Drill-Bit Parity:

supply-side links in oil and gas markets [*]

by

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Abstract

Previous economic analyses focused on relationships between crude oil and natural gas markets centered primarily on demand-side connections. We provide evidence that two important supply-side connections play important roles in understanding crude oil and natural gas market integration in the U.S. First, crude oil production and natural gas production require common inputs: rotary drilling rigs and the specialized labor necessary to operate them. Competition for these inputs should result in comovement in oil prices and gas prices. Second, crude oil wells produce associated gas, while natural gas wells often produce associated oil or oil substitutes. This latter supply-side connection could cause divergence in crude oil prices and natural gas prices, because a price shock for one commodity will increase associated production of the other commodity. We construct a theoretical model to delineate the roles played by these supply-side factors in oil and gas market integration. Commodity specific drilling is negatively related to cross-commodity price shocks when input competition is the dominant supply-side link. Input competition leads commodity prices to increase as a result of cross-commodity price shocks leading to comovement in oil and gas prices. Commodity specific drilling is ambiguously related to cross-commodity price shocks when associated commodity flows form the dominant supply-side links. Associated commodity flows lead commodity prices to decrease as a result of cross-commodity price shocks leading to divergence on oil and gas price, which occurred in the U.S. between 2008 and 2015. We test the predictions of the theoretical model using data from more than 100,000 wells from five large oil and gas producing basins in Kansas, Oklahoma, and Texas. We find substantial evidence that rig-competition leads oil drilling to decrease in response to natural gas price shocks across all five basins. We find some evidence that associated oil flows lead gas drilling to increase in response to oil price shocks. Finally, we investigate the impact of these supply-side links on regional price relationships, and find that different supply-side links appear to dominate in different basins.

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1 Introduction

Following the financial crisis of 2008 and throughout the subsequent boom in shale oil production, U.S. crude oil and natural gas prices diverged substantially (see Figure 1). This divergence led to analyses indicating the two price series were no longer linked (Erdős, 2012; Ramberg and Parsons, 2012). Past studies generally alluded to demand-side considerations in explanations of the crude oil, natural gas price relationship, because oil and gas are extracted to the same ultimate end: the production of energy (Brown and Yücel, 2008; Hartley et al., 2008). However, supply-driven linkages are plausible as well, as drilling rigs, and the specialized labor that operates them, are the primary inputs necessary for drilling crude oil wells and natural gas wells, so we might expect cost spillovers from drilling wells. Figure 2 displays time series of the employment of gas-directed drilling rigs and oil-directed drilling rigs from January 1997 to June 2016. Further, joint production of natural gas from crude oil wells (sometimes called associated gas) is so commonplace that many analysts believe changes in oil-related drilling activity directly affect natural gas market equilibrium (Wall Street Journal, 2016; Bloomberg, 2015). Similarly, "wet" natural gas wells produce large amounts of liquid hydrocarbons, including oil. These associated commodity flows could cause oil and gas prices to diverge: if a demand-shock increases the price of one of the commodities, the supply of the other commodity would increase through this channel, decreasing its price. Further, the flow of output from each new well is geologically constrained, and supply decisions are highly persistent as a result.

Production of shale gas and tight oil from onshore reservoirs in the U.S. has become increasingly important since the mid-to-late 2000's, which has likely increased the cost spillovers from one commodity to the other, as scarce capital and labor resources are allocated across the two commodities. At the same time, demand-side substitution, which forms the basis for the demand-side link between oil and gas markets, decreased. Figures 1 and 2 provide anecdotal evidence to this effect. Before 2008, crude oil and natural gas rig allocations tended to move together, while crude oil and natural gas prices also tended to move together, which is consistent with a demand-side dominated relationship. However, crude oil prices recovered and remained elevated relative to natural gas prices after the financial crisis in 2008. This led to a shift of drilling rig allocations from natural gas to oil. Eventually, oil prices decreased dramatically as a result of a worldwide surplus of crude oil production, which can be at least partially explained by increased drilling in the U.S.

We provide a model of the drilling problem, similar to that of Anderson et al. (2014) and Okullo et al. (2015), that clarifies the importance of each of these features in tying U.S. natural gas markets and crude oil markets through the supply side. Our theoretical model and empirical tests focus on onshore crude oil and natural gas production in the United States. The increasing importance of shale gas and tight oil supply from onshore wells in the U.S. tp the global crude oil market, and increasingly global natural gas market, implies that these supply-side links have global implications. Future studies should aim to analyze supply-side links in other oil and gas producing regions around the world.

The theoretical model implies drilling for commodity i may be negatively or positively related to the price of the cross-commodity j in a supply-side regime, depending on whether the supply-side relationship is primarily dictated by rig competition or by associated commodity flows. We estimate crude oil drilling responses to natural gas price shocks, and natural gas drilling responses to crude oil price shocks using data from five crude oil and natural gas producing basins in Texas and Oklahoma. We find that oil drilling responds negatively to natural gas price shocks in all five sample basins implying that rig competition dominates associated gas flows in these basins. Gas drilling responds negatively to oil price shocks in the Permian Basin, our largest sample basin in terms of total wells drilled and total production, implying that rig competition dominates associated oil flows there. On the other hand, gas drilling responds positively to oil price shocks in the Anadarko Basin implying that associated oil flows dominate rig competition there. We also investigate the impact of these supply-side links on regional cross-commodity prices, and find that price movements are generally consistent with the predictions of the theoretical model.

2 Background

Understanding the relationship between oil and natural gas markets is important for several reasons. The shale boom made the U.S. the largest producer of both crude oil and natural gas,[1] and helped dampen the 2007-2009 recession and expedite the subsequent recovery in shale-rich regions (Lim, 2011; Grunewald and Mahon, 2011; Brown and Yücel, 2013). A large literature documents the impact of oil price shocks on the U.S. economy.[2] More recently Hausman and Kellogg (2015) estimate shale-related production of natural gas created an annual increase of $48 billion in consumer and producer surplus, while Arora and Lieskovsky (2014) find that the impact of natural gas supply on industrial production has increased in the shale era. The increase in production of crude oil and natural gas has also significantly shifted environmental benefits and costs (Johnsen et al., 2016; Knittel et al., 2015; Linn et al., 2014; Muehlenbachs et al., 2013, 2015; Olmstead et al., 2013). Crude oil and natural gas are responsible for a large share of U.S. emissions of carbon dioxide and many other pollutants. Environmental or other policies that directly affect one commodity may indirectly distort the emissions rates of carbon and associated co-pollutants from the other commodity. The extent to which this occurs depends on the mechanisms by which the two markets are linked.

The primary demand-side substitution possibility that has explained the price link between oil and natural gas has declined dramatically since the 1980's: residual fuel oil is the primary oil product that can be substituted with natural gas for electricity generation, and/or space heating, and residual fuel oil supplied to U.S. consumers fell from over a billion barrels per year in 1977 to less than a million barrels in 2015. In 2003, net electricity generation from petroleum liquids was greater than 100,000 Gigawatt hours (GWh), while net generation from natural gas was approximately 650,000 GWh. In 2013, net generation from petroleum liquids had decreased to less than 14,000 GWh, while natural gas generation increased to more than 1.1 Million GWh.[3]

Previous analyses of oil and gas price links focused on robust opportunities for demand-side sub-

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[1] See British Petroleum Statistical Review of World Energy (`http://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html`).

[2] See Balke et al. (2002) and Baumeister and Kilian (2016) for excellent surveys of this literature.

[3] See EIA Table 3.1.A. (`http://www.eia.gov/electricity/annual/html/epa_03_01_a.html`).

stitution. Such substitution possibilities were assumed to derive a “burner-tip parity” condition, by which oil and gas prices had a long-run relationship kept in check by cost-minimizing decisions of energy producers (Brown and Yücel, 2008). If producing energy was less expensive using residual fuel oil than natural gas then energy producers would shift consumption toward fuel oil, thus bidding up its price, and vice versa. The decrease in demand-side substitution possibilities, accompanied by the increase in onshore drilling for oil and natural gas in the continental U.S., motivates our supply-side driven analysis.

The observed divergence between oil and gas prices starting in 2008 led to a new line of research analyzing the instability of the statistical relationship. See for example Erdős (2012), Ramberg and Parsons (2012), Aloui et al. (2014), Atil et al. (2014), Brigida (2014), Hartley and Medlock III (2014), although instabilities had been documented in some of the earlier literature that established the relationship in the first place, e.g., Villar and Joutz (2006), Brown and Yücel (2008) and Hartley et al. (2008). With the exception of Hartley and Medlock III (2014) who provide a model to clarify and test the role of exchange rates in determining the relative price of crude oil and natural gas this research is almost entirely based on reduced-form time series analysis. Such reduced-form analyses may be acceptable if demand-side substitution were the only mechanism tying together oil and gas markets. However, such models are not helpful when the demand-driven relationship breaks down. We provide a structural model to shed light on the supply-side incentives that tie oil and gas markets together.

There are three mechanisms that might create a link between crude oil and natural gas markets: demand-side substitution, supply-side rig competition, and supply-side substitution from associated commodity flows. Responses to price shocks will depend on the relative magnitudes of these effects. In a demand-driven regime, a positive oil price shock would lead consumers to shift their consumption toward natural gas, for example by increasing their use of natural gas relative to oil in electricity generation. This in turn would lead to a higher gas price, and increase gas drilling. A positive natural gas price shock would lead to a higher oil price, and increased oil drilling in a demand-driven regime for the same reasons.

In a supply-driven regime based on rig competition, a positive oil price shock would lead to increased oil drilling, which would increase the marginal cost of natural gas drilling, and decrease natural gas drilling, as our theoretical model will show. This decrease in natural gas drilling would lead to higher natural gas prices. Similarly, a positive natural gas price shock would lead to decreased oil drilling, and a higher oil price in a supply-side regime dominated by drilling-rig competition. Contrarily, if the supply-driven regime is dominated by associated commodity flows, i.e., oil wells produce marketable quantities of natural gas, and/or natural gas wells produce marketable quantities of liquids, then it is possible that a positive natural gas price shock will lead to increased oil drilling, and a positive oil price shock will lead to increased gas drilling. Further, in a regime dominated by associated commodity flows, and absent cost spillovers, cross-commodity price shocks always lead to diverging oil and gas prices. In reality, both demand-side substitution, supply-side competition for drilling rigs and other scarce inputs, and supply-side substitution driven by associated commodity flows all likely occur simultaneously, so the empirical question of interest is which of these factors dominate at any given time and place.

3 Model

Consider a social planner who maximizes total surplus in the economy by allocating drilling rigs to natural gas reservoirs or crude oil reservoirs. Let the state variables $z_o(t)$ and $z_g(t)$ represent the flow of crude oil and natural gas from the wells at time t. At each instant, the planner invests in increasing the flow of each commodity by amounts $q_o(t)$ and $q_g(t)$, which represent the levels of drilling activity, or rig allocations, in crude oil and natural gas reservoirs at time t. The history of these rig allocations determines the current flow of output $z_o(t)$ and $z_g(t)$.

The rates of production from previously drilled oil and natural gas wells, $z_o(t)$ and $z_g(t)$, decline over time due to geological characteristics of the underlying reservoir (Cronquist, 2001).[4] For our purposes, we assume that the geological rate of decline is constant across wells and through time within a well, or that there is a constant exponential decline rate, which may differ between crude

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[4] In Anderson et al. (2014) the state variable is production flow capacity, and actual production flow is a control variable. However, those authors also show that the production flow will be equal to production flow capacity unless there is a steep contango in the commodity market, which is historically rare, so we simplify the model by implicitly assuming that flow capacity is always constrained.

oil and natural gas.[5] Let α_o denote the decline rate for oil wells, and let α_g denote the decline rate for natural gas wells.

We assume the increase in gas flow associated with oil drilling, and the increase in oil flow associated with gas drilling can be characterized by proportions, ψ_g and ψ_o, of the respective drilling levels, where ψ_g and ψ_o are in [0,1). That is, when a firm increases oil flow by $q_o(t)$ through drilling, it also increases gas flow by $\psi_g q_o(t)$, and vice versa. We can now write the dynamics of crude oil and natural gas production as

$$\dot{z}_o(t) = q_o(t) - \alpha_o z_o(t) + \psi_o q_g(t) \tag{1}$$

and

$$\dot{z}_g(t) = q_g(t) - \alpha_g z_g(t) + \psi_g q_o(t).^{6} \tag{2}$$

Equations (1) and (2) indicate that production of the commodities increases with drilling levels and decreases due to geological decline. The exponential-decline assumption leads production to decline by a constant fraction of total production. Production of oil and natural gas also increases when the firm drills due to associated commodity flows, which are captured by the last terms on the right-hand sides of (1) and (2).

Oil and natural gas wells are drilled into scarce reservoirs. There are a limited number of areas available for drilling, and when an additional well is drilled, the number of drilling prospects available decreases. Suppose the stock of drilling prospects available for crude oil at t is $A_o(t)$, while the stock of drilling prospects available for natural gas is $A_g(t)$, with initial drilling-prospect stocks $A_o(0) = A_{o0}$ and $A_g(0) = A_{g0}$ given exogenously. The dynamics of drilling prospect availability for

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[5] We make this simplifying assumption for tractability: natural gas and crude oil wells often exhibit hyperbolic decline, implying that the decline rate decreases as the producing life of the well increases. However, we believe exponential decline offers a reasonable approximation for our purposes.

[6] The firm's total production of gas at t is

$$z_g(t) = \int_0^t [q_g(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds.$$

Using Leibniz Rule to take the time derivative gives

$$\dot{z}_g(t) = -\alpha \int_0^t [q_o(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds + q_g(t) + \psi_g q_o(t) = -\alpha z_g(t) + q_g(t) + \psi_g q_o(t).$$

oil and gas are then

$$\dot{A}_o(t) = -q_o(t) \tag{3}$$

and

$$\dot{A}_g(t) = -q_g(t). \tag{4}$$

It is costly to drill oil and natural gas wells. We assume that drilling costs for each commodity can be broken into three components: costs that are common to drilling for both commodities, costs that are commodity specific, and costs related to gathering associated-commodity flows. For example, a drilling rig must be allocated whether oil or gas is being drilled for, so the cost of allocating a rig is common to both commodities. Similarly, specialized labor represents a scarce common input to drilling oil and natural gas wells. The upshot is that when these scarce inputs are used to drill a well of one commodity type, costs associated with using these scarce inputs spillover into drilling of the other commodity type. We denote the costs that are common to both commodities $C(q_o(t) + q_g(t))$, where the argument in this cost function is the sum of drilling levels. We assume these costs are increasing and convex in total drilling.

Each commodity also has costs of initiating production that are unique to that commodity. These costs are often associated with takeaway capacity. For example, a new gas well will require pipeline capacity purchases, and may even require the construction of new pipelines, whereas oil from a new oil well can be taken away by truck or other transportation mode which may need to be expanded. The portions of cost that are commodity specific are assumed to be additively separable, and are denoted $C_o(q_o(t))$ for crude oil, and $C_g(q_g(t))$ for natural gas. We assume that these cost functions are increasing and convex in their respective drilling intensities, which represents scarcity of inputs involved in building out takeaway capacity, and other infrastructure necessary for initiating new streams of production.

The third components of drilling costs are related to associated-commodity flows are denoted $C_{\psi_o}(\psi_o q_g)$ for associated oil and $C_{\psi_g}(\psi_g q_g)$ for associated gas. These costs are separated from the commodity-specific flow costs in the model, because collecting associated gas from oil wells may differ in fundamental ways from gathering natural gas from gas wells. For example, gas wells are more likely to be near pre-existing natural gas pipelines than oil wells. We assume these functions are increasing and convex, and that $C_{\psi_o}(0) = C_{\psi_g}(0) = 0$.

Consumer surplus associated with oil and gas is increasing in oil and gas flows, $z_o(t)$ and $z_g(t)$, and is also increasing in exogenous shock parameters, $s_o(t)$ and $s_g(t)$. For example, cold weather increases consumer surplus associated with a fixed supply of natural gas. Similarly, an OPEC oil embargo against the United States would increase consumer surplus associated with U.S. oil production – our geographic market of interest. Let $U_g(z_g(t), s_g(t))$ denote consumer surplus associated with natural gas flow at time t, and let $U_o(z_o(t), s_o(t))$ represent consumer surplus associated with oil flow at time t. We assume consumer surplus associated with each commodity is increasing and concave in flows. The marginal consumer surplus associated with each commodity will be the price of that commodity, so $\dfrac{\partial U_g(z_g(t), s_g(t))}{\partial z_g(t)} = P_g(z_g(t), s_g(t))$ and $\dfrac{\partial U_o(z_o(t), s_o(t))}{\partial z_o(t)} = P_o(z_o(t), s_o(t))$. Further, we assume that demand shocks increase the marginal value of oil and natural gas, or $\dfrac{\partial P_o(z_o(t), s_o(t))}{\partial s_o(t)} > 0$ and $\dfrac{\partial P_g(z_g(t), s_g(t))}{\partial s_g(t)} > 0$.

We also assume that total consumer surplus is the sum of surplus associated with oil and surplus associated with gas. The social planner will maximize consumer surplus less production costs, so the social planner's problem can be written (with time dependence suppressed)

$$\begin{aligned}
\max_{q_o, q_g} \quad & \int_{t=0}^{\infty} \left[U_o(z_o, s_o) + U_g(z_g, s_g) - C(q_o + q_g) - C_o(q_o) - C_g(q_g) - C_{\psi_o}(\psi_o q_g) - C_{\psi_g}(\psi_g q_o)\right] e^{-rt} dt \\
\text{subject to} \quad & \dot{z}_o = q_o - \alpha_o z_o + \psi_o q_g, \\
& \dot{z}_g = q_g - \alpha_g z_g + \psi_g q_o, \\
& \dot{A}_o = -q_o, \; A_{o0} \text{ given}, \\
\text{and} \quad & \dot{A}_g = -q_g, \; A_{g0} \text{ given},
\end{aligned} \tag{5}$$

where r is the discount rate. The current-value Hamiltonian associated with (5) is

$$\begin{aligned}
\mathcal{H} = {} & U_o(z_o, s_o) + U_g(z_g, s_g) - C(q_o + q_g) - C_o(q_o) - C_g(q_g) - C_{\psi_o}(\psi_o q_g) - C_{\psi_g}(\psi_g q_o) \\
& - \theta_o q_o - \theta_g q_g + \mu_o(q_o - \alpha_o z_o + \psi_o q_g) + \mu_g(q_g - \alpha_g z_g + \psi_g q_o)
\end{aligned} \tag{6}$$

The θ_o and θ_g variables are the shadow values of oil and gas drilling prospects, and represent the opportunity cost associated with using up a scarce drilling prospect at the current instant. The marginal value of production of oil and gas is more than their respective prices in this model, because a marginal unit of flow added in the current instant through drilling represents a stream of future production. The present values of these production streams at t are μ_o and μ_g.

Assuming an interior solution for drilling in both commodities, the first-order necessary conditions for surplus-maximizing rig allocations associated with (5) are

$$-C'(q_o+q_g)-C'_o(q_o)-\psi_g C'_{\psi_g}(\psi_g q_o)-\theta_o+\mu_o+\psi_g\mu_g=0, \tag{7}$$

$$-C'(q_o+q_g)-C'_g(q_g)-\psi_o C'_{\psi_o}(\psi_o q_g)-\theta_g+\mu_g+\psi_o\mu_o=0, \tag{8}$$

$$-P_o(z_o,s_o)+\alpha_o\mu_o=\dot{\mu}_o-r\mu_o, \tag{9}$$

$$-P_g(z_g,s_g)+\alpha_g\mu_g=\dot{\mu}_g-r\mu_g, \tag{10}$$

$$\dot{\theta}_o-r\theta_o=0, \tag{11}$$

and

$$\dot{\theta}_g-r\theta_g=0. \tag{12}$$

The following transversality conditions are also necessary for the optimality of a drilling program

$$\lim_{t\to\infty}\theta_o(t)A_o(t)=0, \tag{13}$$

$$\lim_{t\to\infty}\theta_g(t)A_g(t)=0, \tag{14}$$

$$\lim_{t\to\infty}\mu_o(t)z_o(t)=0, \tag{15}$$

and

$$\lim_{t\to\infty}\mu_g(t)z_g(t)=0. \tag{16}$$

3.1 Interpretation of Necessary Conditions

Equations (7) and (8) indicate that the social planner should balance the marginal cost associated with allocating additional rigs to the marginal benefits. The first four terms on the left-hand sides of (7) and (8) represent the marginal costs of rig allocation. The first term in each equation, $C'(q_o+q_g)$ is the marginal cost of allocating an additional rig regardless of commodity choice. The second term in each equation is the commodity-specific marginal cost, which represents the increase in cost associated with allocating an additional rig towards oil in (7), and the marginal cost of allocating an additional rig towards natural gas in (8). Further, we assumed that associated-commodity flows increase costs. The marginal costs from increased associated commodity flows are represented by the third terms in (7) and (8). The initiation of a marginal unit of oil flow will be accompanied by an increase in natural gas flow of size ψ_g. This increase in natural gas flow will increase costs by $\psi_g C'_{\psi_g}(\psi_g q_o)$. Oil-drilling costs will similarly be increased via associated oil from natural gas wells. The fourth terms in (7) and (8) represent the user costs associated with converting a drilling prospect to commodity flow. These costs derive from the fact that drilling prospects are exhaustible resources.

The last two terms in the left-hand sides of (7) and (8) represent the marginal benefits associated with initiating marginal units of oil and gas flow through rig-allocation decisions. For example, μ_o is the marginal benefit of having an additional unit of oil flow at time t by definition. Thus, when an additional unit of oil flow is initiated through rig allocation at time t, the marginal benefit is μ_o. Further, the natural gas flow will increase by ψ_g when an additional unit of oil flow is initiated, and $\psi_g \mu_g$ is the marginal benefit of this additional associated-gas flow. In order for maximum surplus to be achieved through strictly positive rig-allocations, it is necessary that the marginal costs and marginal benefits described here sum to zero.

Equations (9) and (10) define the inter-temporal marginal value of oil flow and natural gas flow. A unit of oil or gas production flow at time t implies an infinite stream of future production whose value will decrease at instantaneous rate r due to discounting. The amount of flow will decrease at instantaneous rate α_i, for $i = \{o, g\}$ due to the natural geologic decline in production. Thus, production flow is discounted at instantaneous rate $r + \alpha_i$ when discounting and production decline are both taken into account. Equations (9) and (10) can be rearranged into the form $\mu_i = (P_i(z_i, s_i) + \dot{\mu}_i)/(\alpha_i + r)$. The value of a marginal unit of commodity flow is similar to the value of a perpetuity due to the associated infinite stream of production. Inter-temporal changes in the value of flow, possibly associated with changes in demand, or the scarcity of drilling prospects, are fed into the marginal value of flow through the $\dot{\mu}_i$ term.

Equations (11) and (12) imply that drilling prospects should be managed in such a way that their value increases at the discount rate, so that drilling prospects are competitive with other assets in the economy.

The transversality conditions represented in (13) and (14) show that drilling prospects must be completely depleted, or have zero marginal value in the long run. In the case that a finite number of drilling prospects are available, the combination of (11) and (13) imply that oil drilling prospects will be completely depleted in the long run, while the combination of (12) and (14) imply that gas drilling prospects will be completely depleted in the long run. The depletion of oil or gas drilling prospects imply that (15) and (16) will hold, as after the last drilling prospects are converted through drilling, production will decrease at the constant exponential decline rates and approach zero in the limit.

3.2 Infinite Well Model Results

In order to maintain focus on supply-side links between gas and oil markets in this paper, we analyze the "infinite well" case.[7] In this case, the social planner (or competitive market) behaves as though there are an infinite number of oil and gas drilling prospects available for conversion to production through drilling. This need not mean that market participants actually believe there are an infinite number of drilling prospects available, but rather could mean that market participants believe lower-cost backstop technologies will become available prior to depletion of oil and gas drilling prospects. The assumption of infinite drilling prospect availability implies that $\theta_o(t)$ and $\theta_g(t)$ must go to zero in the limit by (13) and (14), which in conjunction with (11) and (12) implies that

$$\theta_o(t) = \theta_g(t) = \dot{\theta}_o(t) = \dot{\theta}_g(t) = 0 \quad \text{for all} \quad t. \tag{17}$$

The infinite drilling prospect assumption leads to the possibility of achieving a steady-state in which drilling rate, production rates, and shadow values remain constant. In particular, in a steady-state, we have $\dot{\mu}_o = \dot{\mu}_g = 0$, and we can rewrite (9) and (10) as

$$\mu_o = \frac{P_o(z_o, s_o)}{\alpha_o + r} \tag{18}$$

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[7] See Anderson et al. (2014).

and

$$\mu_g = \frac{P_g(z_g, s_g)}{\alpha_g + r}. \tag{19}$$

In a steady-state, prices and the value of production will not change. Therefore, the value of a marginal unit of production flow is exactly the value of a perpetuity discounted at the sum of the discount rate and the commodity-specific decline rate. Equations (18) and (19) can be substituted into (7) and (8) to get a full accounting of the marginal benefits and marginal costs of drilling decisions in the steady state. We have

$$\underbrace{\frac{P_o(z_o, s_o)}{\alpha_o + r} + \frac{\psi_g P_g(z_g, s_g)}{\alpha_g + r}}_{\text{Marginal Benefit } (q_o\uparrow)} - \underbrace{C'(q_o + q_g) - C_o'(q_o) - \psi_g C_{\psi_g}'(\psi_g q_o)}_{\text{Marginal Cost } (q_o\uparrow)} = 0 \tag{20}$$

$$\underbrace{\frac{P_g(z_g, s_g)}{\alpha_g + r} + \frac{\psi_o P_o(z_o, s_o)}{\alpha_o + r}}_{\text{Marginal Benefit } (q_g\uparrow)} - \underbrace{C'(q_o + q_g) - C_g'(q_g) - \psi_o C_{\psi_o}'(\psi_o q_g)}_{\text{Marginal Cost } (q_g\uparrow)} = 0. \tag{21}$$

Equations (20) and (21) show that the planner will balance marginal benefits and marginal costs in the steady-state of an infinite-well optimal drilling program. The first terms in (20) and (21) represent the direct marginal benefit of initiating a marginal unit of oil or gas flow through drilling. This marginal benefit is the initiation of a perpetuity with payoff $P_o(z_o, s_o)$ or $P_g(z_g, s_g)$ per instant. The perpetuity is discounted at $r + \alpha_o$ or $r + \alpha_g$ where the latter terms result from the natural decline in production from oil or gas wells.

The initiation of a marginal unit of oil or gas production through drilling is accompanied by the initiation of ψ_g units of gas production or ψ_o units of oil production through the associated commodity channel. The second terms in equations (20) and (21) account for the marginal benefit from associated commodity flows. The initiation of associated commodity flows will result in infinite payments similar to those of the direct commodity flow, however, this marginal benefit is weighed by ψ_g or ψ_o.

The third terms in (20) and (21) represent the marginal cost of allocating additional units of drilling inputs, irrespective of commodity. Costs will increase for both commodities with additional drilling for either commodity. The fourth terms in (20) and (21) represent the increase in commodity-specific costs associated with drilling for oil or gas, and the fifth terms represent the increase in costs that takes place as a result of initiating associated commodity flows. These last costs are weighed by the associated commodity parameters, ψ_g and ψ_o.

We can substitute the z_o and z_g terms out of (20) and (21) under the assumption that drilling has converged to an infinite-well steady state. In an infinite-well steady state, we have $\dot{z}_o = \dot{z}_g = 0$, substituting these values into (1) and (2), and solving for z_o and z_g gives

$$z_o = \frac{q_o + \psi_o q_g}{\alpha_o} \tag{22}$$

and

$$z_g = \frac{q_g + \psi_g q_o}{\alpha_g}. \tag{23}$$

Substituting these steady-state production values into (20) and (21) gives

$$f_o: \ \frac{P_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o + r} + \frac{\psi_g P_g\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g + r} - C'(q_o + q_g) - C_o'(q_o) - \psi_g C'_{\psi_g}(\psi_g q_o) \equiv 0 \tag{24}$$

and

$$f_g: \ \frac{P_g\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g + r} + \frac{\psi_o P_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o + r} - C'(q_o + q_g) - C_g'(q_g) - \psi_o C'_{\psi_o}(\psi_o q_g) \equiv 0 \tag{25}$$

Equations (24) and (25) are two identities in two choice variables, q_o and q_g, defined at each instant t, when evaluated on the optimal drilling paths.[8] Thus, the implicit-function theorem can be applied in order to predict optimal drilling responses to parameter changes. The comparative statics of primary interest are the optimal steady-state drilling responses to own-commodity and cross-commodity demand shocks, i.e., the changes in optimal oil drilling rates in response to gas-market demand shocks and vice versa. In the Mathematical Appendix, we show that

$$\frac{\partial q_g}{\partial s_g} = \frac{\frac{-P_g^s}{\alpha_g+r}\left[\frac{(1-\psi_g\psi_o)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C''_{\psi_g}\right]}{|H|}, \tag{26}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\frac{-P_o^s}{\alpha_o+r}\left[\frac{(1-\psi_g\psi_o)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' - C_g'' - \psi_o^2 C''_{\psi_o}\right]}{|H|}, \tag{27}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o+r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C''_{\psi_g}\right]}{|H|} \tag{28}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g+r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C''_{\psi_o}\right]}{|H|}. \tag{29}$$

where $|H| > 0$ is the determinant of the Hessian matrix associated with (24) and (25). The own-price derivatives, defined in equations (26) and (27), are positive as expected as a result of the

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[8] The Mathematical Appendix section also confirms that under the assumptions of the model, the identities (24) and (25) characterize a maximum solution to the planner's problem.

concavity of the utility function and convexity of the cost functions, while the signs of the cross-price derivatives, defined in (28) and (29), are indeterminate and will depend on the magnitudes of the associated-commodity parameters, and the degree of convexity of the various drilling cost functions. Note that if $C'' = \psi_g = \psi_o = 0$, the cross-price derivatives evaluate to zero, and supply-side links are totally absent.

We define a *cost-spillover regime* as a regime in which cost spillovers, articulated in the model with the $C(q_o + q_g)$ function, play an important role in oil and gas market, while associated-commodity flows play an arbitrarily small role. In such a regime, we have $C'' > 0$ and $\psi_g = \psi_o = 0$. Similarly, we define an *associated-commodity regime* as a regime in which associated-commodity flows play an important role in oil and gas markets, while cost spillovers play an arbitrarily small role. In such a regime, we have $\psi_g > 0$ and/or $\psi_o > 0$, and $C'' = 0$. These definitions lead us to two testable propositions.

Proposition 1: *In a cost-spillover regime, a positive oil price shock decreases the steady-state rate of natural gas drilling, and a positive natural gas price shock decreases the steady-state rate of oil drilling.*

Proof. If we substitute $\psi_g = \psi_o = 0$ from the definition of a cost-spillover regime into the comparative statics defined by equations (28) and (29), we have

$$\frac{\partial q_g}{\partial s_o} = -\frac{P_o^s C''}{\alpha_o + r} \Big/ |H| < 0 \tag{30}$$

and

$$\frac{\partial q_o}{\partial s_g} = -\frac{P_g^s C''}{\alpha_g + r} \Big/ |H| < 0. \tag{31}$$

The signs of the derivatives defined by equations (30) and (31) result from the assumption that $C'' > 0$ in a cost-spillover regime. □

Proposition 1 indicates that we should expect drilling rates to diverge when a positive gas or oil price shock occurs if supply-side links between oil and gas markets are primarily characterized by drilling-cost spillovers.

In an associated-commodity regime, a positive oil price shock has an ambiguous effect on the steady-state rate of natural gas drilling, because there are two opposite effects. Increased oil drilling associated with the price shock will increase the supply of associated gas, and thus lower the price of natural gas. On the other hand, associated oil produced from gas wells will be more valuable. The situation is analogous in the case of a natural-gas price shock. We present Proposition 2 in two parts based on which of these effects dominates.

Proposition 2a: *In an associated-commodity regime, if*

$$\left|\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)}\right| > \psi_o\left[C_o'' + \psi_g^2 C_{\psi_g}''\right] \tag{32}$$

then a positive oil price shock leads to a decrease in the steady-state rate of natural gas drilling, and if

$$\left|\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)}\right| > \psi_g\left[C_g'' + \psi_o^2 C_{\psi_o}''\right] \tag{33}$$

then a positive natural gas price shock leads to a decrease in the steady-state rate of oil drilling.

Proof. If we substitute $C'' = 0$ from the definition of an associated-commodity regime into the comparative statics defined by equations (28) and (29), we have

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o+r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]}{|H|} \tag{34}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g+r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]}{|H|}. \tag{35}$$

The terms in the brackets on the right-hand sides of (34) and (35) are negative by (32) and (33). Therefore, under the assumptions of Proposition 2a cross-price drilling rate responses are negative. □

Proposition 2b: *In an associated-commodity regime, if*

$$\left|\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)}\right| < \psi_o\left[C_o'' + \psi_g^2 C_{\psi_g}''\right] \tag{36}$$

then a positive oil price shock leads to an increase in the steady-state rate of natural gas drilling, and if

$$\left|\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)}\right| < \psi_g\left[C_g'' + \psi_o^2 C_{\psi_o}''\right] \tag{37}$$

then a positive natural gas price shock leads to an increase in the steady-state rate of oil drilling.

Proof. The proof of Proposition 2b follows directly from the proof of Proposition 2a. □

Proposition 2 indicates that drilling rates may move in the same direction or diverge when a positive gas or oil price shock occurs in an associated-commodity regime. Intuitively, a gas price shock will increase the steady-state gas drilling rate (see equation (26)), and will thus lead to increased associated-oil flow, thus decreasing the price of oil and disincentivising oil drilling. On the other hand, associated-gas flows from oil wells will be more valuable as a result of the positive gas price shock. The larger ψ_g, the larger the positive terms in (35), and the more likely oil drilling will increase as a result of a positive natural gas price shock. Oil and gas drilling responses to cross-commodity price shocks are estimated in the empirical section below.

The derivatives of drilling rates with respect to commodity shocks allow us to investigate the impact of a cross-commodity price shocks on steady-state oil and gas prices, i.e., the effect of an oil price shock on natural gas prices and vice versa, in addition to deriving the impact of a price shock on the own-commodity steady-state price after accounting for supply responses. In the Mathematical Appendix, we show that

$$\frac{dP_g}{ds_g} = P_g^s \left\{ 1 - \frac{P_g'}{\alpha_g(\alpha_g + r)} \left[\frac{(1-\psi_o\psi_g)^2 P_o'}{\alpha_o(\alpha_o + r)} - (1-\psi_g)^2 C'' - C_o'' - \psi_g^2 (C_g'' + C_{\psi_g}'' + \psi_o^2 C_{\psi_o}'')] \Big/ |H| \right\}, \quad (38)$$

$$\frac{dP_o}{ds_o} = P_o^s \left\{ 1 - \frac{P_o'}{\alpha_o(\alpha_o + r)} \left[\frac{(1-\psi_o\psi_g)^2 P_g'}{\alpha_g(\alpha_g + r)} - (1-\psi_o)^2 C'' - C_g'' - \psi_o^2 (C_o'' + C_{\psi_o}'' + \psi_g^2 C_{\psi_g}'') \right] \Big/ |H| \right\}, \quad (39)$$

$$\frac{dP_g}{ds_o} = \frac{P_o^s P_g'}{\alpha_g(\alpha_o + r)} \left[\psi_o C_o'' + \psi_g C_g'' + \psi_o \psi_g^2 C_{\psi_g}'' + \psi_g \psi_o^2 C_{\psi_o}'' - (1-\psi_g)(1-\psi_o) C'' \right] \Big/ |H|, \quad (40)$$

and

$$\frac{dP_o}{ds_g} = \frac{P_g^s P_o'}{\alpha_o(\alpha_g + r)} \left[\psi_g C_g'' + \psi_o C_o'' + \psi_g \psi_o^2 C_{\psi_o}'' + \psi_o \psi_g^2 C_{\psi_g}'' - (1-\psi_g)(1-\psi_o) C'' \right] \Big/ |H|. \quad (41)$$

Equations (38) and (39) indicate that initial commodity-price shocks will be subdued by supply responses to the increased prices. The number one inside the braces in these comparative statics accounts for the increased price associated with the shock, while the remaining terms inside the braces account for the drilling responses. Equations (40) and (41) indicate that price shocks will have an ambiguous effect on the cross-commodity steady-state price, which is important, as analyses that do not account for supply-side links generally assume that natural gas and crude oil prices will move together as a result of demand-side substitution. The comparative statics defined by

(40) and (41) lead us to our final two propositions, which present the changes in steady-state prices associated with cross-commodity price shocks.

Proposition 3: *In a cost-spillover regime an oil price shock leads to a higher steady-state natural-gas price, and a natural-gas price shock leads to a higher steady-state oil price.*

Proof. If we substitute $\psi_g = \psi_o = 0$ from the definition of a cost-spillover regime into the comparative statics defined by equations (40) and (41), we have

$$\frac{dP_g}{ds_o} = \frac{-P_o^s P_g' C''}{\alpha_g(\alpha_o + r)} > 0 \tag{42}$$

and

$$\frac{dP_o}{ds_g} = \frac{-P_o^s P_g' C''}{\alpha_o(\alpha_g + r)} > 0. \tag{43}$$

The right-hand sides of both (42) and (43) are positive due to the definition of a price shock, the concavity of the utility functions in commodity flows, and the convexity of the joint-cost function in drilling rates. □

Proposition 3 indicates that crude-oil and natural-gas prices will move in the same direction when a price shock to one of the commodities occurs. An oil price shock will lead to increased oil drilling per (27), and decreased natural gas drilling per Proposition 1. The latter will result in lower natural gas supply, as there is no associated-gas flowing from oil wells in a cost-spillover regime, thus increasing the price of natural gas.

Proposition 4: *In an associated-commodity regime, a positive oil price shock decreases the steady-state natural gas price, while a natural gas price shock decreases the steady-state oil price.*

Proof. If we substitute $C'' = 0$ from the definition of an associated-commodity regime into the comparative statics defined by equations (40) and (41), we have

$$\frac{dP_g}{ds_o} = \frac{P_o^s P_g'}{\alpha_g(\alpha_o + r)} \left[\psi_o C_o'' + \psi_g C_g'' + \psi_g \psi_o^2 C_{\psi_o}'' + \psi_o \psi_g^2 C_{\psi_g}''\right] < 0 \tag{44}$$

and

$$\frac{dP_o}{ds_g} = \frac{P_o^s P_o'}{\alpha_o(\alpha_g + r)} \left[\psi_o C_o'' + \psi_g C_g'' + \psi_g \psi_o^2 C_{\psi_o}'' + \psi_o \psi_g^2 C_{\psi_g}''\right] < 0. \tag{45}$$

The terms outside the brackets in equations (44) and (45) are negative, while the expressions inside the brackets are positive by the convexity of the various drilling cost functions. Therefore, both of these derivatives are negative. □

Proposition 4 indicates that prices will diverge in response to price shocks in an associated-commodity regime. Oil and gas prices diverge in an associated-commodity regime, because increased drilling of the commodity experiencing the price shock will lead to increased associated commodity flows without affecting the marginal cost of drilling the cross-commodity, thus increasing the supply of the cross-commodity.

4 Empirical Analysis

In this section, we estimate elasticities of drilling with respect to prices in five large crude oil and natural gas producing basins in Oklahoma and Texas to test for the existence of the supply-side links laid out in our theory. The basins used in our estimation include the Anadarko Basin (OK and TX), The Chautauqua Platform (OK), the East Texas Basin (TX), the Fort Worth Basin (TX), and the Permian Basin (TX). We use the natural log of new oil wells and the natural log of new gas wells as the dependent variables in the elasticity estimates, because drilling represents the marginal choice made by oil and gas producers as shown by Anderson et al. (2014) and Mason and Roberts (2018).

We employ a three-stage least squares strategy to identify the effects of cross-commodity price shocks in each of the basins. Simultaneous estimation of basin-level equations accounts for the likely possibility that capital and labor move between these basins, which might lead to correlated errors in the drilling equations across basins. Instrumenting for crude oil prices and natural gas prices remedies problems associated with endogeneity.

We use four categories of instruments for crude oil and natural gas prices in our drilling equation estimation. Roberts and Schlenker (2013) suggest using lagged shocks to storage levels to identify supply and demand parameters for storable commodities. Hausman and Kellogg (2015) implement such a strategy for natural gas using weather data, as weather will cause exogenous shocks to natural gas storage levels – natural gas is used for home heating in the winter, and is used for electricity generation during the summer. Weather exogenously shifts demand for natural gas, and can be used to identify supply parameters. We adopt a similar strategy to identify our drilling equation parameters using deviations from normal population-weighted cooling-degree days (CDDs) and heating-degree days (HDDs).[9] We use a single-month lag and lags of cumulative deviations from degree days over the prior 12-month period. The sums capture the cumulative affect of weather shocks on storage levels (Hausman and Kellogg, 2015). The cooling- and heating- degree day variables are of primary importance as a natural gas demand shifter. Second, we include a hurricane variable, because hurricanes exogenously shift the supply of natural gas and crude oil in the U.S., as gulf coast gas and oil production is often reduced as a result of hurricanes. Third, we use shocks to refinery inputs (thousands of barrels per day) as measured by errors from an autoregression of refinery inputs with three lags, and another autoregression of refinery with three lags and a time trend variable, which represent surprises to crude oil demand associated with refinery maintenance or shutdowns. We use cumulative surprises over the previous 12-month period of both of these variables. Finally, we use lags of the Brent crude oil price to identify global shocks to the crude-oil market, which are unlikely to be affected by drilling in our five estimation basins. All instrumental variables are included in both the first-stage oil price equation and the first stage gas price equation in order to account for the potential impact of demand-side substitution between crude oil and natural gas.

4.1 Data

The data for our estimation of cross-price drilling elasticities were collected from *Drilling Info*, the *Energy Information Administration (EIA)*, the *National Oceanic and Atmospheric Administration (NOAA)*, and the *National Hurricane Center (NHC)* websites.[10] The data used for our empirical analysis include: real crude oil prices, real natural gas prices, basin-level crude oil well counts,

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[9] Cooling-degree days measure the difference in the average daily temperature from 65 degrees if the temperature is greater than 65 degrees, while cooling-degree days are zero if the temperature is below 65 degrees. Heating-degree days measure the difference in the average daily temperature from 65 degrees if the temperature is below 65 degrees, while heating-degree days are zero if the average daily temperature is above 65 degrees.

[10] `www.eia.gov`, `www.noaa.gov`, `www.nhc.noaa.gov`, and `www.bls.gov`.

basin-level natural gas well counts, deviations from normal population-weighted heating-degree days, deviations from normal population-weighted cooling-degree days, hurricane occurrence and intensity, refinery inputs, and Brent crude oil prices. All data are monthly observations. The limiting datasets of the analysis are the basin-level well counts from Drilling Info, which we have from January 2005 to August 2016, which results in 137 observations per equation in the 3SLS model after accounting for the inclusion of lags. Summary statistics for our regression data are presented in Table 1.

The price data are spot prices.[11] The crude oil price is the West Texas Intermediate price (WTI) measured in dollars per barrel, which is the U.S. benchmark price for oil, while the natural gas price series is the Henry Hub price (HH) measured in dollars per MMBTU, which is the U.S. benchmark price for natural gas. These price series are inflated to April 2016 U.S. dollars using the BLS Producer Price Index (PPI) for all commodities.

Hurricane data is collected from the National Hurricane Center. This variable is zero if no hurricane made land fall on the U.S. Gulf Coast, and is the maximum Saffir-Simpson Hurricane Wind Scale (commonly known as hurricane category) if one or more hurricanes made land fall on the U.S. Gulf Coast in a particular month.

We constructed monthly counts of new natural gas wells and new oil wells in each basin from the Drilling Info data set as an approximation of natural gas and crude oil drilling activity. These data are collected from January 2005 to August 2016 and are the limiting variables in our sample. Logs of new well counts are used to estimate drilling elasticities. The Drilling Info dataset also allowed us to estimate monthly decline rates (α_g and α_o), associated commodity flows (ψ_g and ψ_o) (need to update these with regression estimates), and the proportion of oil wells that were connected to natural gas gathering infrastructure.

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[11] Estimation was also performed with prompt-month futures and four-month futures for each commodity, and the results were very similar. Results from that analysis are available upon request.

4.2 Results and Discussion

Table 2 displays the total number of wells in each of our sample basins, the proportion of total wells that were gas or oil directed, the estimated decline rates, the estimated associated-commodity parameters, and the proportion of crude oil wells that are connected to natural gas gathering infrastructure (gg). The last two columns, labeled β_{og} and β_{go} display the basin-level estimated drilling elasticities with respect to cross-commodity price. Table 3 shows the results of the three-stage least squares regression of drilling on prices in our five sample basins. Table 4 presents the results of the first-stage price equations.

In Table 2 the β_{og} column shows the estimated elasticities of oil drilling with respect to natural gas prices, while the β_{go} column shows the estimated elasticities of natural gas drilling with respect to crude oil price. Both elasticities of oil drilling with respect to gas price, and elasticities of gas drilling with respect to oil price are statistically significant in the Anadarko Basin, the Fort Worth Basin and the Permian Basin. These results imply that supply-side links play an important role in producers decisions in these three basins.

Our estimates of the elasticity of oil drilling with respect to natural gas price are all negative, and four of the five basin-level estimates are statistically significant at at least the 10% level. The negativity of these elasticity estimates is consistent with a supply-side regime dominated by input competition, as laid out in ***Proposition 1***, and/or an associated oil regime characterized by ***Proposition 2a***.

Unfortunately, these negative elasticity estimates do not allow us to concretely distinguish between input competition effects and associated oil effects. However, the parameters displayed in Table 2 offer some guidance. Add more here once have regression estimates of associated parameters.

Four out of our five estimates of the elasticity of natural gas drilling with respect to oil prices are positive, and two of these four positive estimates are statistically significant at at least the 5% level. The positivity of these estimates is consistent with an associated commodity regime characterized by ***Proposition 2b***. The estimate of the elasticity of natural gas drilling with respect to oil price in the Permian Basin is negative and statistically significant in contrast to the other basins used in our estimation.

We find that gas drilling increases by 4.4% when oil prices increase by 10% in the Anadarko Basin, which is consistent with the large associated oil parameter in the Anadarko Basin of 0.084 relative to the other basins. We find that gas drilling increases by 3% when oil prices increase by 10% in the Fort Worth Basin (see condition (36)). Although the associated oil parameter is relatively low in the Fort Worth Basin compared to our other sample basins, and to the associated gas parameter there, it is likely very costly to market associated gas from the Fort Worth Basin due to the relatively low proportion of oil wells that are connected to gas gathering infrastructure there.

The Permian Basin stands out as our only sample basin where both the elasticity of oil drilling with respect to gas price and the elasticity of gas drilling with respect to oil price are negative and statistically significant. The Permian Basin has a large number of both oil and natural gas wells, and experienced a drilling boom over our sample period. Therefore, we believe it is likely that input competition is driving supply-side links there.

5 Conclusion

6 Tables and Figures

Figure 1: *Monthly spot prices of crude oil and natural gas. Source: EIA*

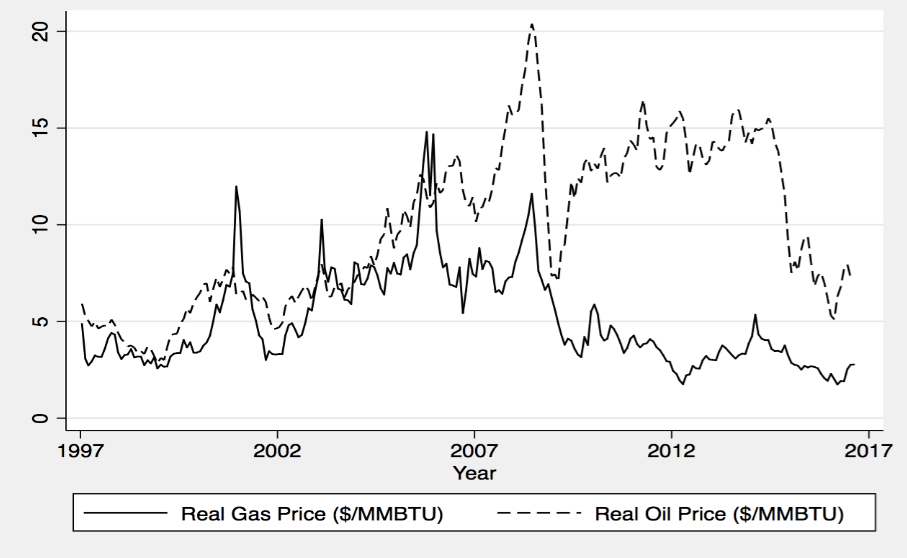

Figure 2: *Monthly crude oil directed rigs and natural gas directed rigs. Source: EIA*

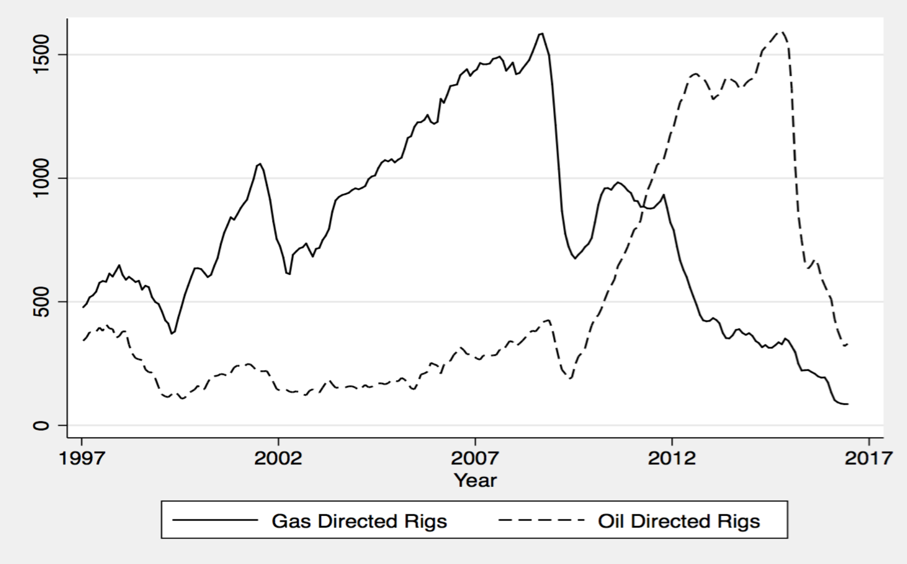

Table 1: Summary statistics.

Variable	Mean	Std Dev	Min	Max
Anadarko Basin New Oil Wells	69.78	48.74	14	205
Anadarko Basin New Gas Wells	103.18	54.00	12	224
Chautauqua Platform New Oil Wells	27.41	17.11	2	120
Chautauqua Platform New Gas Wells	24.21	18.75	1	90
East Texas Basin New Oil Wells	9.59	5.98	2	47
East Texas Basin New Gas Wells	93.51	72.17	6	224
Fort Worth Basin New Oil Wells	23.21	9.52	5	47
Fort Worth Basin New Gas Wells	127.87	76.04	8	292
Permian Basin New Oil Wells	184.72	65.08	89	528
Permian Basin New Gas Wells	68.51	56.68	10	180
WTI Real Spot Price (div 6)	12.45	3.05	5.10	20.40
Henry Hub Real Spot Price	5.11	2.77	1.74	14.80
Brent Oil Price	80.53	26.48	30.70	132.72
Deviation from Normal CDDs	6.20	19.08	-56	63
Deviation from Normal HDDs	-10.99	56.19	-242	153
12-month Cumulative Deviation from Normal CDDs	59.51	89.99	-129	195
12-month Cumulative Deviation from Normal HDDs	-109.47	258.49	-721	305
12-month Cumulative Refinery Input Surprises	328.12	1309.69	-2511.10	3040.26
12-month Cumulative Refinery Input Surprises (w/ time trend)	-324.15	1575.17	-3616.57	2416.63
Hurricane	0.086	0.42	0	3

Table 2: Crude-oil and natural-gas well parameters for various basins.

Basin	Total Wells	% Gas	% Oil	α_g	α_o	ψ_g	ψ_o	gg	β_{og}	β_{go}
Anadarko Basin	24,257	59.6	40.4	0.018 (0.0001)	0.022 (0.0001)	0.515 (0.0092)	0.084 (0.0014)	0.76	-0.480*** (0.1017)	0.440*** (0.1032)
Chautauqua Platform	7,238	47.0	53.0	0.015 (0.0003)	0.013 (0.0003)	0.228 (0.0107)	0 (.)	0.63	-0.172* (0.0950)	0.289 (0.2050)
East Texas Basin	14,478	90.6	9.4	0.020 (0.0001)	0.018 (0.0001)	0.158 (0.0217)	0.018 (0.0004)	0.63	-0.099 (0.0938)	0.142 (0.1047)
Fort Worth Basin	21,225	84.6	15.4	0.017 (0.0001)	0.022 (0.0002)	0.237 (0.0180)	0 (.)	0.57	-0.127** (0.0616)	0.306** (0.1263)
Permian Basin	36,002	26.8	73.2	0.014 (0.0001)	0.015 (0.0002)	0.359 (0.0021)	0.013 (0.0005)	0.86	-0.283*** (0.0479)	-0.251** (0.1236)

Standard errors in parentheses

Standard errors clustered at well level in well-level fixed-effects estimation of α_i

Bootstrapped standard errors from sample median of ψ_g and ψ_o

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ for β coefficients

Table 3: Crude-oil and natural-gas drilling three-stage least squares results.

	Anadarko Basin		*Chautauqua Platform*		*East Texas Basin*	
	ln $(q_{o,t})$	**ln $(q_{g,t})$**	**ln $(q_{o,t})$**	**ln $(q_{g,t})$**	**ln $(q_{o,t})$**	**ln $(q_{g,t})$**
ln(Henry Hub)	**-0.480*****	0.362***	**-0.172***	1.296***	**-0.099**	0.545***
	(0.102)	(0.067)	(0.095)	(0.146)	(0.094)	(0.091)
ln(WTI)	0.720***	**0.440*****	0.684***	**0.289**	0.678***	**0.142**
	(0.163)	(0.103)	(0.201)	(0.205)	(0.215)	(0.105)
ln$(q_{i,t-1})$	0.589***	0.611***	0.473***	0.043	0.161**	0.705***
	(0.064)	(0.059)	(0.069)	(0.080)	(0.081)	(0.046)
Constant	0.571*	0.092	0.021	0.101	0.242	0.041
	(0.321)	(0.237)	(0.491)	(0.494)	(0.475)	(0.247)
$\mathbf{R^2}$	0.81	0.84	0.34	0.60	0.15	0.92

	Fort Worth Basin		*Permian Basin*	
	ln $(q_{o,t})$	**ln $(q_{g,t})$**	**ln $(q_{o,t})$**	**ln $(q_{g,t})$**
ln(Henry Hub)	**-0.127****	0.181***	**-0.282*****	0.562***
	(0.062)	(0.057)	(0.048)	(0.124)
ln(WTI)	0.841***	**0.306****	0.568***	**-0.251****
	(0.145)	(0.126)	(0.088)	(0.124)
ln$(q_{i,t-1})$	0.385***	0.850***	0.461***	0.681***
	(0.067)	(0.043)	(0.065)	(0.060)
Constant	-0.022	-0.359	1.792***	0.999***
	(0.311))	(0.242)	(0.304)	(0.325)
$\mathbf{R^2}$	0.44	0.91	0.70	0.90

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Crude-oil and natural-gas drilling equations first-stage results for Henry Hub and WTI prices.

	ln(Henry Hub)	**ln(WTI)**
Lag Brent Oil Price	0.0036***	0.0071***
	(0.0008)	(0.0006)
Lag Deviation from Normal CDDs	0.0003	-0.0001
	(0.0008)	(0.0007)
Lag Deviation from Normal HDDs	0.0010	0.0002
	(0.0003)	(0.0002)
Lag 12-month Cumulative Deviation from Normal CDDs	-0.0004*	0.0002
	(0.0002)	(0.0002)
Lag 12-month Cumulative Deviation from Normal HDDs	0.0001*	0.00005
	(0.00007)	(0.00006)
12-month Cumulative Refinery Input Surprises	-0.0003***	0.0002***
	(0.00009)	(0.00007)
12-month Cumulative Refinery Input Surprises (w/ time trend)	0.0003***	-0.0002***
	(0.00007)	(0.00005)
Time Trend	-0.0079***	-0.0058***
	(0.0011)	(0.0010)
Constant	2.779***	2.739***
	(0.682)	(0.129)
$\mathbf{R^2}$	0.86	0.65

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

7 Mathematical Appendix

The Hessian matrix associated with identities (24) and (25) is

$$H = \begin{bmatrix} f_{oo} & f_{og} \\ f_{go} & f_{gg} \end{bmatrix}$$

where

$$f_{oo} = \frac{P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} + \frac{\psi_g^2 P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} - C''(q_o+q_g) - C_o''(q_o) - \psi_g^2 C''_{\psi_g}(\psi_g q_o) \le 0, \tag{46}$$

$$f_{gg} = \frac{P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} + \frac{\psi_o^2 P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} - C''(q_o+q_g) - C_g''(q_g) - \psi_o^2 C''_{\psi_o}(\psi_o q_g) \le 0 \tag{47}$$

and

$$f_{og} = f_{go} = \frac{\psi_o P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} + \frac{\psi_g P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} - C''(q_o+q_g) \le 0, \tag{48}$$

and where $P_i'(z_i, s_i) = \dfrac{\partial P_i'(z_i, s_i)}{\partial z_i}$ for $i = \{o, g\}$. The sufficient conditions for a maximum require that $f_{oo} \le 0$, $f_{gg} \le 0$, and $f_{oo}f_{gg} - f_{og}^2 \ge 0$.[12] The conditions on (46), (47) hold as a result of the concavity of consumer surplus and the convexity of the cost functions. For the non-negativity of the determinant of the Hessian we have (dropping dependence from the price and cost functions)

$$\left[\frac{P_o'}{\alpha_o(\alpha_o+r)} + \frac{\psi_g^2 P_g'}{\alpha_g(\alpha_g+r)} - C'' - C_o'' - \psi_g^2 C''_{\psi_g}\right]\left[\frac{P_g'}{\alpha_g(\alpha_g+r)} + \frac{\psi_o^2 P_o'}{\alpha_o(\alpha_o+r)} - C'' - C_g'' - \psi_o^2 C''_{\psi_o}\right]$$
$$- \left[\frac{\psi_o P_o'}{\alpha_o(\alpha_o+r)} + \frac{\psi_g P_g'}{\alpha_g(\alpha_g+r)} - C''\right]^2$$

$$= (1-\psi_o\psi_g)^2 \frac{P_o' P_g'}{\alpha_o\alpha_g(\alpha_o+r)(\alpha_g+r)} - (1-\psi_o)^2 \frac{P_o' C''}{\alpha_o(\alpha_o+r)} - (1-\psi_g)^2 \frac{P_g' C''}{\alpha_g(\alpha_g+r)}$$
$$- \psi_o^2 \left[\frac{P_o'(C_o'' + C''_{\psi_o})}{\alpha_o(\alpha_o+r)} - C''C''_{\psi_o} - C_o''C''_{\psi_o}\right] - \psi_g^2 \left[\frac{P_g'(C_g'' + C''_{\psi_g})}{\alpha_g(\alpha_g+r)} - C''C''_{\psi_g} - C_g''C''_{\psi_g}\right]$$
$$- \psi_o^2\psi_g^2 \left[\frac{P_g' C''_{\psi_o}}{\alpha_g(\alpha_g+r)} \frac{P_o' C''_{\psi_g}}{\alpha_o(\alpha_o+r)} - C''_{\psi_o} C''_{\psi_g}\right] \ge 0,$$

where the last inequality results from the concavity of consumer surplus, the convexity of the cost functions, and the fact that $\psi_o, \psi_g \in [0, 1)$. Thus, the sufficient conditions for maximization are met, and we have that $|H| > 0$.

Now, we are interested in the comparative statics $\dfrac{\partial q_g}{\partial s_g}$, $\dfrac{\partial q_o}{\partial s_o}$, $\dfrac{\partial q_g}{\partial s_o}$ and $\dfrac{\partial q_o}{\partial s_g}$, which give the optimal drilling responses to own-commodity and cross-commodity demand shocks. Using the implicit

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[12] We have ruled out the possibility that surplus can be increased by inter-temporal reallocation of drilling decisions by assuming that a steady state is reached, thus choosing steady-state drilling rates that maximize surplus at each instant is sufficient for maximizing the value of a drilling program.

function theorem we have

$$\frac{\partial q_g}{\partial s_g} = \frac{\begin{vmatrix} f_{oo} & -f_{os_g} \\ f_{og} & -f_{gs_g} \end{vmatrix}}{|H|}, \tag{49}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\begin{vmatrix} f_{gg} & -f_{os_o} \\ f_{og} & -f_{gs_o} \end{vmatrix}}{|H|}, \tag{50}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\begin{vmatrix} f_{oo} & -f_{os_o} \\ f_{og} & -f_{gs_o} \end{vmatrix}}{|H|}, \tag{51}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\begin{vmatrix} f_{gg} & -f_{gs_g} \\ f_{og} & -f_{os_g} \end{vmatrix}}{|H|}, \tag{52}$$

where f_{os_o} is the derivative of the oil-drilling identity, (24), with respect to an oil-market demand shock, and f_{gs_o} is the derivative of the gas-market identity, (25), with respect to an oil-market demand shock, and similarly for f_{gs_g} and f_{os_g}. Letting $P_i^s = \dfrac{\partial P_i(z_i, s_i)}{\partial s_i}$ for $i = \{o, g\}$, we have

$$f_{os_o} = \frac{P_o^s}{\alpha_o + r}, \tag{53}$$

$$f_{gs_o} = \frac{\psi_o P_o^s}{\alpha_o + r}, \tag{54}$$

$$f_{gs_g} = \frac{P_g^s}{\alpha_g + r}, \tag{55}$$

and

$$f_{os_g} = \frac{\psi_g P_g^s}{\alpha_g + r}. \tag{56}$$

Substituting these values into (49), (50), (51) and (52), and simplifying, gives

$$\frac{\partial q_g}{\partial s_g} = \frac{\frac{-P_g^s}{\alpha_g + r}\left[\frac{(1-\psi_g\psi_o)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]}{|H|}, \tag{57}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\frac{-P_o^s}{\alpha_o + r}\left[\frac{(1-\psi_g\psi_o)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' - C_g'' - \psi_o^2 C_{\psi_o}''\right]}{|H|}, \tag{58}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o + r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]}{|H|} \tag{59}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g + r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]}{|H|}. \tag{60}$$

Given the above comparative statics for optimal drilling responses to demand shocks we can derive the total derivatives of prices with respect to own-commodity and cross-commodity demand shocks, $\dfrac{dP_o(z_o, s_o)}{ds_o}$, $\dfrac{dP_o(z_o, s_g)}{ds_o}$, $\dfrac{dP_g(z_g, s_g)}{ds_g}$, and $\dfrac{dP_g(z_g, s_g)}{ds_g}$. We have

$$
\begin{aligned}
\frac{dP_o(z_o, s_o)}{ds_o} &= \frac{dP_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{ds_g} = \frac{\partial P_o}{\partial s_o} + \frac{\partial P_o}{\partial q_o}\frac{\partial q_o}{\partial s_o} + \frac{\partial P_o}{\partial q_g}\frac{\partial q_g}{\partial s_o} \\
&= P_o^s + \left(\frac{P_o'}{\alpha_o}\left\{\frac{-P_o^s}{\alpha_o + r}\left[\frac{(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g + r)} - (1-\psi_o)C'' + C_g'' + \psi_o^2 C_{\psi_o}''\right]\right\}\right. \\
&\qquad \left.\frac{\psi_o P_o'}{\alpha_o}\left\{\frac{P_o^s}{\alpha_o + r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g + r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]\right\}\right) \Bigg/ |H| \\
&= P_o^s\left\{1 - \frac{P_o'}{\alpha_o(\alpha_o + r)}\left[\frac{(1-\psi_o\psi_g)^2 P_g'}{\alpha_g(\alpha_g + r)} - (1-\psi_o)^2 C'' - C_g'' - \psi_o^2(C_o'' + C_{\psi_o}'' + \psi_g^2 C_{\psi_g}'')\right] \Bigg/ |H|\right\}
\end{aligned}
\tag{61}
$$

and

$$
\begin{aligned}
\frac{dP_o(z_o, s_o)}{ds_g} &= \frac{dP_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{ds_g} = \frac{\partial P_o}{\partial q_o}\frac{\partial q_o}{\partial s_g} + \frac{\partial P_o}{\partial q_g}\frac{\partial q_g}{\partial s_g} \\
&= \left(\frac{P_o'}{\alpha_o}\left\{\frac{P_g^s}{\alpha_g + r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o + r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]\right\}\right. \\
&\qquad \left. - \frac{\psi_o P_o'}{\alpha_o}\left\{\frac{P_g^s}{\alpha_g + r}\left[\frac{(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o + r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]\right\}\right) \Bigg/ |H| \\
&= \frac{P_g^s P_o'}{\alpha_o(\alpha_g + r)}\left[\psi_g C_g'' + \psi_o C_o'' + \psi_g\psi_o^2 C_{\psi_o}'' + \psi_o\psi_g^2 C_{\psi_g}'' - (1-\psi_g)(1-\psi_o)C''\right] \Bigg/ |H|.
\end{aligned}
\tag{62}
$$

Similarly,

$$
\frac{dP_g}{ds_g} = P_g^s\left\{1 - \frac{P_g'}{\alpha_g(\alpha_g + r)}\left[\frac{(1-\psi_o\psi_g)^2 P_o'}{\alpha_o(\alpha_o + r)} - (1-\psi_g)^2 C'' - C_o'' - \psi_g^2(C_g'' + C_{\psi_g}'' + \psi_o^2 C_{\psi_o}'')\right] \Bigg/ |H|\right\}
\tag{63}
$$

and

$$
\frac{dP_g}{ds_o} = \frac{P_o^s P_g'}{\alpha_g(\alpha_o + r)}\left[\psi_o C_o'' + \psi_g C_g'' + \psi_o\psi_g^2 C_{\psi_g}'' + \psi_g\psi_o^2 C_{\psi_o}'' - (1-\psi_g)(1-\psi_o)C''\right] \Bigg/ |H|.
\tag{64}
$$

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