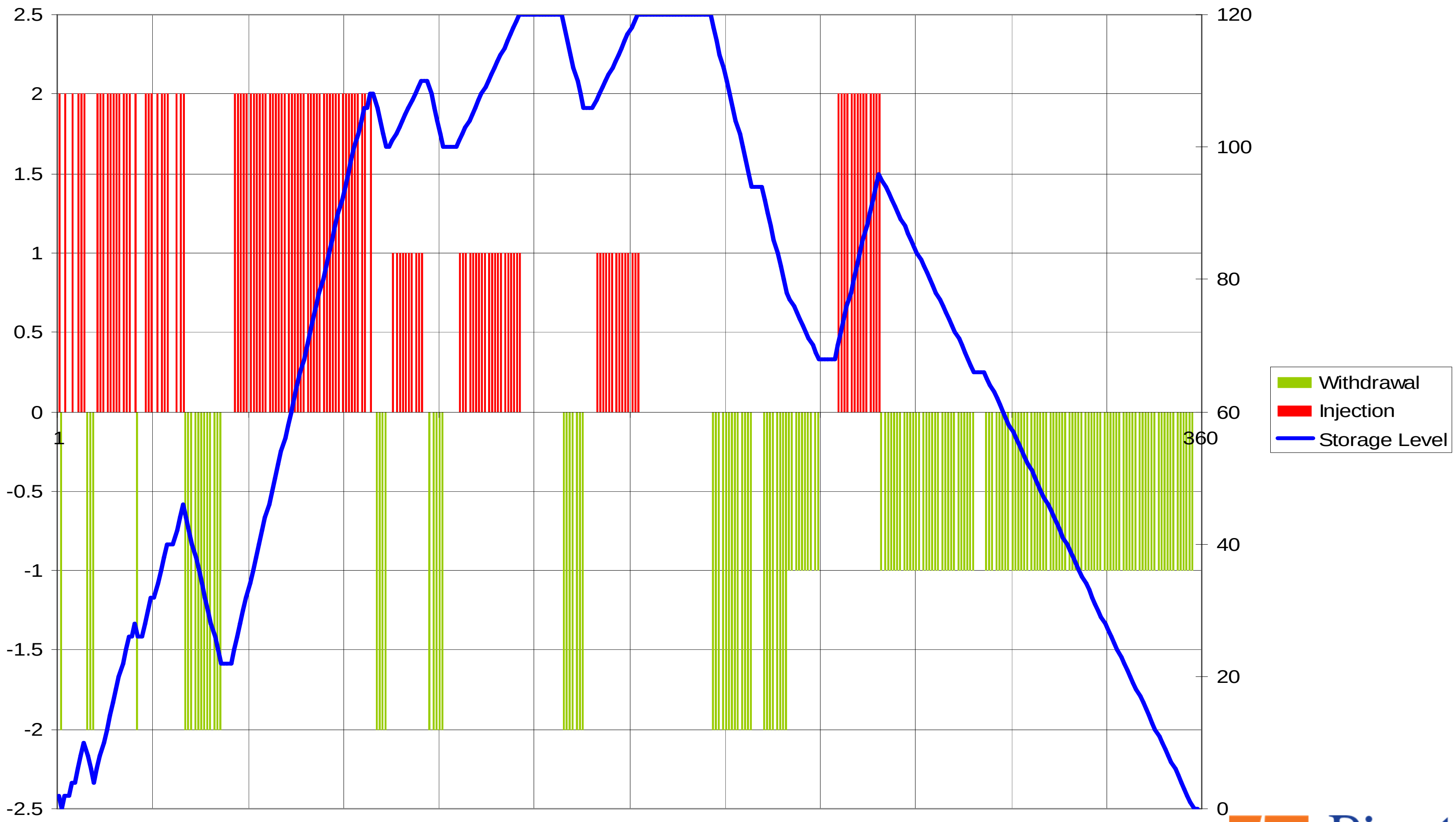
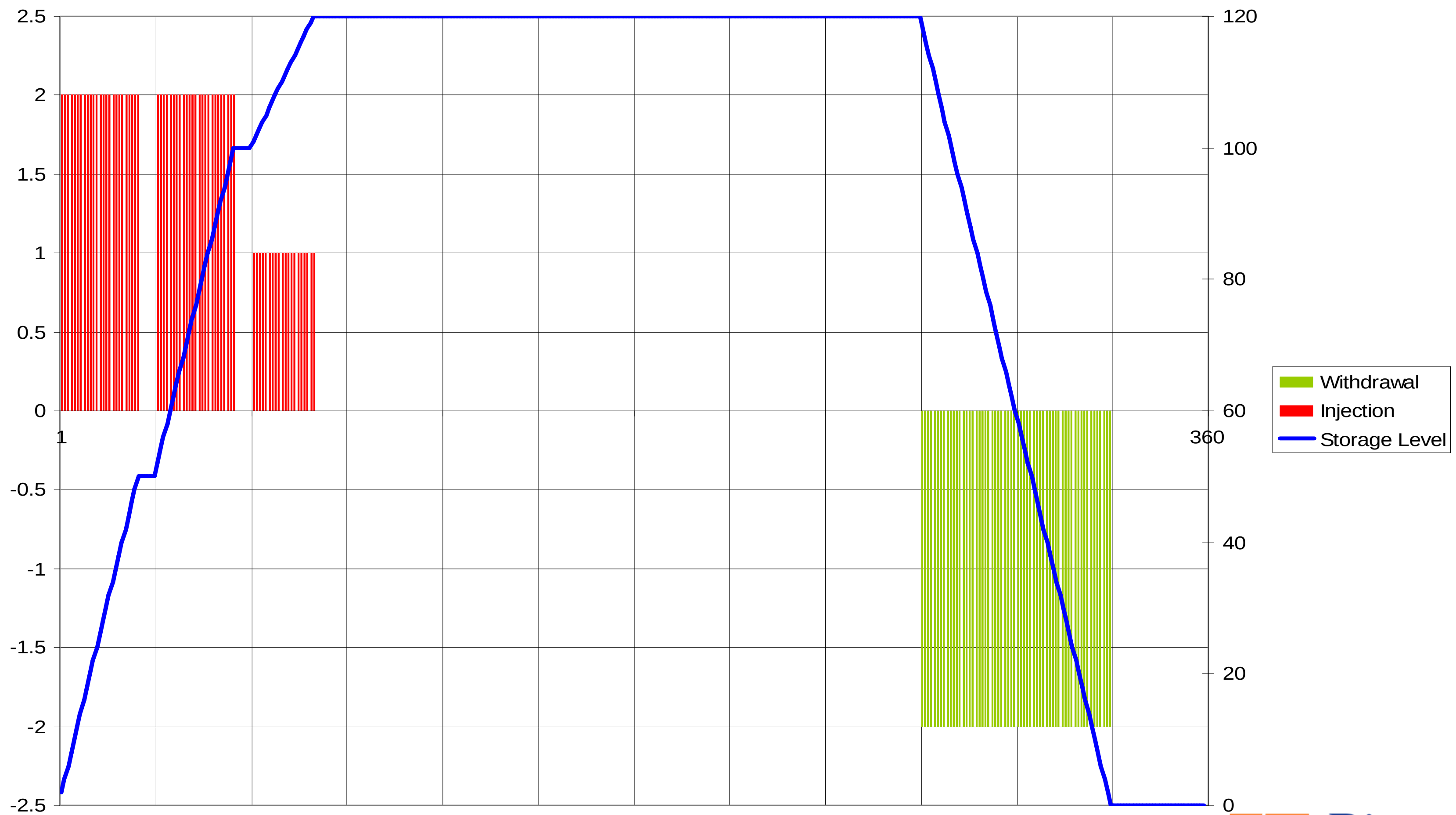
After 360 days of injection and withdrawal



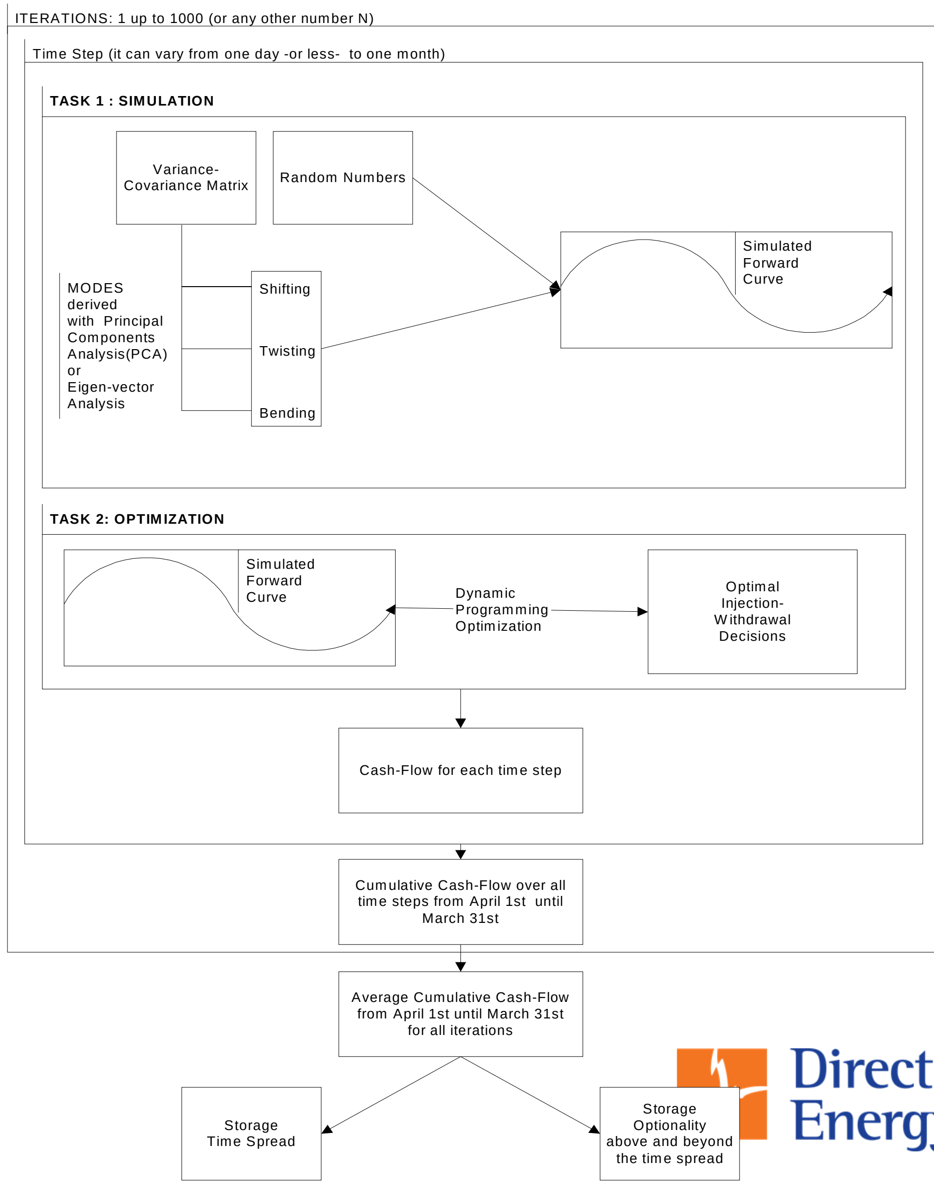
Full Hedging on the first Day of Operations



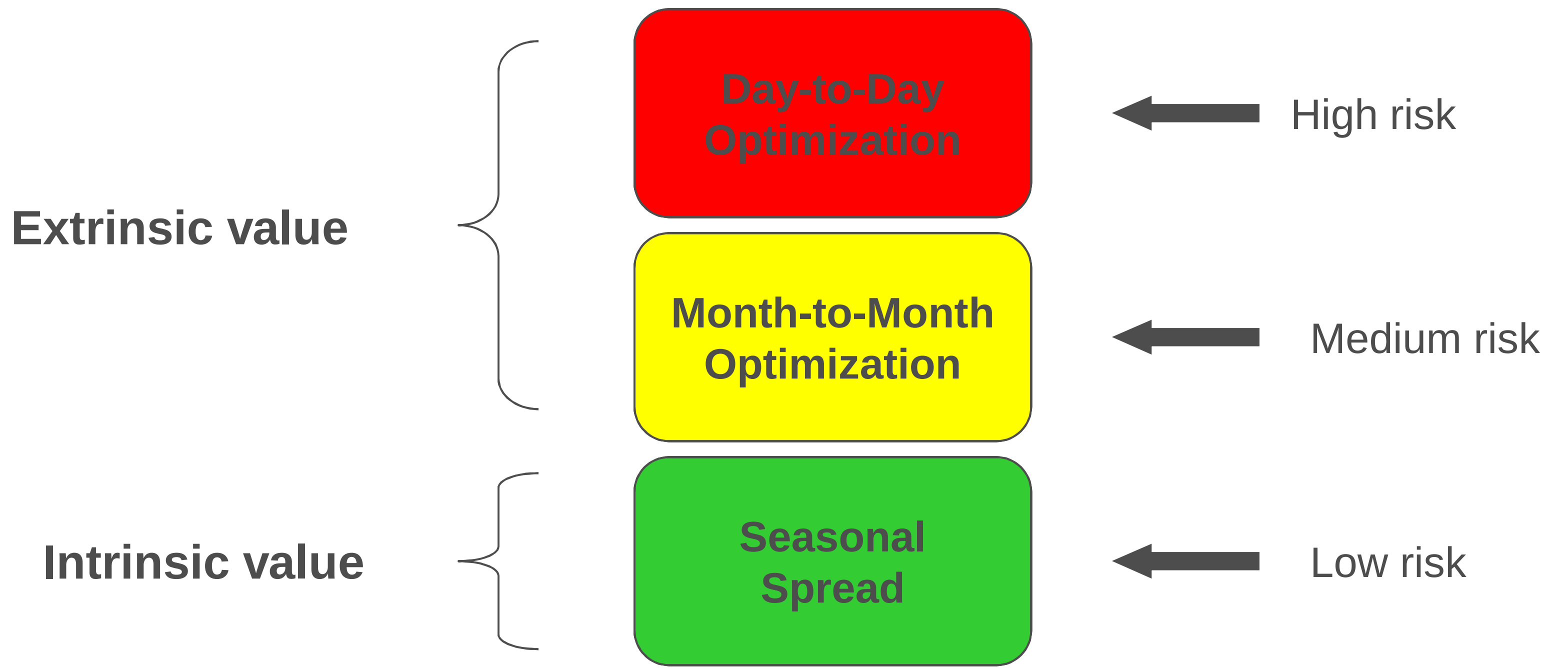
Storage Valuation Simulation

Spyros Maragos, "Valuation of the Operational Flexibility of Natural gas Storage Reservoirs" *Real Options and Energy Management*, Editor, Ehud I. Ronn, 2002, Risk Publications, London, Chapter 14, pp. 431-456

Figure 6: Graph of the Storage Valuation Simulation



Storage Value Components and Associated Risk



SPARK SPREAD: MARGRABE MODEL

Spark Spread: Margrabe option formula

- Variation of the familiar Black76 formula
- Option on the Spark Spread
  - max(E-H*G, 0)
  - E = Power Price is the Underlying
  - H*G = Cost of Production = Plant Heat Rate * Gas Price, is the Strike Price
- What is different than the Black-Scholes / Black76 model?
  - the spark spread is lognormally distributed
  - Volatility: The volatility is the volatility of the spread taking into account the correlation of the two underlying commodities

Spark Spread Options

$$CALL_{BS} = e^{-rt} \cdot \left[EL \cdot N(d_1) - h \cdot NG \cdot N(d_2)\right]$$

where

$$d_1 = \frac{\ln\left(\frac{EL}{h \cdot NG}\right) + 0.5 \cdot \sigma^2 \cdot t}{\sigma \cdot \sqrt{t}}$$

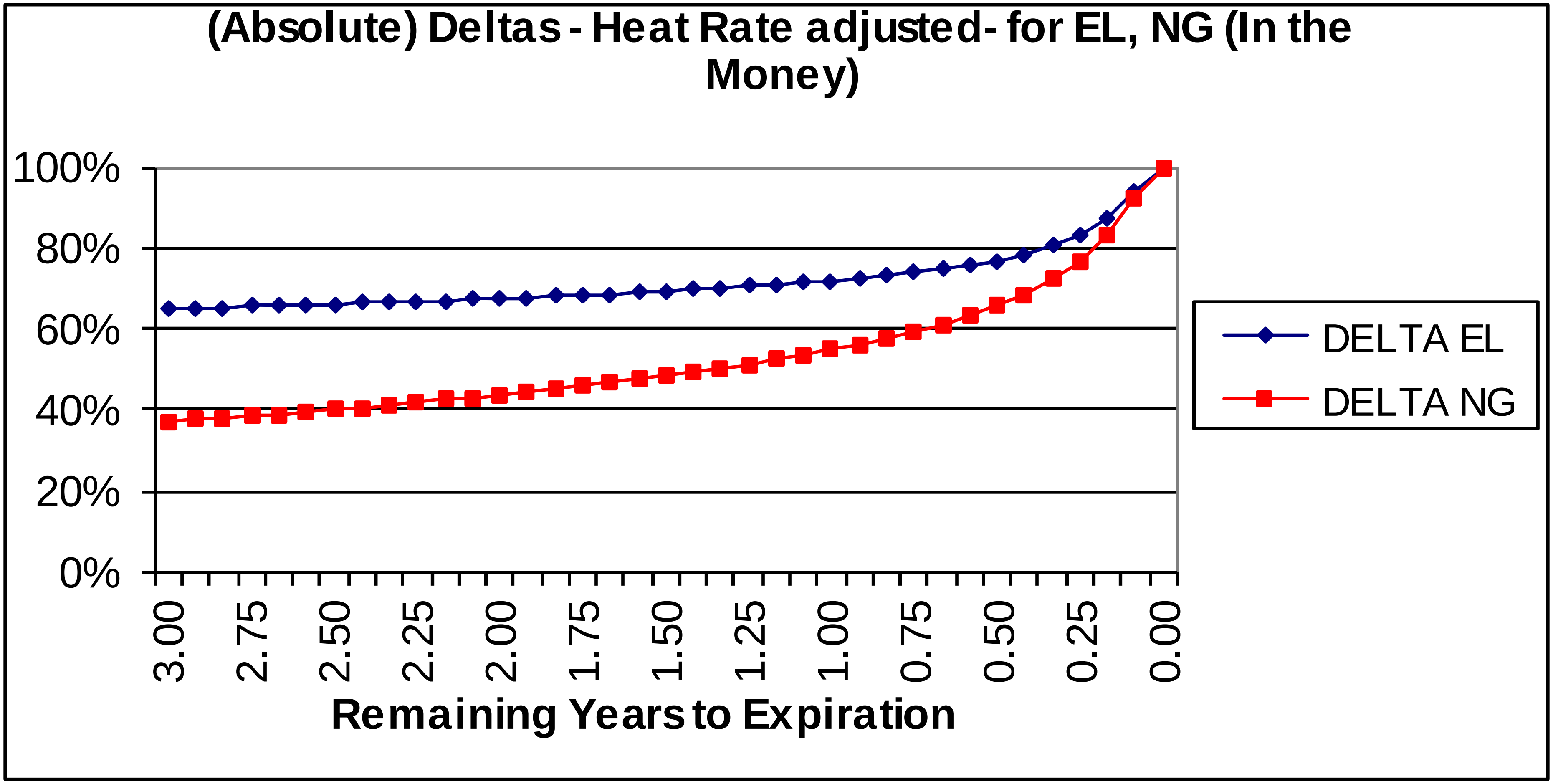
$$\sigma = \sqrt{\sigma_{EL}^2 + \sigma_{NG}^2 - 2 \cdot \rho \cdot \sigma_{EL} \cdot \sigma_{NG}}$$

- EL = power price
- NG = natural gas price
- h = heat rate of the plant
- σ = volatility of the spark-spread.
- σ_{EL} = electricity volatility
- σ_{NG} = gas volatility
- ρ = correlation of gas and electricity

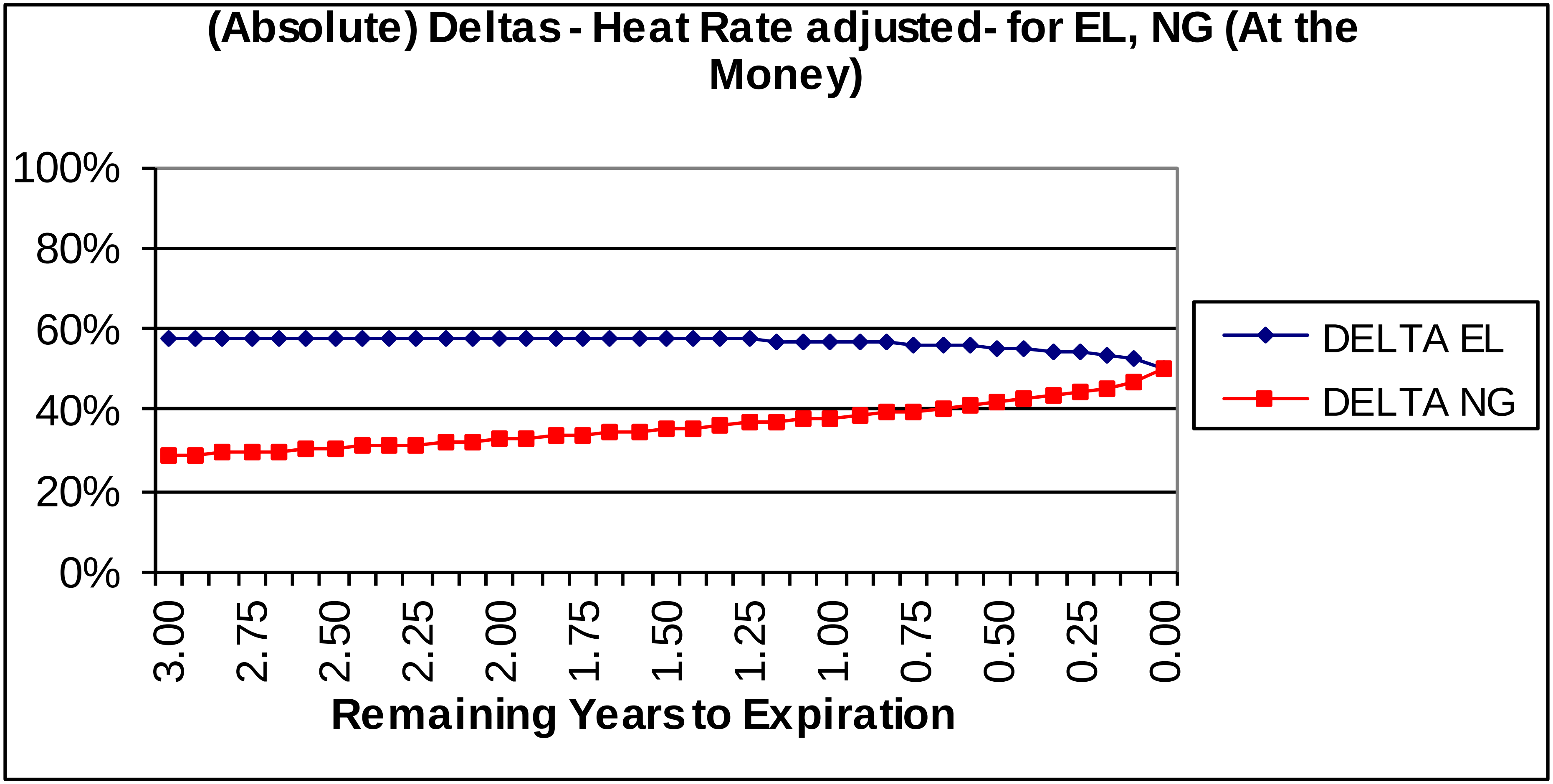
Example

- Price of electricity three years out is $50/MWh,
- Spread volatility is 50%
- Price of NG is 5 $/mmBtu

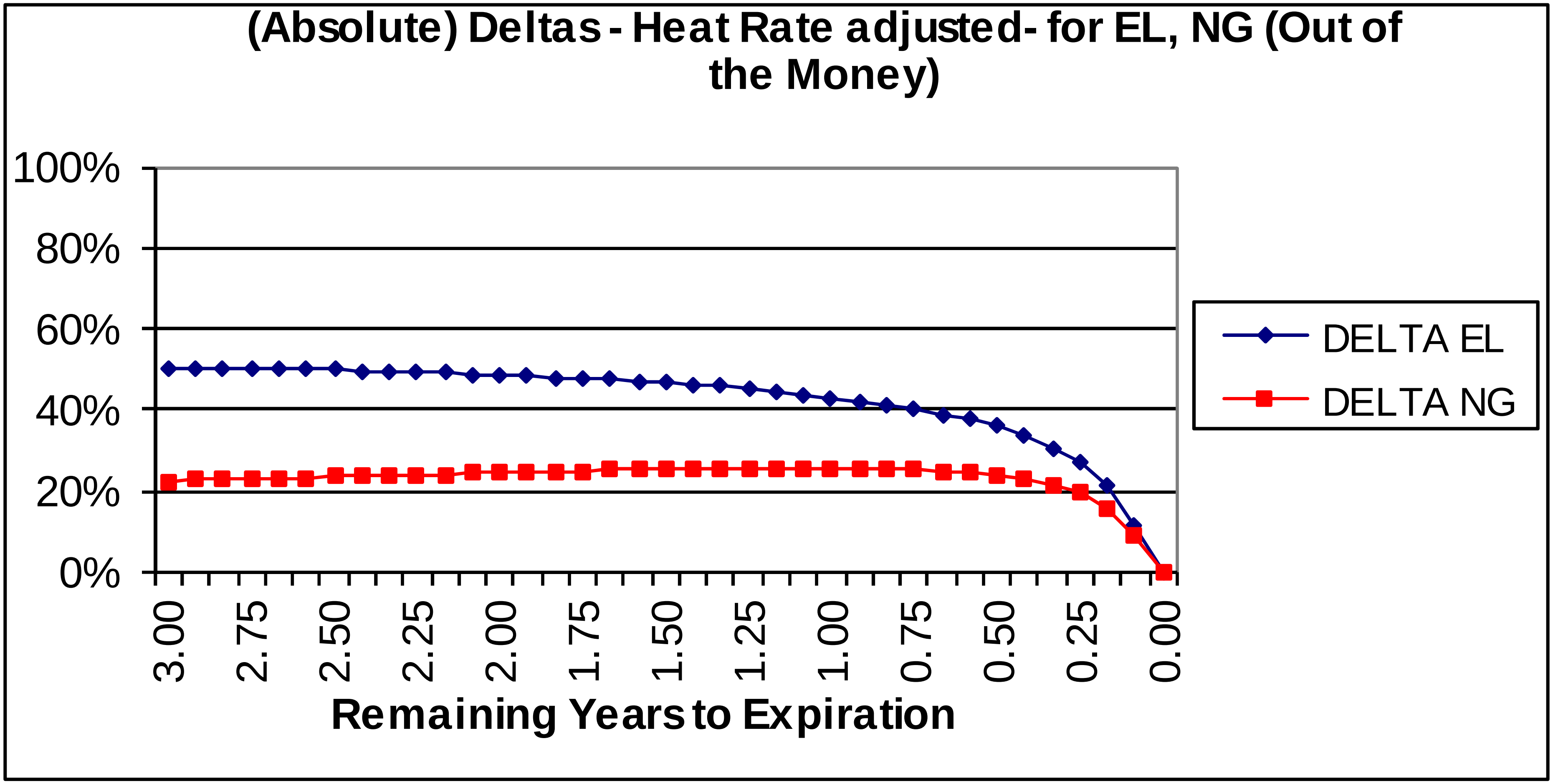
In-the-money spark-spread



At-the-money spark-spread

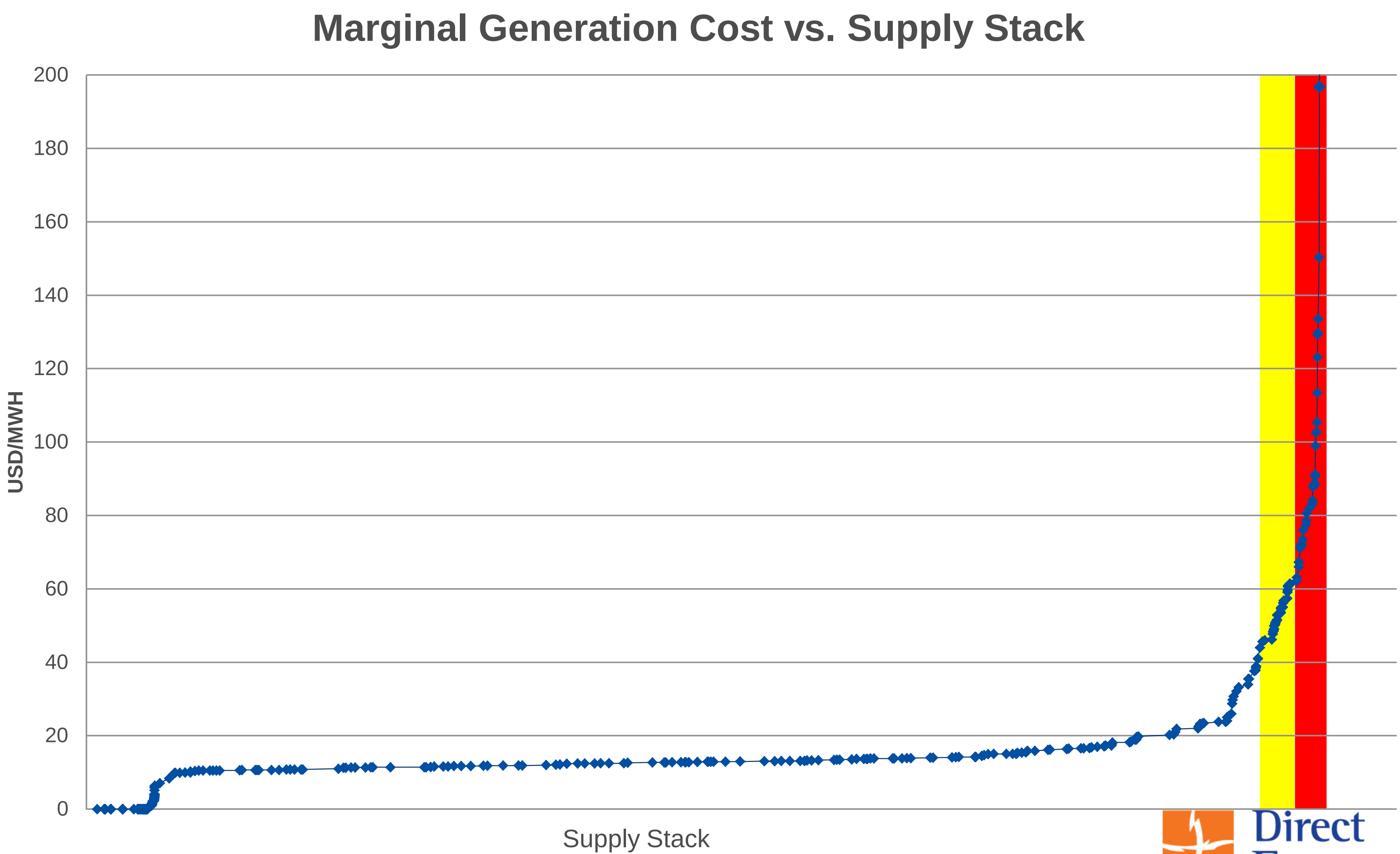


Out-of-the-money spark-spread

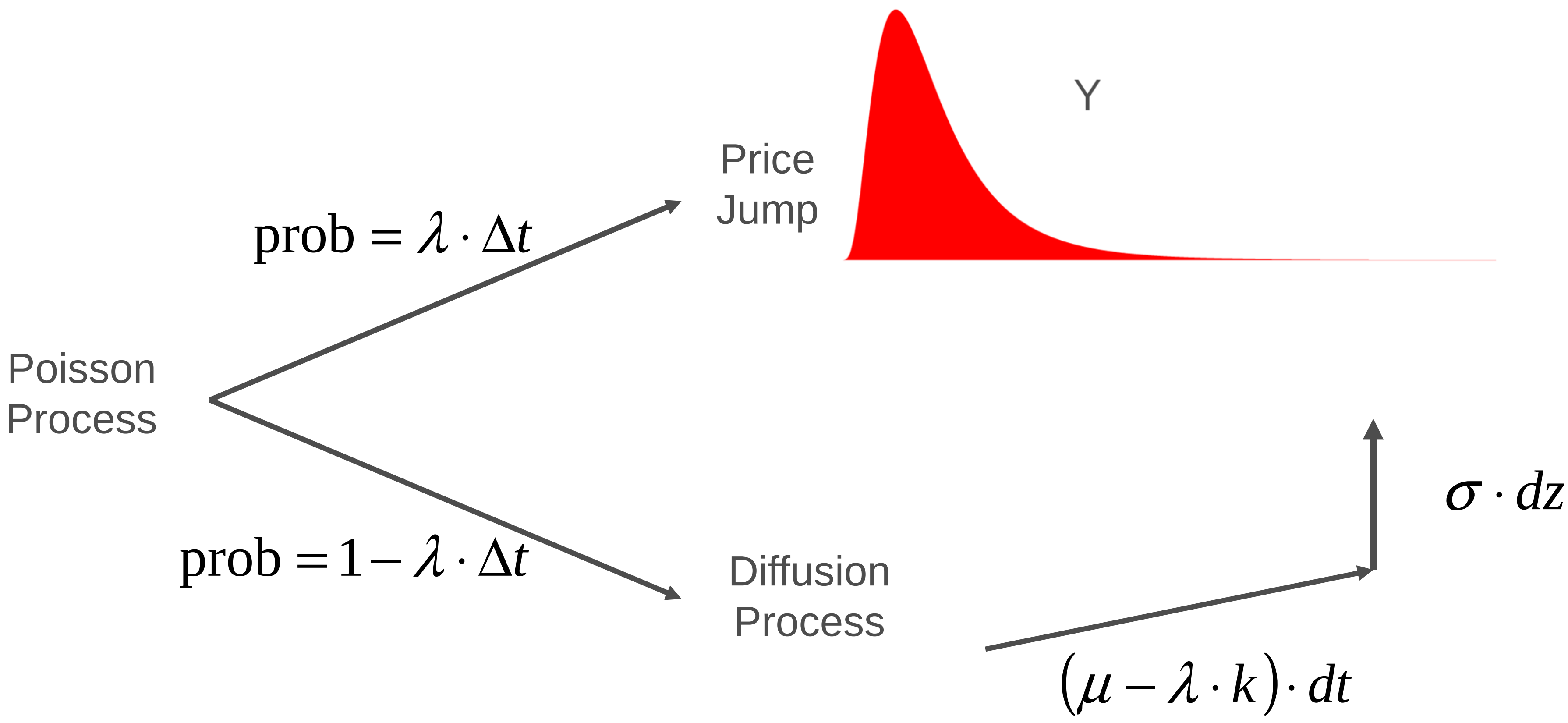


POWER PRICE MODELLING: JUMP-DIFFUSION

Marginal Cost in the Power Markets or why Power Prices are not log-normally distributed

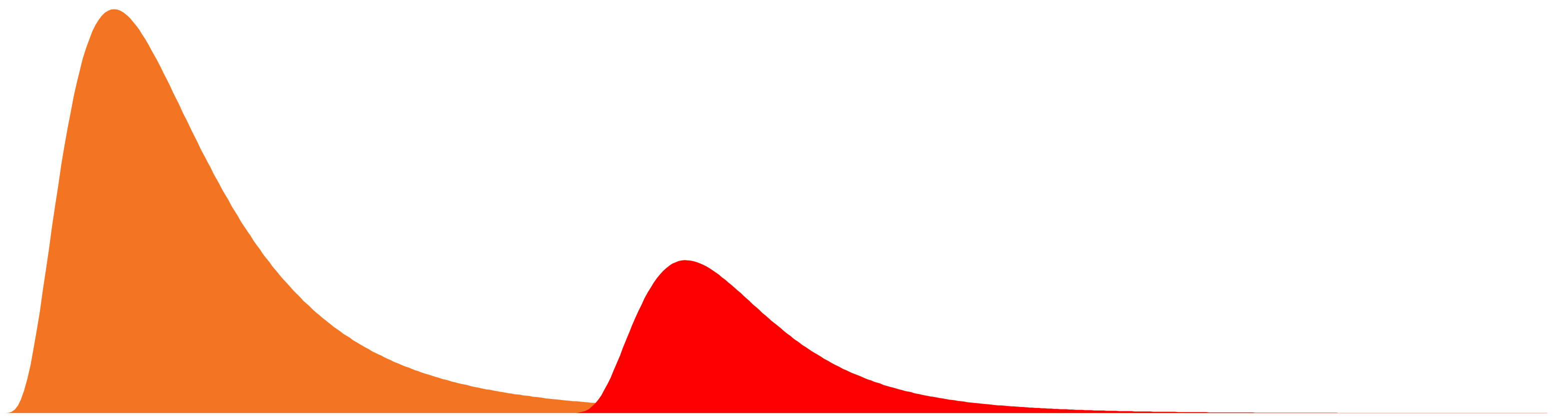


Jump-Diffusion (Merton model 1976)

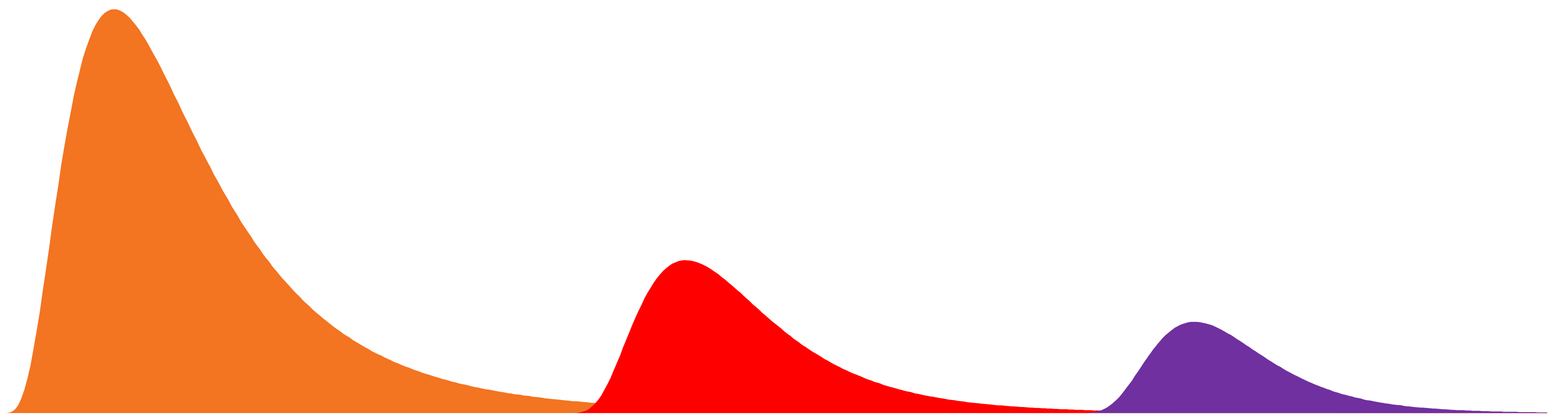


Jump Diffusion (no jump yet)

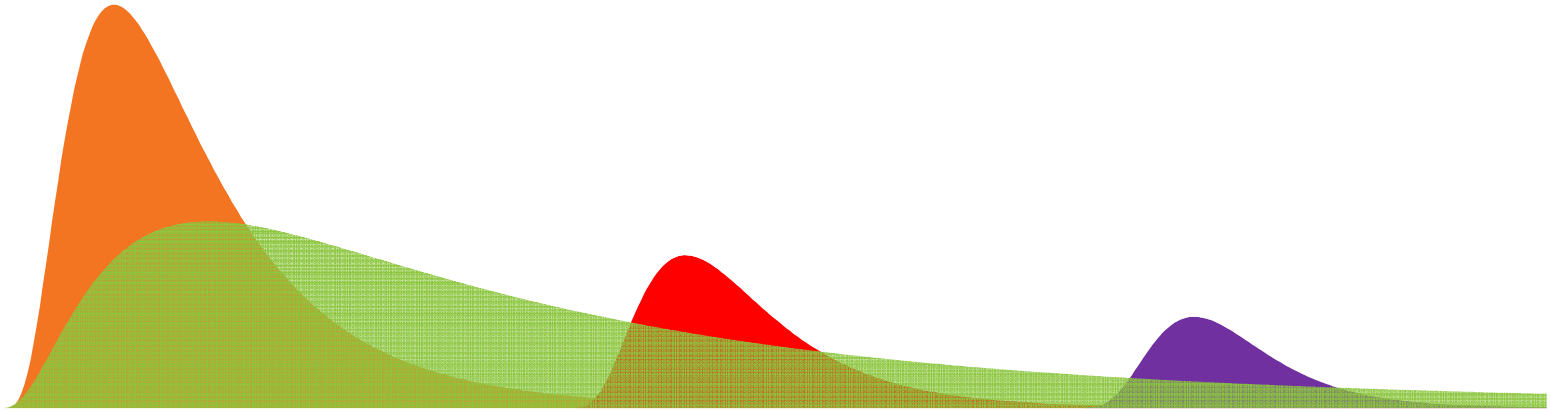
Jump Diffusion (first jump)



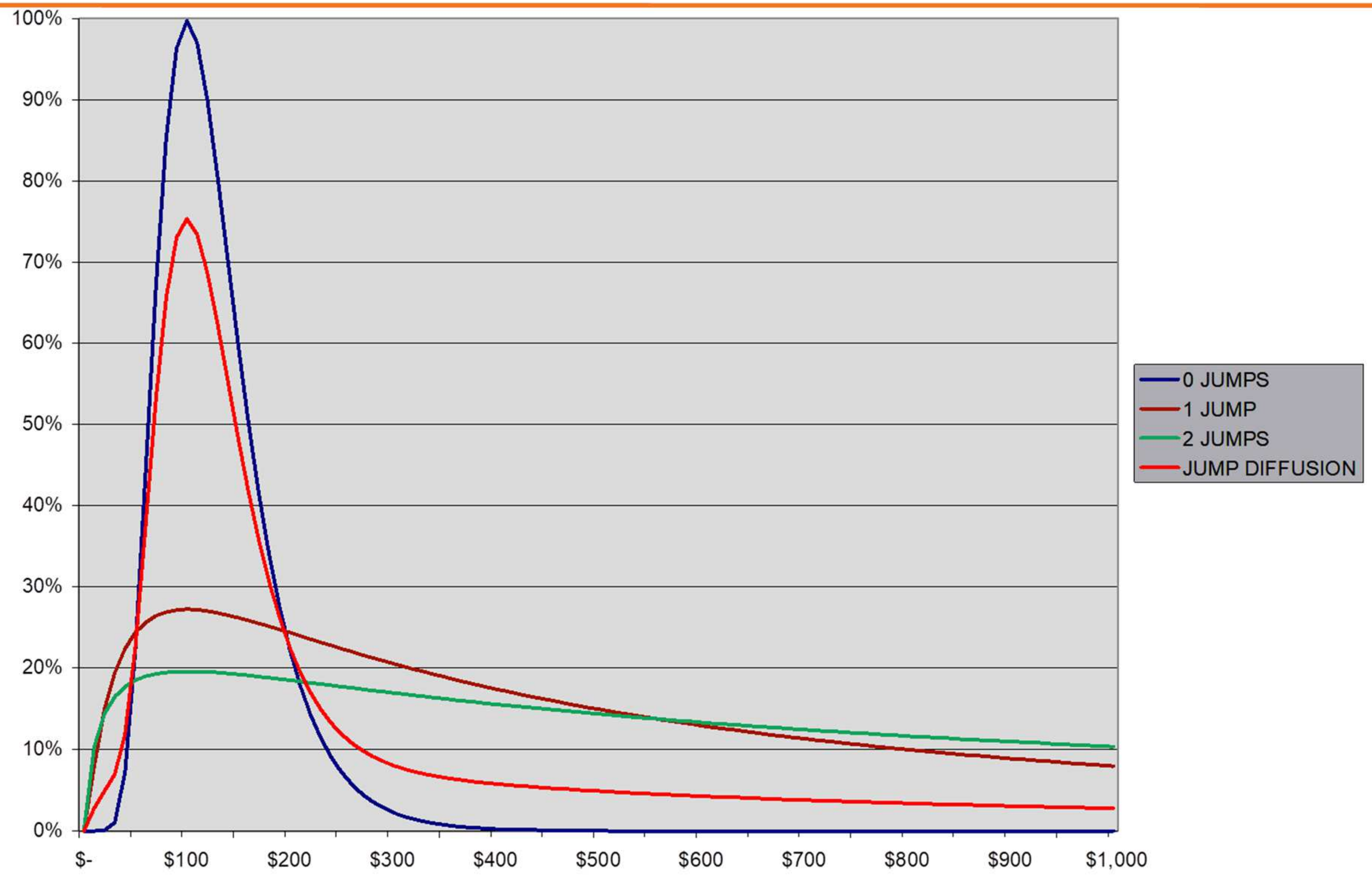
Jump Diffusion (second jump)



Jump Diffusion



Jump-Diffusion Example (Distribution)



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Jump-Diffusion Example

VOL_SIMPLE	L	T	E(Y)	vol(Y)	R_SIMPLE	PRICE
40%	20%	1	2	2	5%	$ 100
					Black-Scholes	$ 15.08
52.4%					**Black-Scholes with 52.4%**	$ 19.65
JUMPS	L*	VOL(N)	R(N)	PROBABILITY	VALUE_IND	VALUE
0	0.4	40%	5%	67.03%	$ 15.08	$ 10.11
1	0.4	147%	54%	26.81%	$ 31.23	$ 8.37
2	0.4	204%	124%	5.36%	$ 20.10	$ 1.08
3	0.4	248%	193%	0.72%	$ 11.41	$ 0.08
4	0.4	286%	262%	0.07%	$ 6.15	$ 0.00
5	0.4	319%	332%	0.01%	$ 3.23	$ 0.00
					Jump-Diffusion	$ 19.65

POWER PLANT VALUATION

Real Options: Power Peaking Plants Valuation

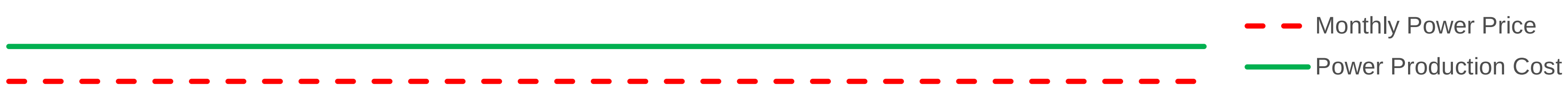
- Peaking plants depend on high power prices/margins that appear over a relatively small number of hours
- Power price profile
  - Seasonal Variation
  - On-Peak 5x16, usually 7am-11pm
  - Off-Peak
  - Weekday vs. weekend
- Power and Fuel Prices are volatile
- Operational
- Strategic

Power Plant Valuation Challenges

- The annual value of the power plant is modelled as a series of 8,760/8,784 call spread options corresponding to every hour of the year
  - Multi-commodity price movements
  - Operational Cost
  - Maintenance down time
  - Limitation of the annual hours of operation
  - Limitation of number of Daily/Annual Starts
  - Emissions limitation/cost
  - Minimum up and down time
  - Minimum Ramp-up time
  - Minimum Generation Levels
  - Variable Heat Rates
  - Variable Start-Up cost
  - Startup Costs (Cold/Warm Start)
    - Operational Considerations
    - Risk Profile
  - Valuation of thousands of Deep out of the money hourly options

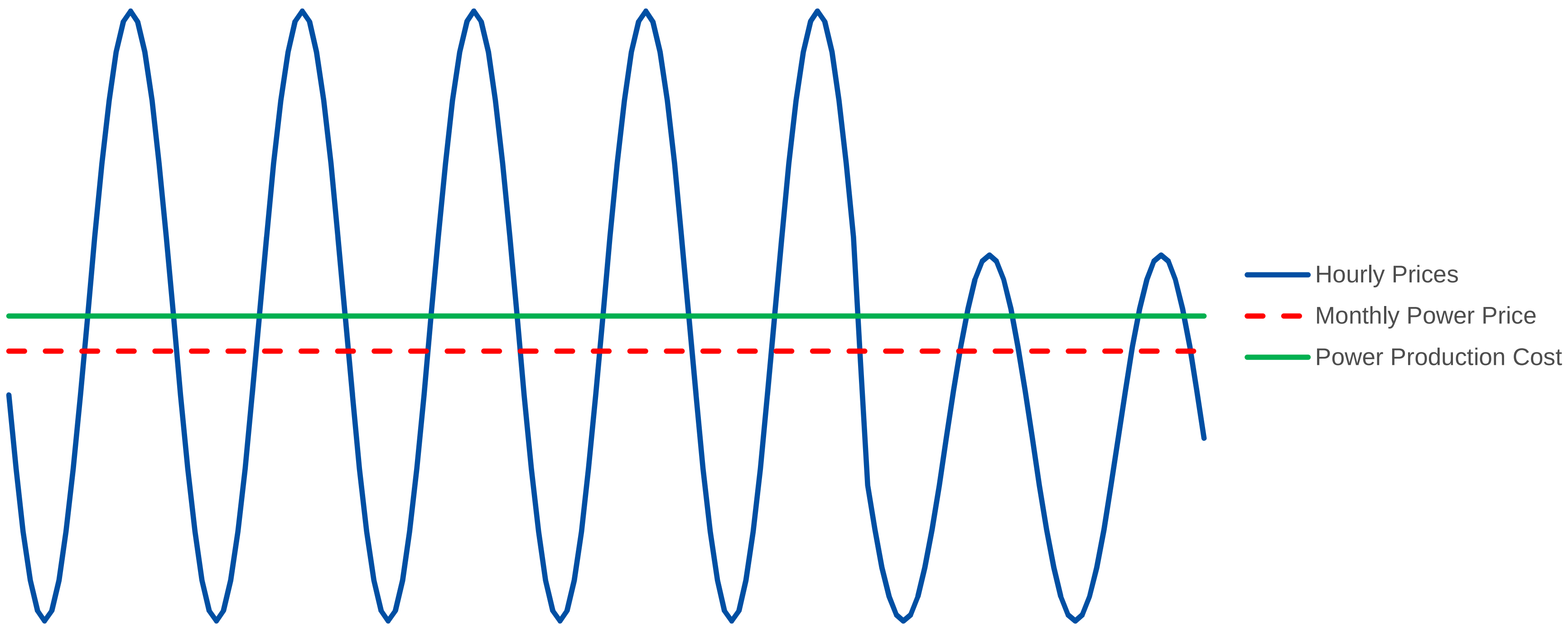
Peaking Generation Unit Economics

Hourly Power Price vs. Power Production Cost



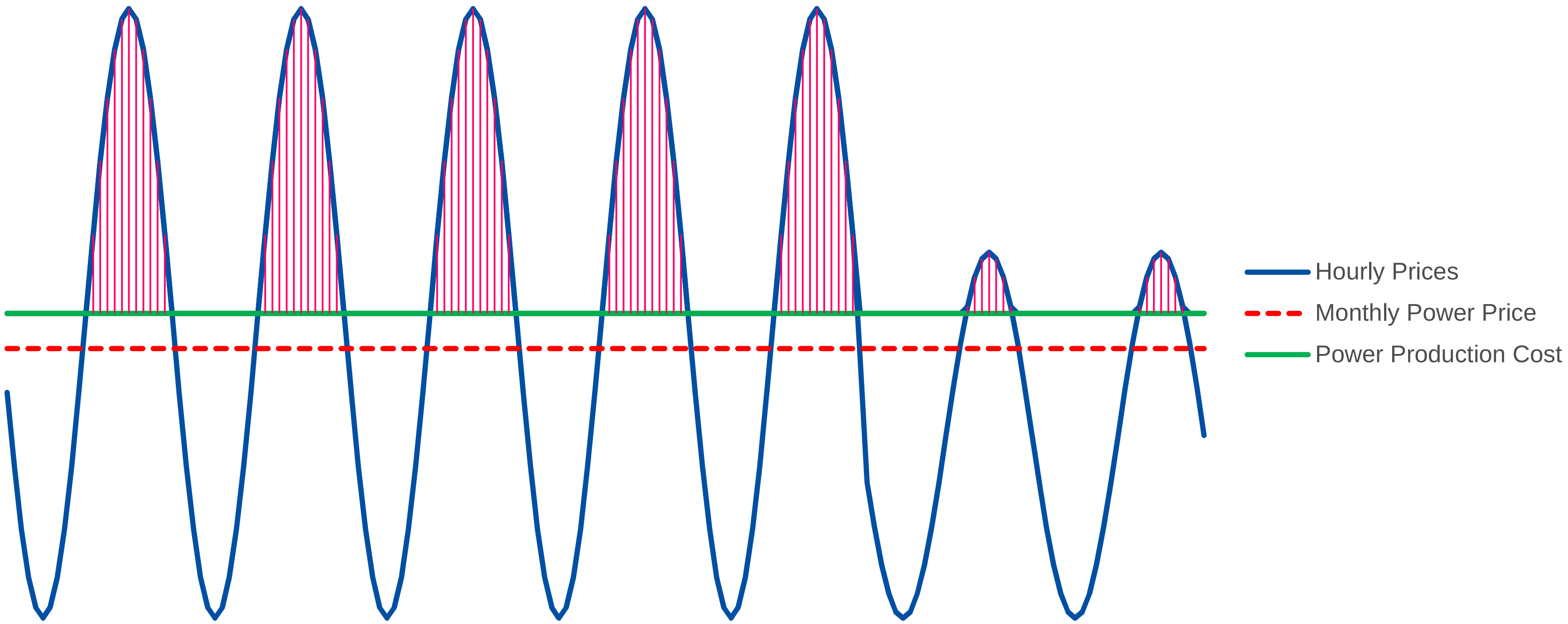
Peaking Generation Unit Economics (cont.)

Hourly Power Price vs. Power Production Cost



Peaking Generation Unit Economics (cont.)

Hourly Power Price vs. Power Production Cost



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Valuation of a power plant

- Valuation of Hourly Options
  - Analytical Methodologies
    - Spark Spread, Margrabe
    - Jump-diffusion
  - Simulation of Commodities (electricity and gas)
    - Monte Carlo
- Optimization of Operations, Optimization Techniques
    - Dynamic Programming
    - Binary Optimization
    - Mixed Integer Programming

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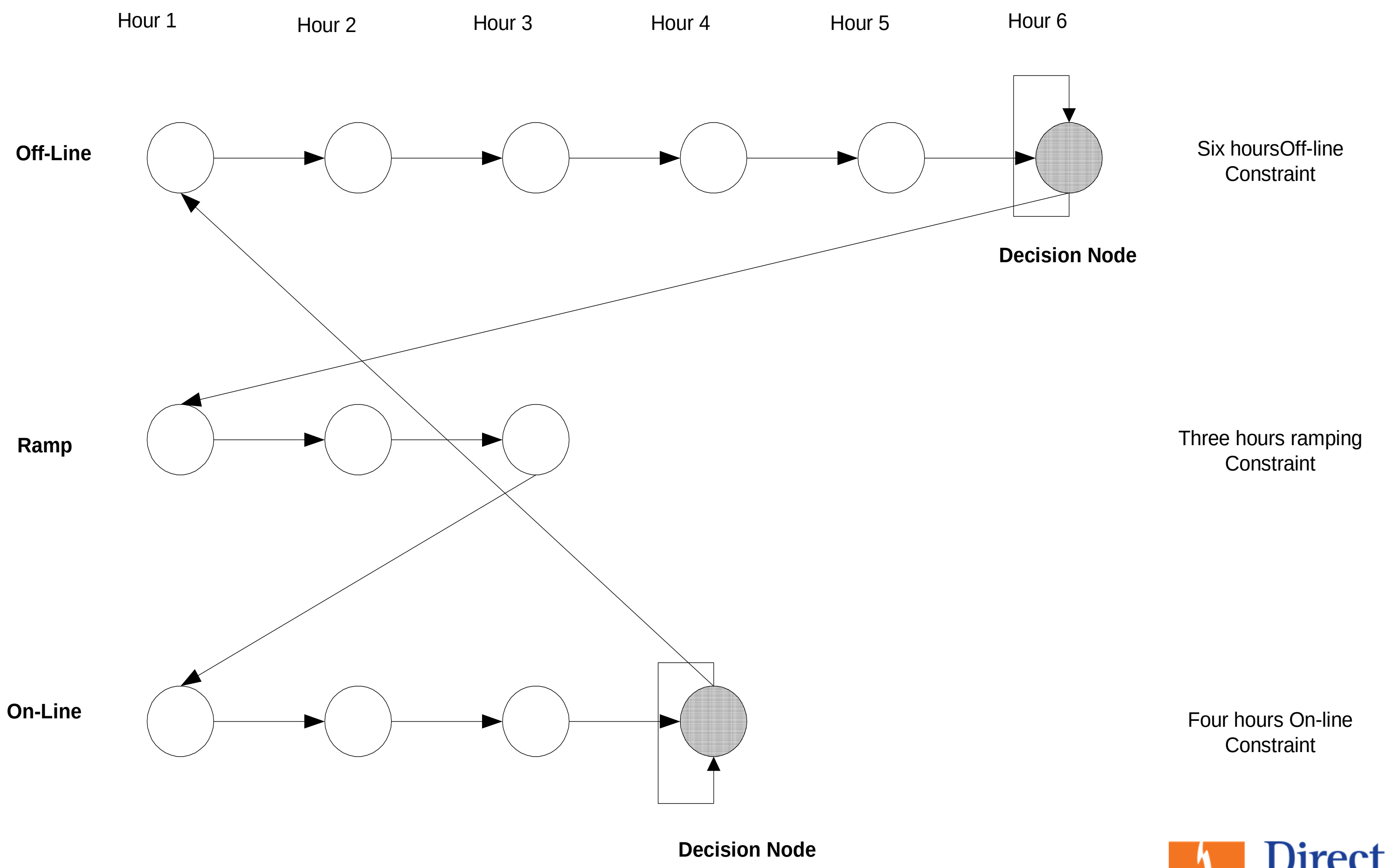
States and Stages of Power Plant Operation



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Cautionary Tale: You live by the sword …

- From $1.8 B valuation to two Bankruptcies and $0.5 B Market Value

Conclusions

- Financial Options inspired and assisted the development of the Real Options
- Flexibility is value, for both operational and strategic purposes
- Reminder: Transfer Financial Option Know How, but not the Risk Free Interest Rate (think CAMP)
- By using real option valuation methodology as a roadmap, real option and asset owners can establish a process to assign and extract this “real” optionality from the asset.
- …and increase their overall return on a sustained basis.

APPENDIX

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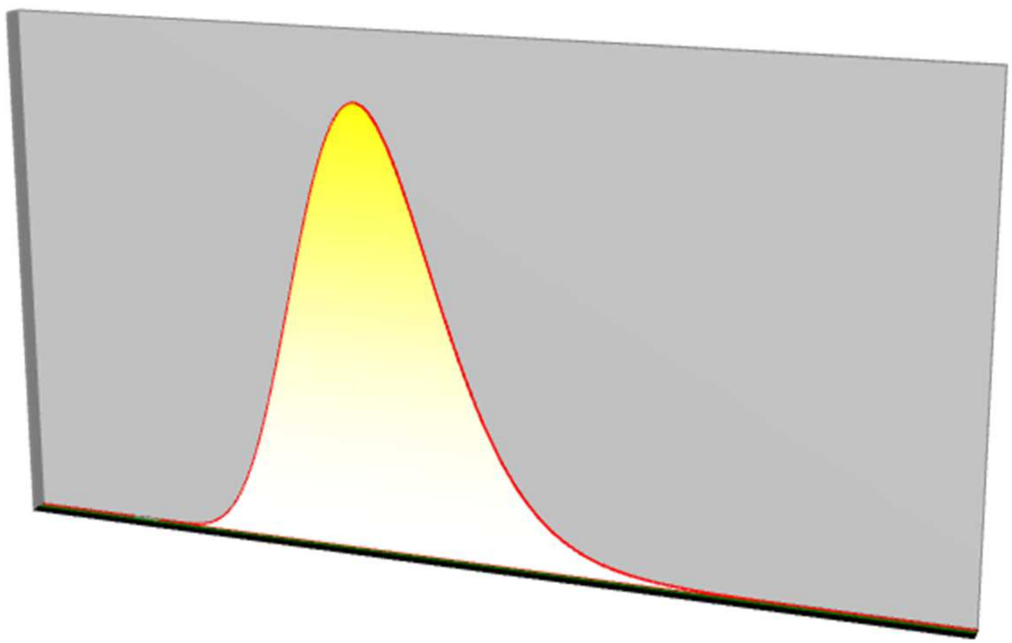
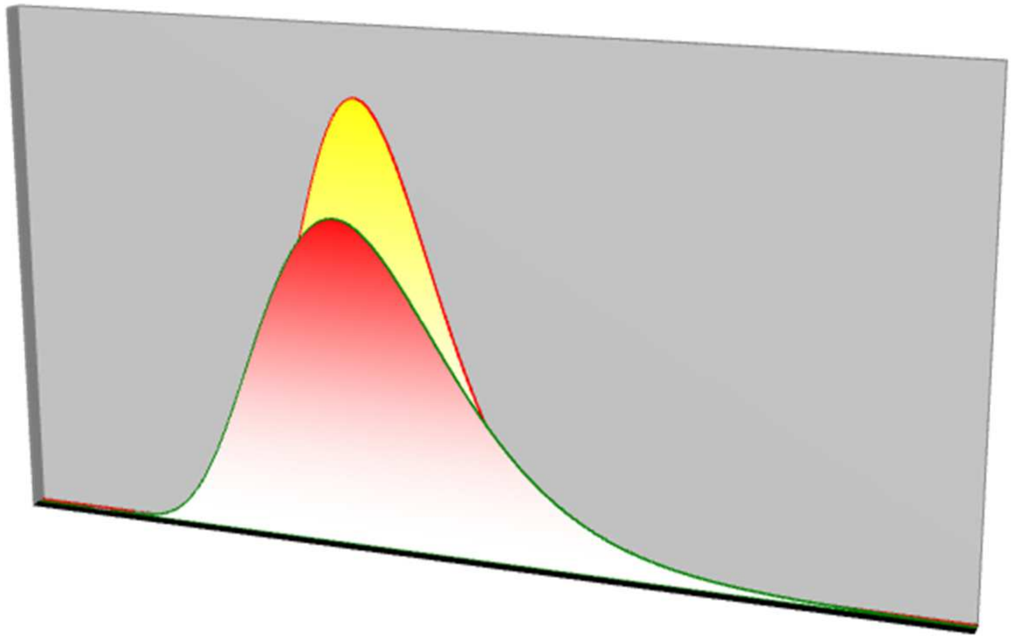
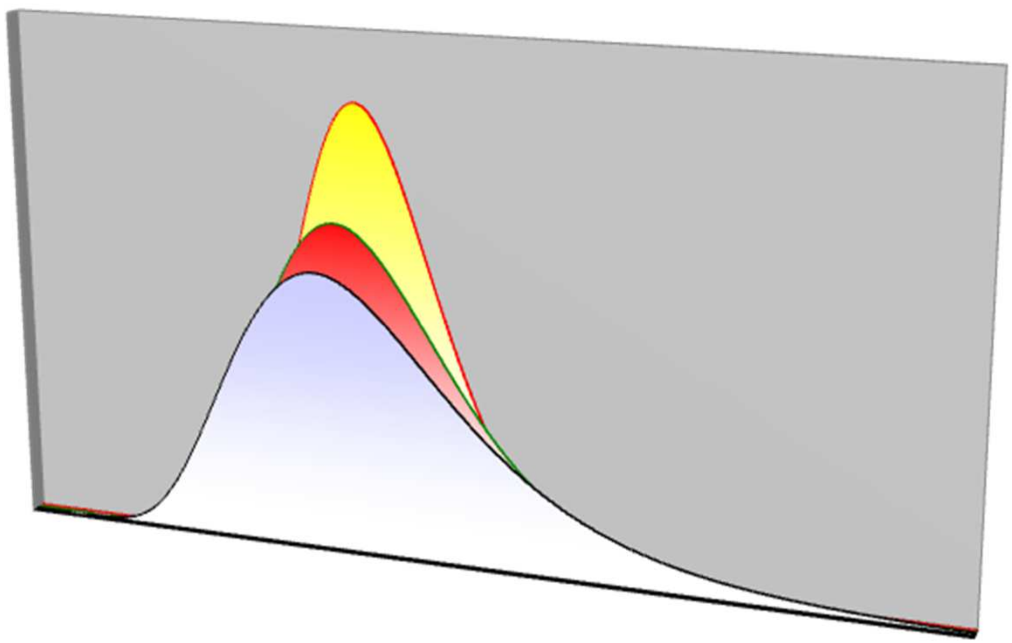
Log-Normal Distribution over Time



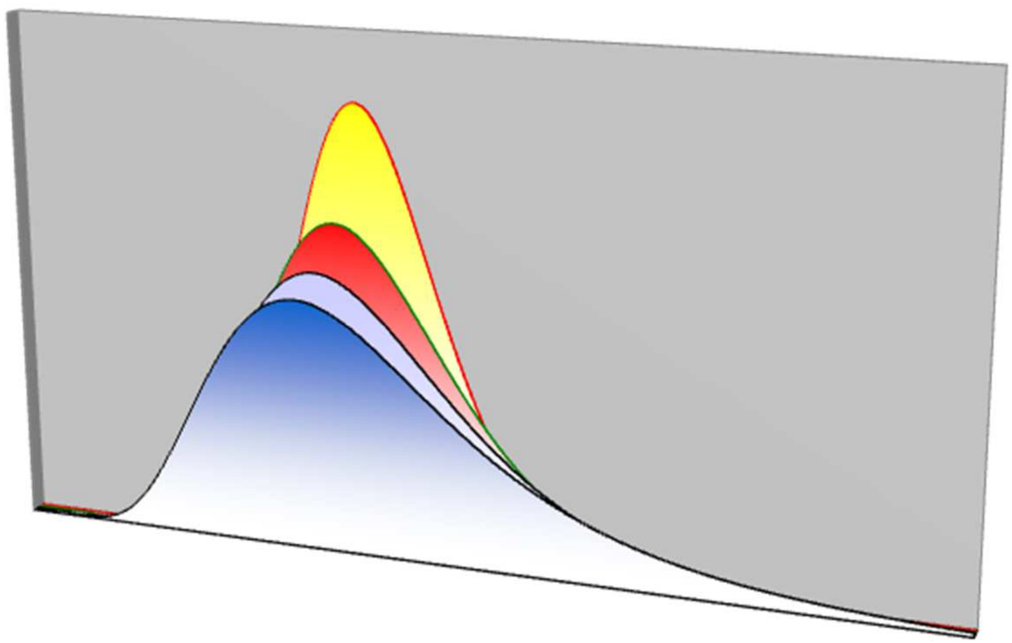
Log-Normal Distribution over Time



Log-Normal Distribution over Time



Log-Normal Distribution over Time



GREEKS

Delta summary

– Delta is the price sensitivity of an option with respect to spot

– Delta is the hedge factor to hedge the option price against small movements in the spot price

– An intuitive way of thinking of delta (for European vanillas only) is as the “probability” of finishing in-the-money.

Gamma summary

- Gamma is the delta sensitivity with respect to spot and measures the sensitivity of the option price to larger movements in spot
- When you buy/(sell) an option you have positive/(negative) Gamma
- Gamma measures how to adjust the hedge to remain "delta-neutral" as the exchange rate changes
- Options with high Gamma need more frequent re-hedging
- Gamma is high if the option is close to expiry, close to the strike price or has low volatility

Theta summary

- Theta is the sensitivity of the option price to a one day change in time. It is also known as the time decay
- When you buy/(sell) an option, you have a negative/(positive) Theta position
  - If you buy/(sell) and option you “pay”/ “earn” decay
- When Theta is positive, Gamma is negative and vice-versa
- Theta is large if the option is close to expiry, close to ATM or has high volatility

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Vega summary

- Vega is the sensitivity of the option price to a change in volatility
- When you buy/(sell) an option, you have a positive/(negative) Vega position
- Vega is larger if the option is long-dated or close to ATM

The Twelve Attribution Components

- Greeks used for Attribution
  - First Order Greeks (all eight of them):
    - » Delta (EL & NG)
    - » Vega (EL & NG)
    - » Theta
    - » Rho
    - » Correlation Delta (Chi)
    - » Strike Price Delta
  - Second Order Greeks (three out of thirty six possible)
    - » Gamma EL, Gamma NG, Cross Gamma for both EL & NG
  - Higher Order Greeks

Second & Higher order Greeks

- Second Order Greeks (Partial List)
  - Vanna, Vomma, Charm, Dvega,Dtime, Vera (or Rhova), etc.
- Third Order Greeks (Partial List)
  - Color (or gamma decay), Speed, Ultima, Zomma, etc.
- Multi-Asset Greeks (Partial List)
  - Cross Gamma
  - Cross Vanna
- Fourth Order Greeks, Fifth Order Greeks etc.
    - Diminishing attribution
- One can come up with more option greeks (of diminishing attribution) than there are Greeks in Greece (about 11 million according to the 2012 census)

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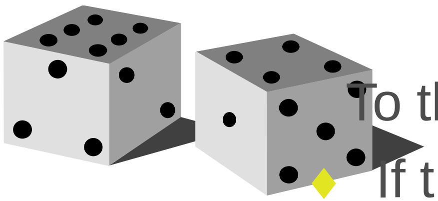
The value of an option is the NPV of the Expected Value of all the potential outcomes of the Option

PRICING OF OPTIONS

How much would you pay today for the right .



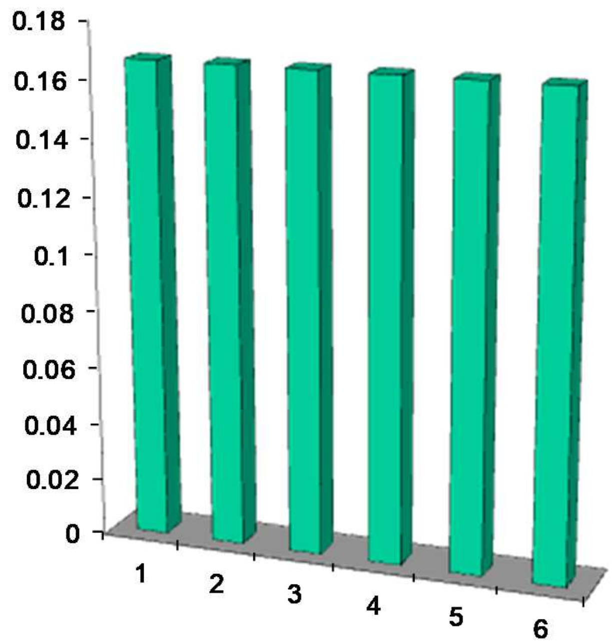
To throw the dice in one year

- If the outcome is greater than 3 you are paid the difference in dollars
- If the outcome is 3 or less, you are being paid nothing
- Interest Rate = 6%

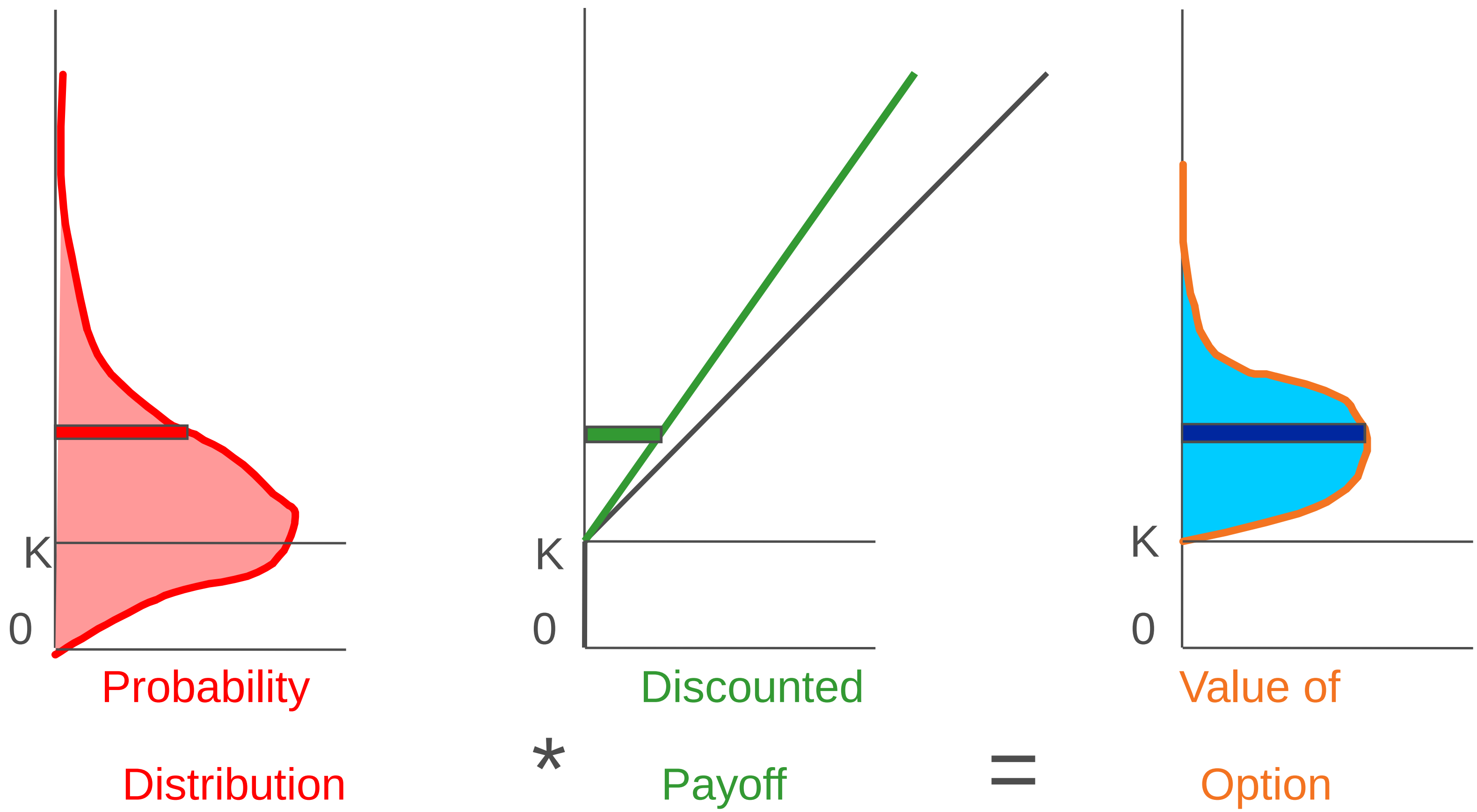
$$\text{Value} = (6-3)*\frac{1}{6}*\frac{1}{1.06} + (5-3)*\frac{1}{6}*\frac{1}{1.06} + (4-3)*\frac{1}{6}*\frac{1}{1.06} + (3-3)*\frac{1}{6}*\frac{1}{1.06} + 0 + 0$$

$$\text{Value} = \$0.94$$

Dice Outcome Distribution

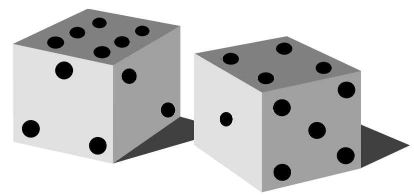


Value of an (not necessarily Black-Scholes) Option



The Dice problem and Black-Scholes

- F=$3, X=$3
- Time to Expiration = 1 year
- Volatility = 265%
- Black-Scholes Value = $2.30
- Distribution of Probabilities



OUTCOME	PROBABILITIES
0	75%
1	11%
2	3%
3	3%
4	1%
5	1%
6	1%
>6	5%

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Option on a NG Futures contract

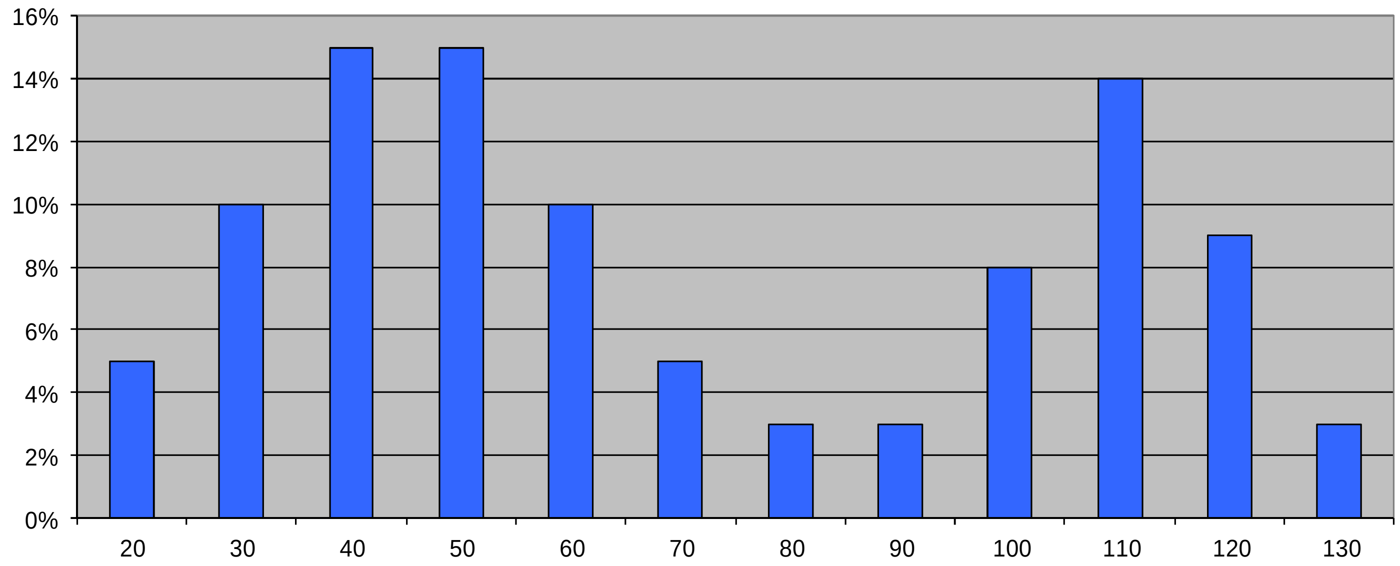
- F = $2.40/mmBtu
- X = $2.40/mmBtu
- Volatility = 40%
- Time to Expiration = 6 months
- Interest Rate = 6%
- Black-Scholes Value = $0.2619

Option on a NG Futures contract (cont.)

- Implicit Assumptions
  - Expected contract price at settlement = $2.40/mmBtu
  - Median Price at settlement = $2.30/mmBtu
  - “Mode” Price at settlement = $2.12/mmBtu
  - Probability distribution of prices

RANGE OF PRICES	PROBABILITIES
0<P<0.5	0%
0.5<P<1	0%
1<P<1.5	6%
1.5<P<2	24%
2<P<2.5	30%
2.5<P<3	21%
3<P<3.5	10%
3.5<P<4	4%
4<P<4.5	1%
4.5<P<5	1%
5<P	1%

Example: Electricity Option



Example: Electricity Option (cont.)

TIME TO EXPIRATION =6 MONTHS	PRICE	PROBABILITY	PAYOFF	DISCOUNT FACTOR	OUTCOME
STRIKE PRICE = $40	20	5%	0	0.97	0
INTEREST RATE = 6%	30	10%	0	0.97	0
	40	15%	0	0.97	0
	50	15%	10	0.97	1.46
	60	10%	20	0.97	1.94
	70	5%	30	0.97	1.46
	80	3%	40	0.97	1.16
	90	3%	50	0.97	1.46
	100	8%	60	0.97	4.66
	110	14%	70	0.97	9.51
	120	9%	80	0.97	6.98
	130	3%	90	0.97	2.62
				VALUE OF THE OPTION	$31.23

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JUMP-DIFFUSION

Jump-Diffusion

$$\frac{dF}{F} = (\mu - \lambda k) \cdot dt + \sigma \cdot dz + dq$$

$$dz = \varepsilon \cdot \sqrt{dt}$$

$$\varepsilon = N(0,1)$$

$$q = \begin{cases} \text{prob(jump } Y \text{ occurs } \textbf{exactly} \text{ once in time interval } dt) = \lambda \cdot dt \\ \text{prob(otherwise)} = 1 - \lambda \cdot dt \end{cases}$$

$$Y = \text{percentage jump of the price of the commodity, at time } t$$

$$F(t + \Delta t) = F(t) \cdot Y \Rightarrow \underbrace{F(t + \Delta t) - F(t)}_{\text{price change}} = Y - 1$$

$$k = E(Y - 1)$$

Jump-Diffusion

$$\frac{dF}{F} = \begin{cases} (\mu - \lambda k) \cdot dt + \sigma \cdot dz & \text{with probability} = 1 - \lambda \cdot \text{dt} \\ (\mu - \lambda k) \cdot dt + \sigma \cdot dz + (Y - 1) & \text{with probability} = \lambda \cdot \text{dt} \end{cases}$$

$$\frac{F_T}{F_0} = \mathbf{W}(\mathbf{n}) \cdot \exp\left[(\mu - \frac{\sigma^2}{2} - \lambda \cdot k) \cdot T + N\left(0, \sigma \cdot \sqrt{T}\right) \right]$$

$$n = \text{number of jumps}$$

$$\mathbf{W}(\mathbf{n}) = \begin{cases} 1 & \text{if n} = 0 \\ \prod_{j=1}^{n} Y_j & n \geq 1 \end{cases}$$

Jump-Diffusion

- If the jumps are log-normally distributed
  - Then the final prices are log-normally distributed

- What is the distribution of the jumps (n) ?

$$pr(n) = \frac{\exp(-\lambda \cdot T) \cdot (\lambda \cdot T)^n}{n!} \qquad \text{(Poisson Distribution)}$$

Jump-Diffusion (Log-normal jump)

$$JD(F,T) = \sum_{n=0}^{\infty} \frac{\exp\left(-\lambda^* \cdot T\right) \cdot \left(\lambda^* \cdot T\right)^n}{n!} \cdot BS\left(F, S, T, \nu_n, r_n\right)$$

$$\lambda^* = \lambda \cdot \left(1 + k\right) = \lambda \cdot E(Y)$$

$$\nu_n = \sqrt{\sigma^2 + \frac{n \cdot \delta^2}{T}}$$

$$\nu_n = \text{volatility of returns when we experience } n \text{ jumps}$$

$$\delta = \text{volatility of each log-normal jump } Y_j$$

$$r_n = r - \lambda \cdot k + \frac{n \cdot \ln\left(1 + k\right)}{T}, \quad \text{adjusted rate of return}$$

Jump-Diffusion

- Parameter Estimation
  - Kendall and Stewart (1977)
  - Beckers (1981)
  - Ball and Torous (1985)
  - Kremer and Roenfeldt (1992)

$$\left(\lambda, k = E(Y), \delta^2 = \operatorname{var}(Y)\right)$$

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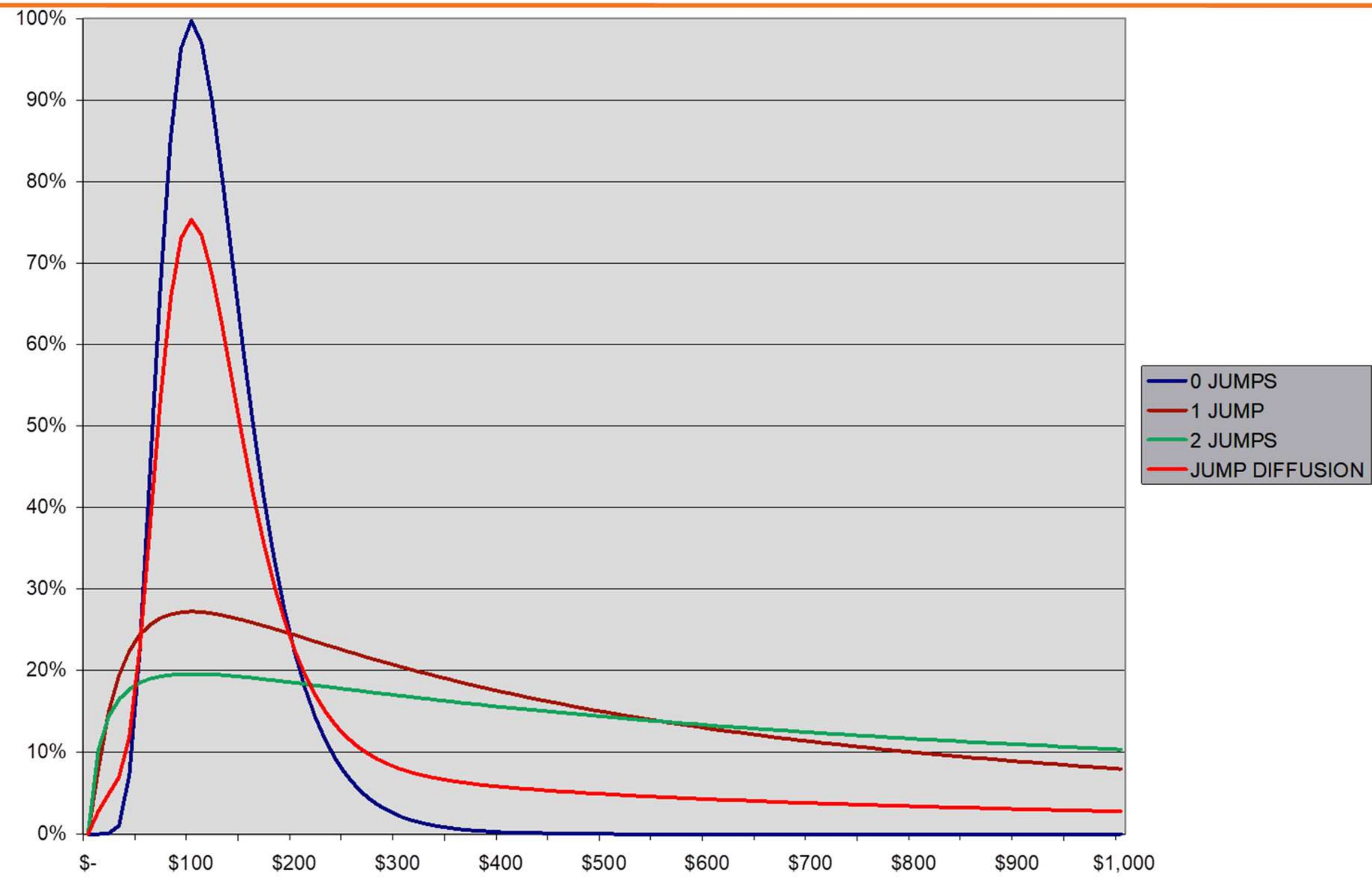
Jump-Diffusion Example

VOL_SIMPLE	L	T	E(Y)	vol(Y)	R_SIMPLE	PRICE
40%	20%	1	2	2	5%	$ 100
					Black-Scholes	$ 15.08
52.4%					**Black-Scholes with 52.4%**	$ 19.65
JUMPS	L*	VOL(N)	R(N)	PROBABILITY	VALUE_IND	VALUE
0	0.4	40%	5%	67.03%	$ 15.08	$ 10.11
1	0.4	147%	54%	26.81%	$ 31.23	$ 8.37
2	0.4	204%	124%	5.36%	$ 20.10	$ 1.08
3	0.4	248%	193%	0.72%	$ 11.41	$ 0.08
4	0.4	286%	262%	0.07%	$ 6.15	$ 0.00
5	0.4	319%	332%	0.01%	$ 3.23	$ 0.00
					Jump-Diffusion	$ 19.65

Jump-Diffusion Example (Distribution)



Deltas of Spark Spread Call Options

$$\text{DELTA}_{\text{ELECTRICITY}} = e^{-rt} \cdot N(d_1)$$

$$\text{DELTA}_{\text{NATGAS}} = -e^{-rt} \cdot h \cdot N(d_2)$$

$$N(d_2) = \text{probability that the option will be in the money at expiration}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{t}$$

Conclusions

- When away from expiration, the delta of NG is -in absolute terms- about half the delta of electricity. Use of the "simple" Black-Scholes delta found in finance textbooks, for hedging of the NG exposure of the spark spread will lead to over-hedging. Depending on the remaining time to expiration, this over-hedging can create an exposure to NG prices that is higher than the original exposure before the hedging

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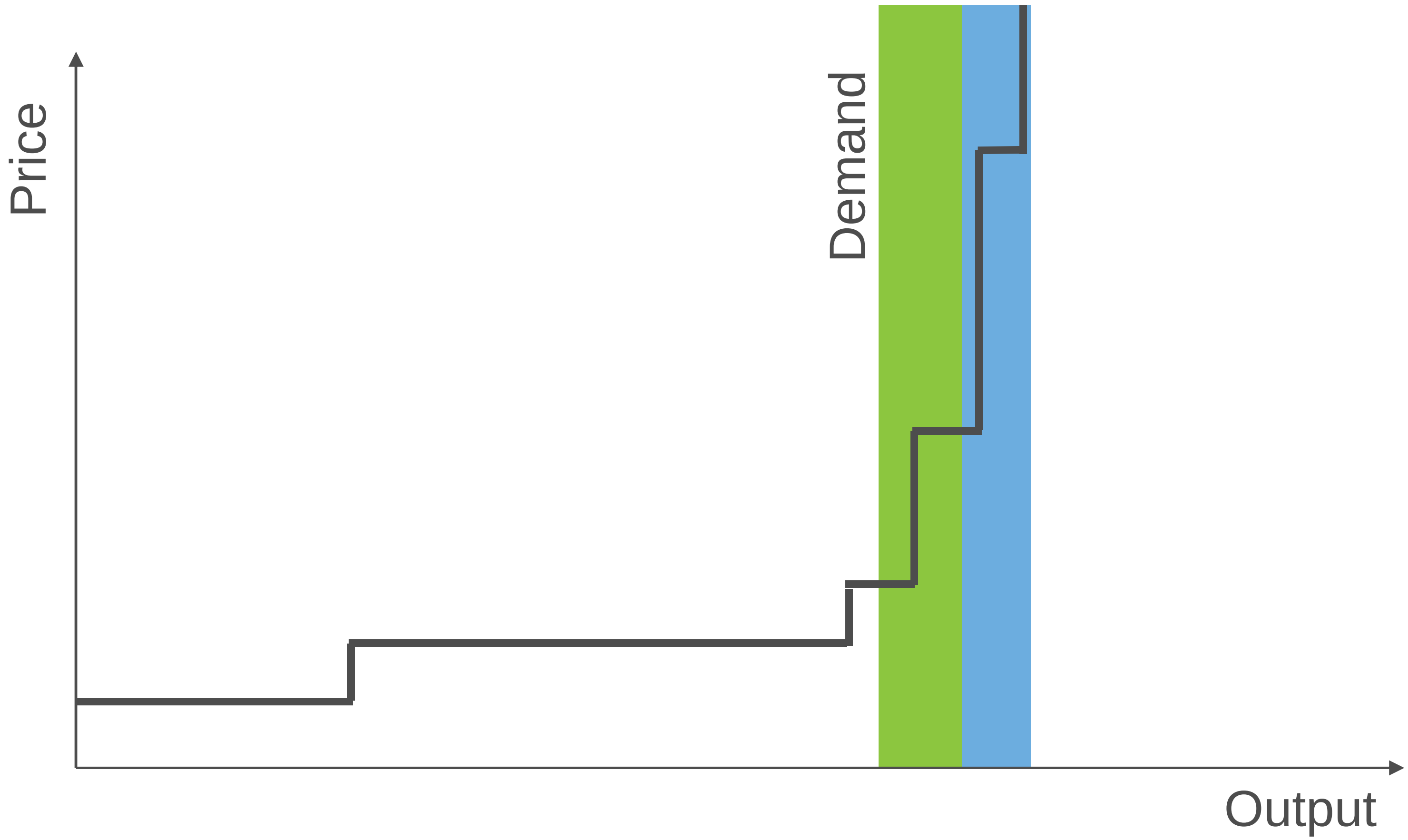
Supply Curve for Electricity (why the spikes ?)



SWING OPTION

SWING OPTION

- *Q* = the total amount of gas that the customer has to purchase during T days (i.e. Q=100 mmBtus during 100 days)
- *K*= the strike price at which the customer buys the gas.
- *q* = the maximum quantity of gas that the customer can buy during any trading day.
- *P* = penalty for each unit of gas that the customer fails to purchase.

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Constraints and Assumptions

$$0 \le x_1 + x_2 + \quad + x_T \le Q$$

$$0 \le x_i \le q, \quad i = 1, \quad , T$$

$$N = \frac{Q}{q}$$

- x is the daily exercised amount
- N is the minimum number of days during which the customer has to purchase gas.
- Assumption:

  P<K

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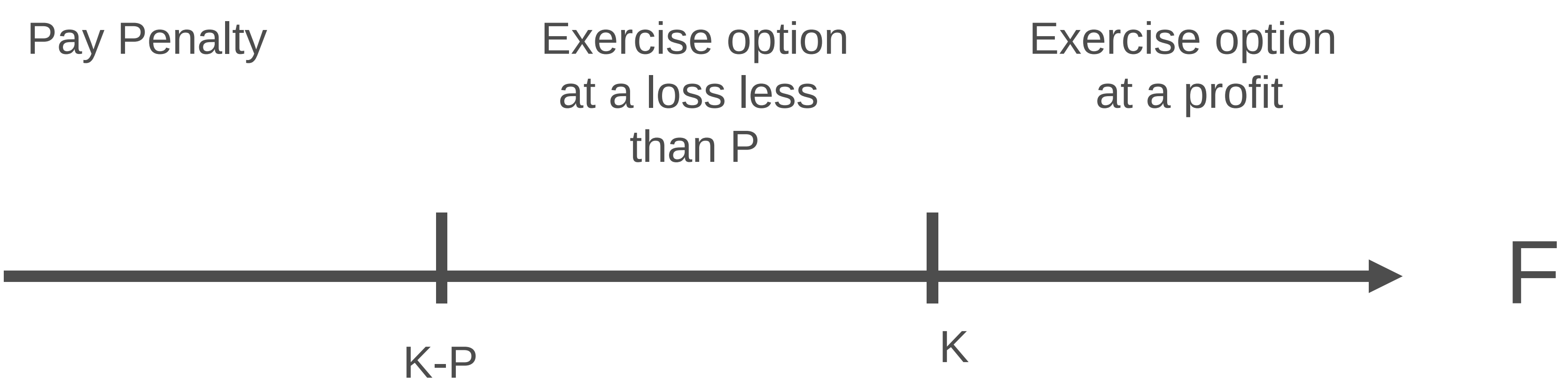
During the last N days



At inception of Contract

$$At \quad time \quad t=0$$

$$MARKET\_MAKER \quad sells$$

$$swing \quad option$$

$$SW = \sum_{t=T-N+1}^{T}\left[Call(F,t,vol,r,strike=K-P) - P\cdot e^{-rt}\right]$$

$$MARKET\_MAKER$$

$$borrows$$

$$B = \sum_{t=T-N+1}^{T} P\cdot e^{-rt}$$

$$MARKET\_MAKER \quad buys$$

$$american \quad options$$

$$A = \sum_{t=T-N+1}^{T} Call(F,t,vol,r,strike=K-P)$$

$$TOTAL$$

$$=0$$

During the first T-N days of the contract

At time t <= T-N	**F>K** Customer wants to exercise the swing option	**F<K, F-K>-P** Customer has to exercise the swing option	**K-F>P** Customer has to pay the penalty
DYNEGY revenues from the swing option it sold	-(F-K)	NO	NO
DYNEGY pays loan	-P	NO	NO
DYNEGY exercises american option	F-(K-P)	NO	NO
TOTAL	=0	=0	=0

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During the last N days of the contract

At time t >= T-N+1	**F>K** Customer wants to exercise the swing option	**F<K, F-K>-P** Customer has to exercise the swing option	**K-F>P** Customer prefers to pay the penalty
DYNEGY revenues from the swing option it sold	-(F-K)	-(F-K)	P
DYNEGY pays loan	-P	-P	-P
DYNEGY exercises american option	F-(K-P)	F-(K-P)	NO
TOTAL	=0	=0	=0

Equivalence: Swing and American options

- Proved: American Option hedges Swing
- Need to prove: Swing hedges American Option

During the first T-N days of the contract

At time t <= T-N	F>K Customer wants to exercise the swing option	F<K, F-K>-P Customer has to exercise the swing option	K-F>P Customer has to pay the penalty
DYNEGY revenues from the swing option it sold	-(F-K)	-(F-K)	NO
DYNEGY pays loan	-P	-P	NO
DYNEGY exercises american option	F-(K-P)	F-(K-P)	NO
TOTAL	=0	=0	=0

Swing Valuation Example

- A customer wants to buy a swing option
  - Q= 100 contracts
  - T=the next 50 trading day
  - q=20 contracts per day
  - N=100/20=5 American options, expiring from the 46th to the 50th day of trading.
  - F=$2/mmBtu, K=$2/mmBtu, P=$0.5/mmBtu
  - r=9%, vol=80%.

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Swing Valuation Example

Option	Expiration Time (t)	Formula	Option Value /mmBtu	Volume (mmBtu's)	Option Value
1	t1=0.184(=46/250)	Call(strike=K-P,t1)-P*exp(-rt1)	0.0651	20	1.302
2	0.188	Call(strike=K-P,t2)-P*exp(-rt2)	0.0668	20	1.336
3	0.192	Call(strike=K-P,t3)-P*exp(-rt3)	0.0685	20	1.37
4	0.196	Call(strike=K-P,t4)-P*exp(-rt4)	0.0702	20	1.404
5	0.2	Call(strike=K-P,t5)-P*exp(-rt5)	0.072	20	1.44
TOTAL					6.852

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