Effects of Allowances Allocation Schemes on the Emission Leakage and Contract Shuffling in Electric Markets

Yihsu Chen* and Andrew L. Liu†

May 4, 2009

Abstract

This paper examines the effects of emissions allowances allocation schemes on the extent of the GHG (greenhouse gas) leakage and the contract shuffling. Two allocation methods, grandfathering and the output-based method, are considered. Whereas grandfathering separates future allowances allocation from today's decisions, the output-based approach links the future awarded allowances to today's output level. The latter effectively subsidizes generators' production costs, encourages more output and consequently elevates GHG allowances prices in current period. In this paper, we first analyze the effect of output-update allocation on the leakage and shuffling using a stylized duopoly model, which abstracts from details of point-of-regulation. A single-stage computable model, which allows representing source-based, load-based and first-seller-based programs, is then applied to examine the implications. The latter model is equivalent to a two-stage formulation with perfect foresight. Our results suggest that the magnitude of GHG leakage is inversely associated with per MWh of the future allowances awarded for today's output. The power prices under output-update approach could be either higher or lower compared to the grandfathering. Yet, the equivalence among source-, load-based and first-deliverer approaches remains valid only under certain conditions.

1 Introduction

Climate change is an unprecedented challenge faced by our society toady. Numerous resources and policies have been devoted to controlling greenhouse gas emissions (GHG) from the power sector and other energy-intensive sectors. Among many instruments,

---

*Y. Chen is with the School of Engineering, School of Social Sciences, Humanities and Arts, and Sierra Nevada Research Institute, University of California, Merced, Merced, CA95343, USA. e-mail: `yihsu.chen@ucmerced.edu`. This research is partially supported by the research funding from the Graduate Research Council(GRC), University of California, Merced.

†A. L. Liu is currently a PhD candidate at the Department of Applied Mathematics and Statistics, The Johns Hopkins University, Baltimore, MD 21218, USA. e-mail: `lliu@jhu.edu`. He will be with the School of Industrial Engineering at Purdue University.

cap-and-trade program emerges as the widely accepted and preferred approach. Although emissions trading is not a new concept, the economic impact of GHG polices is expected to be far reaching than the previous programs that target on other pollutants (e.g., SO_2 and NO_x). This is in part because both baseload and peakload units would incur substantial emissions costs. The pioneer EU ETS (European Union Emissions Trading Scheme), which began in 2005 and then expanded to 27 EU member countries in 2007, has produced some encouraging results [9].

In the United States, due to lack of federal leadership, a number of states have taken actions to control GHG emissions. The efforts by the eastern and western states are called the Regional Greenhouse Gas Initiative (RGGI)[15] and the Western Climate Initiative (WCI)[16], respectively. RGGI's policy goal is to reduce CO_2 emissions ten percent below the current level by 2019. The compliance schedule set forth is that the CO_2 emissions will be capped at the current level during 2009-2015, followed by a gradual decline to 10 percent below the current level by 2019. Fossil-fuelled generating units (e.g., gas, oil and coal) with a name capacity greater than or equal to 25 MW fall under the cap.

Among the WCI states, the State of California is the first state to adopt legislation limiting GHGs (including six species). On September 27, 2006, the state of California passed a comprehensive bill – AB32, "The Global Warming Solutions Act" – that aims at reducing in-state GHG emissions from various sectors to 1990 levels by 2020, which is equivalent to a decrease of approximately 174 $MMTCO_2E$ (million metric tons CO_2 equivalent) per year [2]. AB32 is the first climate change legislation in the U.S. that would regulate most polluting sectors in an economy. Led by the California Energy Commission (CEC) and the California Air Resources Board (CARB) in consultation with other agencies, a state-wide emission cap is expected to be in effect by 2012. This is expected to be accomplished with a suite of instruments such as a low carbon fuel standard for vehicles that would reduce GHGs of transportation fuels by at least 10% by 2020.

The designs of emissions trading have faced substantial challenges. For instance, while California imported electricity accounts for half of the electric sector emissions, such imports represented only 22% of electricity demand in California in 2006 [4]. Thus, imported electricity is significantly dirtier than in-state generation since a significant portion is generated by coal plants located in neighboring states [5]. A central issue in the recent debate over implementation of California's GHG emissions trading program has been over where along the electricity sector supply chain (i.e., fuel to generation to consumption) should limits be imposed; this is the "point-of-regulation" issue. Three proposals are on the table: source-based, load-based and first-deliverer (-seller) approaches.

- Source-based systems are most popular elsewhere. The conventional point-of-regulation in the electric sector cap-and-trade systems (e.g., under the SO_2 Acid Rain Program under the 1990 Clean Air Act Amendments and the USEPA NO_x SIP Call) is at the generator.
- Load-based systems have received much attention in the California debate. The

basic load-based program requires LSEs to track the emissions associated with the electricity they purchase from generators and third parties and to ensure that the sales-weighted carbon emissions rate of its purchases does not exceed a target established by the regulator.[1]

- The final of the three approaches is the first-deliverer (seller) proposal [8]. A first-deliverer is defined as the entity that first contracts to sell electricity in California. In parctice, the schedule coordinators (SCs) that contract with generators in other states to import electricity into California could be designated as the first-deliverer instead of the LSE [1]. More recently, the CPUC (California Public Utility Commission) released a draft report and expressed their support of this approach. It is an variant of the first-seller system and the entity that possesses the ownership of the electricity at the first point of delivery in California would be the point of regulation[7].

In a previous work, Chen et al. [3] have established the equivalence among three proposals with respect to electricity prices, nodal sales, social welfare, emissions leakage, contract shuffling, etc. In this paper we extend our analysis to consider the effects of allowances allocations on the GHG leakage, contract shuffling, electricity prices, allowances prices, etc. We examine two allocation schemes: grandfather and output-update. The first approach is chosen as the benchmark because of its popularity in other emissions trading programs. The second approach has received significant attention, and CPUC has been in favor of this approach due to two reasons. First, it effectively suppresses electricity prices because of the expansion of output from producers. Second, it mitigates leakage since in-state output would increase in response to the economic benefit of awarded allowances. Our results suggest while the GHG leakage is indeed mitigated given out-of-state generators are relatively dirty, electricity price might be higher due to higher GHG prices in current period as a result of output expansion. The change in electricity prices depends on tightness of GHG cap and demand elasticity. Finally, the equivalence among source-, load-based and first-deliverer approaches is only valid under certain conditions.

The remainder of the paper is organized as follows. In Section 2, an analytical duopoly model is developed to study how might the extent of GHG leakage be altered under output-update scheme. While this model abstracts from point-of-regulation, it helps to generate hypotheses that would be further tested in the numerical example in the Section 3. In Section 3, we present numeric examples of three proposals that are based on Chen et al. [3] and elaborated to account for output-update allowances allocation. In contrast to the duopoly model in Section 2, the formulation of models in Section 3 allows for detail modelling of different designs in emissions trading programs. Consistent with Section 2, we deploy a single-stage framework coupled with inter-temporal arbitrage condition on allowances prices. This is equivalent to a two-stage model with

---

[1] A variant unbundles emissions certificates and power and requires the LSE to purchase an amount of emissions certificates. This is represented by two proposals: the CO_2RC proposal [14] and the "Tradable Emission Attribute Certificates" (TEAC) proposal [12]. Further discussion of CO2RC and TEAC can be found elsewhere[cite] [13, 17].

perfect foresight. We will address the limitations and possible extension of such approach in the context of two-stage modelling. We discuss the results of the numerical examples in Section 4, focusing on economic and emissions outcomes as well as the issues of GHG emissions leakage and contract shuffling. Conclusions and policy implications are addressed in Section 5. (Theoretical proofs and data used in the economic and emissions analysis are included in the Appendix.)

2 A Duopoly Model

We begin our analysis with a duopoly model of two price-taking producers i and j (e.g., perfect competition), representing an in-state supplier and an importer, respectively. We assume that firm i owns a natural gas-fired plant, whereas firm j owns a coal-fired plant. Thus, the GHG emission rate E_i is less than E_j. Let $c_i(q)$ and $c_j(q)$ denote the cost function of firm i and j, and assume that they are convex quadratic and twice differentiable. More specifically,

$$\begin{aligned} c_i(q) &= a_i q + \frac{1}{2} b_i q^2, \ a_i, b_i > 0, \text{ and,} \\ c_j(q) &= a_j q + \frac{1}{2} b_j q^2, \ a_j, b_j > 0. \end{aligned} \tag{1}$$

Assume that (aggregated) consumers' willingness-to-pay for a certain quantity Q is captured by a linear inverse demand function $p(Q)$, with price and quantity intercepts of P^0 and Q^0, respectively. Let K denote the slope of the linear function; i.e., $K = P^0/Q^0$. Then $p(Q) = P^0 - KQ$. To rule out uninteresting cases, we assume that $P^0 > \max\{a_i,\ a_j\}$.

The profit optimization problem faced by i or j under grandfathering allocation is as follows.

$$\max_{q_i \geq 0} \quad pq_i - c_i(q_i) - p^{GHG}(E_i q_i - A_i), \tag{2}$$

where p^{GHG} and A_i denotes GHG allowances price and possessed allowances, respectively. We assume that importers are also subject to the emissions cap, which is broadly consistent with several proposals considered by WCI[16]. Under an output-based scheme, per MWh output in period t is rewarded with B tons of allowances in period $t+1$ [2]. Distinguish the GHG allowances prices for periods t and $t+1$ and assume that the discount factor is δ. Then firm i's profit optimization problem under the output based scheme can be written as follows:

$$\max_{q_i \geq 0} \quad pq_i - c_i(q_i) - p_t^{GHG}(E_i q_i - A_i) + \delta p_{t+1}^{GHG} B q_i. \tag{3}$$

---

[2] A variant, which has been endorsed by California Public Utility Commission, is called technology-based update [6]. This is equivalent to a program with a technology-specific updating parameter B_h, with h corresponding to a specific electricity generating technology. The parameter explicit defines the benchmark emission rate for a specific technology.

By enforcing the no-arbitrage condition – $p_t^{GHG} = \delta p_{t+1}^{GHG}$, (and omitting the subscript t for conciseness), equation (2) can be simplified as follows:

$$\max_{q_i \geq 0} \quad pq_i - c_i(q_i) - p^{GHG}(E_i q_i - A_i - Bq_i). \tag{4}$$

Similarly, firm j's profit maximization problem is given below.

$$\max_{q_j \geq 0} \quad pq_j - c_j(q_j) - p^{GHG}(E_j q_j - A_j - Bq_j). \tag{5}$$

The market clearing condition that determines the GHG price in time period t is as follows.

$$0 \leq p^{GHG} \quad \perp \quad \mathcal{C} - E_i q_i - E_j q_j \geq 0, \tag{6}$$

where $\mathcal{C}$ denotes the system cap on GHG emissions in period t. $\mathcal{C}$ may or may not equal $A_i + A_j$.

The market clearing price p as in (4) and (5) is determined by the inverse demand function $p(q_i+q_j)$. Then (4), (5), and (6) together define an equilibrium problem. Since the optimization problems (4) and (5) are convex quadratic problems, the first-order optimality condition (FOC) is both necessary and sufficient. By writing out the FOC explicitly through the Karush-Kuhn-Tucker (KKT) conditions, the equilibrium problem can be represented by the following linear complementarity problem (LCP).

$$0 \leq \begin{bmatrix} q_i \\ q_j \\ p^{GHG} \end{bmatrix} \perp \begin{bmatrix} a_i - P^0 \\ a_j - P^0 \\ \mathcal{C} \end{bmatrix} + \begin{bmatrix} K + b_i & K & E_i - B \\ K & K + b_j & E_j - B \\ -E_i & -E_j & 0 \end{bmatrix} \begin{bmatrix} q_i \\ q_j \\ p^{GHG} \end{bmatrix} \geq 0, \tag{7}$$

where the $\perp$ sign means that the product of two vectors is 0. Before analyzing properties of the solutions to the LCP, we first show that a solution exists. Proofs for all the results shown in this section are provided in the Appendix.

Proposition 1. *Given that $B < E_i < E_j$, LCP (7) has a solution.*

To ease arguments, let d denote the vector $(a_i - P^0,\ a_j - P^0,\ \mathcal{C})^T$, and M denote the 3×3 matrix in (7). Let x be a solution to the LCP (7); that is,

$$0 \leq x \perp d + Mx \geq 0. \tag{8}$$

For our discussions, we focus on solutions that $Mx = 0$ with $x \geq 0$; namely,

$$\begin{bmatrix} K + b_i & K & E_i - B \\ K & K + b_j & E_j - B \\ -E_i & -E_j & 0 \end{bmatrix} \begin{bmatrix} q_i \\ q_j \\ p^{GHG} \end{bmatrix} = \begin{bmatrix} P^0 - a_i \\ P^0 - a_j \\ -\mathcal{C} \end{bmatrix}, \text{ with } \begin{bmatrix} q_i \\ q_j \\ p^{GHG} \end{bmatrix} \geq 0. \tag{9}$$

To check that whether a solution of the LCP (7) that satisfies the conditions (9) exist, we can solve the following convex quadratic program.

$$\begin{aligned} \max_{x} \quad & -||x||^2 \\ \text{s.t.} \quad & Mx = d, \\ & x \geq 0. \end{aligned} \tag{10}$$

If the optimization problem (10) is feasible, then a solution of the LCP (7) that satisfies conditions (9) exists. In addition, such a solution is unique, as M is nonsingular under the condition $B < E_i < E_j$. (The determinant of M, under the condition $B < E_i < E_j$, is shown to be nonzero in the proof of Proposition 2 in the Appendix.) For the following analyses, we assume that the LCP (7) has a solution satisfying (9).

To analyze the effects of the output-based allowance allocation rate B on firm i, j's outputs and on GHG price, we apply implicit function theorem on the linear system (9). The results are summarized in the following proposition.

Proposition 2. *Assume that $B < E_i < E_j$. Let $\tilde{x} \equiv (\tilde{q}_i, \tilde{q}_j, \tilde{p}^{GHG})$ denote a solution that satisfies (9) with respect to input parameters $\tilde{B}$ and $\tilde{\mathcal{C}}$. Then their exists an open set U such that $(\tilde{B},\ \tilde{\mathcal{C}}) \in U$ and within which q_i, q_j and p^{GHG} are continuous function with respect to $(B,\ \mathcal{C})$. In addition, their partial derivatives with respect to B and $\mathcal{C}$ exist for $(B,\ \mathcal{C}) \in U$. Particularly, if*

$$\mathcal{C} \neq \frac{\left[(K+b_j)(P^0-a_i) - K(P^0-a_j)\right] E_i + \left[(K+b_i)(P^0-a_j) - K(P^0-a_i)\right] E_j}{K(b_i+b_j)+b_ib_j},$$

then the following is true.

$$\frac{\partial q_i}{\partial B} > 0,\ \frac{\partial q_j}{\partial B} < 0,\ \frac{\partial p^{GHG}}{\partial B} > 0. \tag{11}$$

With the above results, we can analyze the impacts of B on GHG leakage under a regional GHG cap-and-trade program. Within the 2–firm example, our analysis on leakage problem is only partial, as it cannot account for output and emissions changes in regions that are not subject to a GHG cap. Instead, we define a surrogate (denoted as L_j) to indicate emissions leakage as the contribution of $q_j's$ GHG emissions to the total emissions:

$$L_j(B) \equiv \frac{q_j(B)E_j}{q_i(B)E_i + q_j(B)E_j} \times 100\%, \tag{12}$$

where $q_i(B)$ and $q_j(B)$ denote the equilibrium outcome with respect to the updating parameter B. Focusing the equilibrium solutions that satisfy conditions (9), we have that $q_i(B)E_i + q_j(B)E_j = \mathcal{C}$ for every B such that $(B,\ \mathcal{C}) \in U$. Thus, $dq_j/dB < 0$ implies that $L_j(B)$ is a decreasing function with respect to B over $\mathcal{N}_\epsilon(B)$ (a neighborhood at B), indicating that output-update allocation scheme would mitigate leakage. The following example illustrates the above result.

We borrow the idea of the best response function[3] to illustrate how firms might adjust their output and what the resulting equilibrium would be under different GHG price and update parameters B. Figure 1 illustrates the response curves based on solving the system in (9) with the following parameters: $a_i = 2$, $a_j = 5$[\$/MWh], $b_i = 10$, $b_j = 5$[\$/MWh/MW], $e_i = 0.5$, $e_j = 1.5$[tons/MWh], $p^0 = 120$[\$/MWh] and $K = 2$[\$/MWh/MW].

---

[3] Although under the perfect competition assumption, firms are price-takers and will not response to their rivals' decisions [11].

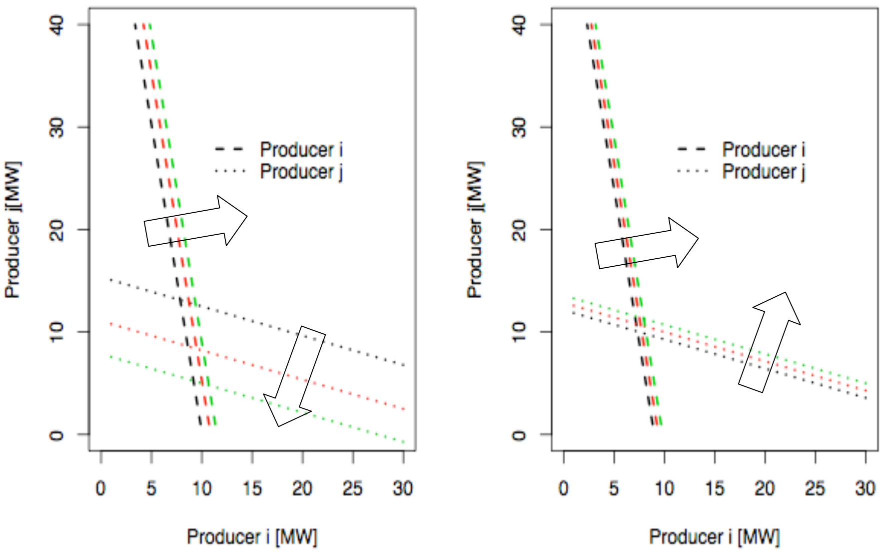

Figure 1: Response curves for producer i and j conditioned on p^{GHG} (0, 50\$/ton, 80 \$/ton) and B (0, 0.25 ton/MWh and 0.5 ton/MWh)

The slope of the best response functions for producers i and j is $\frac{K+b_i}{K}$ and $\frac{K+b_j}{K}$, respectively. If p^{GHG} is unchanged, any change in updating by $\triangle B$ would shift the curves outward by $\frac{\triangle B p^{GHG}}{K}$, effectively encouraging outputs (Fig. 1, right). On the other hand, any change in GHG allowances price by $\triangle p^{GHG}$ can shift the curves by $-\frac{\triangle p^{GHG}(E-B)}{K}$ (Fig. 1, left), where the subscript of E is omitted. Thus, depending on the the emissions relative to B, it could move outward if $E < B$ (i.e., $\triangle q > 0$ as producer i).

The analysis so far ignores various policy designs, such as source-based, load-based or first-deliverer-based cap-and-trade programs, which cannot be examined in this setting. Hence, we use computational models presented in the next section to examine the effect of allocation rule on the emissions leakage and contract shuffling.

3 Power Market Model and Simulations

The model used in this section's simulations is based on Chen et al. [3], and modified to account for output-based allowances update scheme. The full models for three proposals – source-, load- based and first-deliverer – can be found elsewhere [3]. For conciseness, we only elaborate on the formulations that needs to be changed in order to model output-base allowances allocation in Section 3.1. We discuss the setups and the assumptions in Section 3.2

3.1 Models

To model output-based allowances allocation under the single-stage framework, we first denote the GHG prices in period t and $t+1$ as p_t^{GHG} and p_{t+1}^{GHG}, respectively. Since the awarded allowances can only be used to cover the emissions in the period $t+1$, the cap in period t remains unchanged. We illustrates how changes should be made using a generic example. In the absence of output-update, the terms associated of allowances in the objective function of either producers (or LSEs) can be expressed as follows:

$$-p_t^{GHG}\left(\sum_{h\in H_f} E_{fh}x_{fht} - A_{ft}\right), \tag{13}$$

where x_{fht} is output in MWh, E_{fh} is emissions rates in tons/MWh, and A_{ft} is the amount of possessed allowances in tons in period t. Equation (13) represents cost (revenue) if the term within the parenthesis is positive (negative). When the allowances in the period $t+1$ is awarded based on output in the period t, Equation (13) is altered to

$$-p_t^{GHG}\left(\sum_{h\in H_f} E_{fh}x_{fht} - A_{ft}\right) + \frac{p_{t+1}^{GHG}}{1+r}\left(B\sum_{h\in H_f} x_{fht}\right), \tag{14}$$

where B is the amount of allowances awarded in period $t{+}1$ per MWh of output in period t. Equation (14) can be simplified as follows after imposing no arbitrage condition:

$$-p_t^{GHG}\left(\sum_{h\in H_f} (E_{fh}-B)x_{fht} - A_{ft}\right). \tag{15}$$

Thus, literally, the economic value of awarded allowances in period $t+1$ effectively reduces firms' production cost and encourages output in period t.

3.2 A Numeric Example

We deploy a similar setup as Chen et al. [3]: three zones ($i = \{A, B, C\}$) connected with transmission lines with fixed capacities (Figure 2). Zone A is the regulated zone (referred to below as "*in-state*"), and zones B and C are unregulated but trade with zone A (and are referred to as "*out-of-state*"). A number of generating units ($h = \{1, 2..., 10\}$) are located in each zone owned by firms ($f = \{1, 2, 3\}$). We allow firms to own generating assets in different locations. Consumers' demand of electricity is represented by linear inverse demand curves.

The specific input assumptions used in this example are presented in the Appendix of Chen et al.[3]. Figure 2 plots the illustrative marginal cost (solid lines), demand curves and the corresponding CO_2 emissions rates (dash lines) for zones A, B and C, respectively. The marginal cost for zone A ranges between 35-55 \$/MWh with a stable

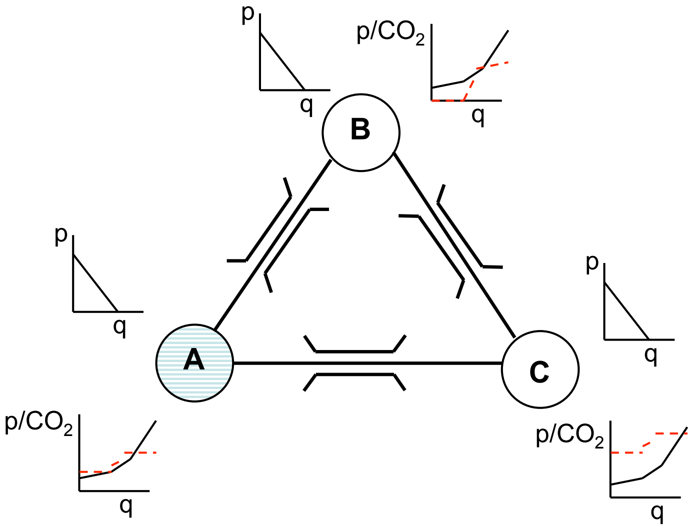

Figure 2: Network, illustrative demand, supply curves and CO_2 emission rates (dash lines) of the numeric example

emissions rate around 550 kg/MWh. Zone B consists of low-cost, clean hydropower with moderately expensive units with a marginal cost and emissions rate of approximately 25 \$/MWh and 550 kgs/MWh, respectively. Zone C has units with higher emissions but lower costs. The capacity-weighted average CO_2 emissions rate is approximately 600, 400 and 1,200 kg/MWh for zones A, B and C, respectively. Under no-cap case, roughly 20% and 10% of the electricity demand in zone A is met by imports from zone B and C, respectively. We assume a demand elasticity of -0.2 at the price-quantity pair in the baseline solution.

We simulate three scenarios for each emissions trading program, varying output-updated allocation parameter B from 0, 0.25 to 0.5 ton per MWh. Whereas first case represents grandfathering or reference scenario, the last two cases are designed to examine the effect of different levels of output-updated allowances allocation on the emissions leakage. In scenarios 1–3, zone A is subject to an emissions cap of 400 tons under three programs: load-based, source-based, and first-deliverer approaches. An additional scenario is designed to benchmark the situation when there is no emissions program. In next section, we report the results, including electricity and CO_2 allowances prices, sales, CO_2 emissions, CO_2 emissions leakage and contract shuffling and welfare outcomes. respectively.

3.3 Results

The main results are summarized by the Tables 2–4, and we also list the results from no-cap case in Table 1. The electricity sales and CO_2 emissions flow are displayed in Figures 3–4, respectively. We first discuss the electricity prices, followed by the results in CO_2 emissions leakage and contract shuffling.

Two counteracting forces jointly determine the equilibrium electricity prices. Whereas expansion of output induced by the anticipated awarded allowances would suppress electricity prices, it is to some extent offset by the increase in the GHG allowances cost, which is blended in the electricity prices. The former dominates the latter one when B=0.25 ton/MWh, and the resulting electricity price in Zone A (Table 3) is lower than that associated with no-updated case in Table 4. Yet, the reverse is true when B=0.5 ton/MWh, and allowances cost becomes sufficiently high, from 61 $/ton in no-update, to 106 and 530 $/ton under B=0.25 and 0.5 ton/MWh, respectively. Thus, the impact of output-based updating on the electricity price in Zone A can be in either direction, depending on the relative effects of output expansion and tightness of GHG emissions cap. In our simulation, consequently, the consumers are better off when B=0.25 ton/MWh.

The pattern of electricity sales among three zones are summarized in Figure 3. While nodal sales are roughly unchanged, the net sale from Zone C to B grows to more than twice from 118 MWh in grandfathering (lower, left) to 246 MWh in B=0.25 ton/MWh. The sales from Zone B to A increase slightly, because the lower-emitting generators in Zone B take advantage of the incentives provides by the awarded allowances. Also consistent with theoretical analysis in Section 2, the output from polluting Zone C decreases marginally by 3 MWh in output-updated allocation compared to grandfathering.

Under current context, contract shuffling describes the situation in which the rearrangement of financial contracts results in no actual emissions reduction in an emissions trading program. Consistent with definitions in Chen el al. [3], we define contract shuffling as the difference between the apparent decrease in emissions associated with power imports to zone A and the actual decrease in emissions in zones B and C. (Further calculation can be found in Chen et al. [3].) As a result, contract shuffling amounts to roughly 95% when B=0.25 and 0.5 ton/MWh.

Also consistent with Chen et al. [3] leakage is defined as the difference between the decrease in regulated emissions and the decrease in total regional emissions. That is, increases in unregulated emissions elsewhere could, to some degree, offset the decline in regulated emissions. We summarize the CO_2 emissions among three nodes in the Figure 4, where the arrows siting on the arcs representing net CO_2 emissions associated sales between two nodes. We focus on a comparison of the no-cap case in Table 1 with other approaches in Tables 2–4. As reported in Chen et al. [3], leakage is roughly 85% under grandfathering (i.e., B=0). The simulation here suggests that CO_2 leakage declines from 83% to 73% when B increases from 0.25 to 0.5 ton/MWh. This is broadly consistent with the conclusions in the theoretical analysis in Section 2.

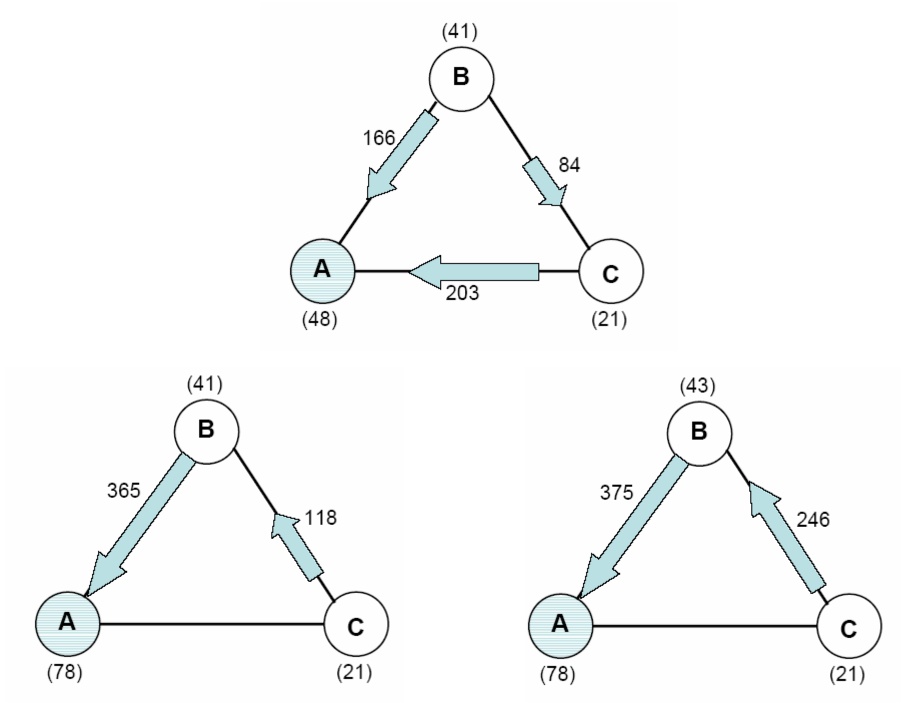

Figure 3: Equilibrium electricity sales of the numeric example: no-cap (upper), B=0 ton/MWh (lower, left) and B=0.25 (lower, right); nodal prices are within the parentheses.

4 Conclusion and Discussion

California and other WCI states are considering three proposals to regulate greenhouse gasses emitted by electric power plants: source-based, load-based and first-seller approaches. These three proposals differ by their point-of-regulation and possibly the "jurisdiction" of the regulation. We extend the previous analysis to consider how might the output-update allowances allocation that have been in favor by the California Public Utility Commission might affect CO_2 emissions leakage in general. We first examine the principles using analytical tractable duopoly setting. This theoretical analysis suggests output-updated allocation could mitigate GHG leakage by encouraging in-state generation from clean sources. However, the impact on electricity price is inconclusive and depends on relative emission rates, update parameter and tightness of the emission cap. These conclusions are then tested using the computable models that allow flexibility in embodying different point-of-regulation.

Our results suggest that majority of in-state emissions reduction (Zone A) could still be due to contract shuffling. However, the leakage could be to some extent mitigated by the output-update allowances allocation scheme. The level of leakage is negatively related to the update parameter B. Yet, the equivalence theorem established before in Chen et al. [3] are not valid when B becomes larger. For future research, further analysis

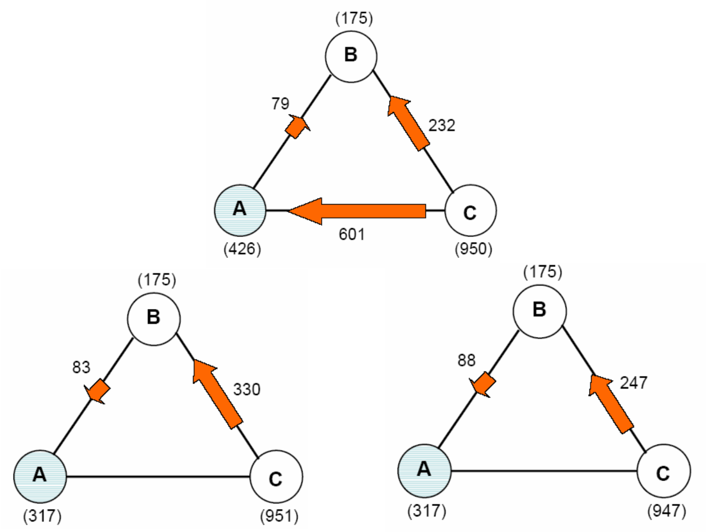

Figure 4: Equilibrium CO_2 flow of the numeric example: no-cap (upper), B=0 ton/MWh (lower, left) and B=0.25 (lower, right); nodal CO_2 emissions are within the parentheses.

is needed to study the property of resulting complementarity problem and to establish the conditions under which the existence and equivalence remain valid.

Appendix

To show the existence of a solution to LCP (7), we first introduce three matrices classes – $\mathbf{P_0}$–matrices, $\mathbf{P_1}$–matrices, and $\mathbf{Q_0}$–matrices, and the relationship among the three.

Definition 1. *A matrix $A \in \Re^{n\times n}$ is a $\mathbf{P_0}$–matrix if all its principal minors are nonnegative. The class of such matrices is denoted $\mathbf{P_0}$.*

Definition 2. *If $A \in \mathbf{P_0} \cap \Re^{n\times n}$, then A is a $\mathbf{P_1}$–matrix if there exists a unique index set $\alpha \subseteq \{1, \ldots, n\}$ such that $det A_{\alpha\alpha} = 0$. The class of $\mathbf{P_1}$–matrices is denoted $\mathbf{P_1}$.*

Definition 3. *For a vector $q \in \Re^n$ and a matrix $A \in \Re^{n\times n}$, if the LCP(q, A) is solvable whenever it is feasible, A is called a $\mathbf{Q_0}$–matrix. The class of such matrices is denoted $\mathbf{Q_0}$.*

Lemma 1. *(Cottle et al. [10], Corollary 4.1.11) $\mathbf{P_1} \subset \mathbf{Q_0}$ (properly).* ☐

The following result shows that the matrix that defines LCP (7) is a $\mathbf{P_1}$–matrix, under the condition $B < E_i < E_j$.

Lemma 2. *Define*

$$M \equiv \begin{bmatrix} K + b_i & K & E_i - B \\ K & K + b_j & E_j - B \\ -E_i & -E_j & 0 \end{bmatrix}.$$

For $B < E_i < E_j$*,* $M \in \mathbf{P_1}$*.*

Proof. Let $\mathcal{I}$ denote the index set of the principal minors of M. Then

$$\mathcal{I} = \{\{1\},\ \{2\},\ \{3\},\ \{1,3\},\ \{2,3\},\ \{1,2,3\}\}.$$

It is straightforward to check that for each $\alpha \in \mathcal{I}$, $\det M_{\alpha\alpha} \geq 0$, given that $B < E_i < E_j$. Hence $M \in \mathbf{P_0}$. In addition, $\det M_{\alpha\alpha} = 0$ only for $\alpha = \{3\}$. By definition, $M \in \mathbf{P_1}$. □

Proof of Proposition 1. By Lemma 1 and 2, it suffices to show that LCP (7) is feasible. Define a vector as follows.

$$\begin{bmatrix} \bar{q}_i \\ \bar{q}_j \\ \bar{p}^{GHG} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \max\{\dfrac{P^0 - a_i}{E_i - B}, \dfrac{P^0 - a_j}{E_j - B}\} \end{bmatrix}$$

By the assumption that $E_j > E_i > B$ and $P^0 > a_i > a_j$, it is trivial to show that the vector $(\bar{q}_i, \bar{q}_i, \bar{p}^{GHG})$ is feasible to LCP (7). □

Proof of Proposition 2. Assume that a solution $(\tilde{q}_i, \tilde{q}_j, \tilde{p}^{GHG})$ satisfying the conditions in (9) exists such that

$$\begin{aligned} f_1(q_i, q_j, p^{GHG}; B, \mathcal{C}) &\equiv (K + b_i)\tilde{q}_i + K\tilde{q}_j + (E_i - \tilde{B})\tilde{p}^{GHG} - (P^0 - a_i) = 0 \\ f_2(q_i, q_j, p^{GHG}; B, \mathcal{C}) &\equiv K\tilde{q}_i + (K + b_j)\tilde{q}_j + (E_j - \tilde{B})\tilde{p}^{GHG} - (P^0 - a_j) = 0 \\ f_3(q_i, q_j, p^{GHG}; B, \mathcal{C}) &\equiv E_i\tilde{q}_i + E_j\tilde{q}_j - \mathcal{C} = 0. \end{aligned} \tag{16}$$

Define a mapping $F : \Re^5 \rightarrow \Re^3$ as $F = (f_1, f_2, f_3)$. The Jacobian determinant of F with respect to (q_i, q_j, p^{GHG}) at $(\tilde{q}_i, \tilde{q}_j, \tilde{p}^{GHG}; \tilde{B}, \tilde{\mathcal{C}})$ is as follows.

$$\begin{aligned} |J|_{(\tilde{B},\tilde{\mathcal{C}})} &\equiv \left| \begin{matrix} \dfrac{\partial f_1}{\partial q_i} & \dfrac{\partial f_1}{\partial q_j} & \dfrac{\partial f_1}{\partial q^{GHG}} \\ \dfrac{\partial f_2}{\partial q_i} & \dfrac{\partial f_2}{\partial q_j} & \dfrac{\partial f_2}{\partial q^{GHG}} \\ \dfrac{\partial f_3}{\partial q_i} & \dfrac{\partial f_3}{\partial q_j} & \dfrac{\partial f_3}{\partial q^{GHG}} \end{matrix} \right|_{(\tilde{B},\tilde{\mathcal{C}})} = \left| \begin{matrix} K + b_i & K & E_i - \tilde{B} \\ K & K + b_j & E_j - \tilde{B} \\ E_i & E_j & 0 \end{matrix} \right| \\ &= -K(E_j - E_i)^2 - b_iE_j(E_j - \tilde{B}) - b_jE_i(E_i - \tilde{B}) \neq 0, \end{aligned} \tag{17}$$

where the last inequality follows trivially under the condition that $B < E_i < E_j$. Then by the Implicit Function Theorem, q_i, q_j, p^{GHG} are continuous functions of $(B, \mathcal{C})$ over set U such that $f_i(q_i(B), q_j(B), p^{GHG}(B); B, \mathcal{C}) = 0$ and the partial derivatives of q_i, q_j, p^{GHG} with respect to B and $\mathcal{C}$ exists. Furthermore, let $(\tilde{B}, \tilde{C}) \in U$, and let $(\tilde{q}_i, \tilde{q}_j, \tilde{p}^{GHG})$ be the corresponding equilibrium that satisfies the condition (9). Then the partial derivatives of (q_i, q_j, p^{GHG}) with respect to B satisfy the following linear system over the set U

$$\begin{bmatrix} K+b_i & K & E_i-\tilde{B} \\ K & K+b_j & E_j-\tilde{B} \\ E_i & E_j & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial q_i}{\partial B} \\ \dfrac{\partial q_j}{\partial B} \\ \dfrac{\partial p^{GHG}}{\partial B} \end{bmatrix} = \begin{bmatrix} \tilde{p}^{GHG} \\ \tilde{p}^{GHG} \\ 0. \end{bmatrix} \tag{18}$$

By solving the above linear system of equations, we obtain the following

$$\begin{aligned} \frac{\partial q_i}{\partial B} &= \frac{E_j(E_i-E_j)\tilde{p}^{GHG}}{-K(E_j-E_i)^2-b_iE_j(E_j-B)-b_jEi(E_i-B)} \\ \frac{\partial q_j}{\partial B} &= \frac{E_i(E_j-E_i)\tilde{p}^{GHG}}{-K(E_j-E_i)^2-b_iE_j(E_j-B)-b_jEi(E_i-B)} \\ \frac{\partial p^{GHG}}{\partial B} &= \frac{-(E_ib_j+E_jb_i)\tilde{p}^{GHG}}{-K(E_j-E_i)^2-b_iE_j(E_j-B)-b_jEi(E_i-B)} \end{aligned} \tag{19}$$

Since $\tilde{p}^{GHG}$ is from an equilibrium that satisfies the condition (9), $\tilde{p}^{GHG} \geq 0$. In addition, by solving the linear system of equations defined in (9), $\tilde{p}^{GHG} = 0$ only if

$$\mathcal{C} = \frac{\left[(K+b_j)(P^0-a_i)-K(P^0-a_j)\right]E_i + \left[(K+b_i)(P^0-a_j)-K(P^0-a_i)\right]E_j}{K(b_i+b_j)+b_ib_j}.$$

Hence, under the condition that the above equality does not hold, $\tilde{p}^{GHG} > 0$. Then by the assumption that $B < E_i < E_j$, we have the following

$$\frac{\partial q_i}{\partial B} > 0,\ \frac{\partial q_j}{\partial B} < 0,\ \frac{\partial p^{GHG}}{\partial B} > 0.$$

□

References

[1] J. Bushnell. The implementation of California AB32 and its impact on wholesale electricity markets. Technical Report WP-170, University of California Energy Institute, 2007.

[2] California Air Resources Board. AB32 fact sheet - California Global Warming Solutions Act.

[3] Y. Chen, A. Liu, and B.F. Hobbs. Economic and emissions implications of load-based, source-based and first-seller emissions trading programs under California AB32. University of California Energy Institute, EPE–022, 2008.

[4] California Energy Commission. 2006 net system power report. Technical Report CEC-300-2007-007, California Energy Commission, 2006.

[5] California Energy Commission. Revised methodology to estimate the generation resources mix of California electricity imports. Technical Report CEC-700-2007-007, California Energy Commission, 2007.

[6] California Public Utility Commission. Final opioion on greenhouse gas regulatory strategies, 2008.

[7] California Public Utility Commission. Interim opioion on greenhouse gas regulatory strategies, 2008.

[8] Market Advisory Committee. Market Advisory Committee to the California Air Resources Board: Recommendation for desgining a greenhouse gas cap-and-trade system for California, June 2007.

[9] F.J. Convery and L. Redmond. Market and price developments in the european union emissions trading schemes. *Review of Environmental Economics and Policy*, 1:88–111, 2007.

[10] R. W. Cottle, J.-S. Pang, and R. E. Stone. *The Linear Complementarity Problem.* Academic Press, Boston, 1992.

[11] Robert Gibbons. *Game Theory for Applied Economists.* Princeton, 1992.

[12] M. Gillenwater and C. Breidenich. Internalizing carbon costs in electricity markets: Using certificates in a load-based emissions trading scheme. Technical report, Science, Technology and Environmental Policy Program, Woodrow Wilson School of Public and International Affairs, Princeton University, Princeton, NJ, 2007.

[13] B. F. Hobbs. Theoretical equivalence of CO_2RC and source-based CO_2 trading. Mimeo, Department of Geography & Environmental Engineering, The Johns Hopkins University, 2007.

[14] S. Michel and J. Nielsen. An alternative load-based CO_2 cap & trade for the West. Western Resource Advocate, October 2007.

[15] Regional Greenhouse Gas Initiative. An Initiative of the Northeast and Mid Atlantic States of U.S.

[16] Western Climate Initiative. Western Climate Initiative.

[17] F. Wolak, J. Bushnell, and B. F. Hobbs. Opinion on load-based and source-based trading of Carbon Dioxide in California. Technical report, Market Surveillance Committee of the California ISO, November 2007.

Table 1: Summary of the results under pure load-based, pure source-based programs and no cap scenario

	Pure Load-Based			Pure-Source-based			No Cap		
Emission Cap [tons]		400				400			0
CS			151201.6			141407.0			160538.7
PS			10751.3			17307.5			5522.0
ISO			8856.8			14339.6			7899.1
LSE			21655.2			NA			NA
SW			192464.9			173054.1			173959.7
Variables\Node	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
Price [$/MWh]	69.3	10.8	18.6	55.2	16.2	18.6	38.0	16.2	18.6
CO2 Emissions [tons]									
CA	131.3	48.2	169.3	189.0	88.3	122.7	236.9	96.1	128.2
NW	268.7	0.0	0.0	135.5	12.5	105.1	134.2	19.8	99.1
SW	0.0	489.4	510.4	432.3	321.8	245.8	462.6	285.5	251.8
Nodal CO2 Emissions [ton]	348.8	268.7	999.8	400.0	253.0	999.8	461.2	253.0	999.8
Total CO2 Emissions [ton]			1617.3			1652.9			1714.1
CO2 Price [$/ton]			54.1			29.6			0.0
Generating Cost [$]			44248.2			47150.9			51158.4
Allowances Rents [$]			21656.0			11844.0			0.0
Electricity Sales (from\to)	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
CA	237.1	80.3	282.2	326.0	149.0	211.3	407.6	162.7	221.3
NW	737.5	0.0	0.0	380.7	36.0	289.4	380.0	51.7	274.4
SW	0.0	397.1	417.9	354.5	261.1	199.4	379.1	231.6	204.3
Total Sale [MWh]	974.6	477.5	700.0	1061.2	446.1	700.0	1166.7	446.1	700.0

Table 2: Summary of the results under load-based, first-seller and source-based programs

	Load-Based			First-Seller			Source-based		
Emission Cap [tons]		400				400			400
CS			107952.5			107952.5			107952.5
PS			25869.2			45211.7			50252.0
ISO			4834.2			4834.2			4834.2
LSE			24382.7			5040.3			0.0
SW			163038.7			163038.7			163038.7
Variables\Node	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
Price [$/MWh]	78.1	40.9	20.8	78.1	40.9	20.8	78.1	40.9	20.8
CO2 Emissions [tons]									
CA	317.3	0.0	0.0	317.3	0.0	0.0	317.3	0.0	0.0
NW	82.7	14.4	78.0	82.7	0.8	91.5	82.7	16.8	75.6
SW	0.0	330.7	620.0	0.0	368.9	581.8	0.0	325.2	625.5
Nodal CO2 Emissions [ton]	317.3	175.0	950.7	317.3	175.0	950.7	317.3	175.0	950.7
Total CO2 Emissions [ton]			1443.0			1443.0			1443.0
CO2 Price [$/ton]			61.0			61.0			61.0
Generating Cost [$]			43358.2			43358.2			43358.2
Allowances Rents [$]			24384.0			24384.0			24384.0
Electricity Sales (from\to)	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
CA	555.1	0.0	0.0	555.1	0.0	0.0	555.1	0.0	0.0
NW	365.4	28.7	155.9	365.4	1.6	183.0	365.4	33.5	151.1
SW	0.0	274.1	527.9	0.0	301.2	500.7	0.0	269.3	532.6
Total Sale [MWh]	920.5	302.8	683.8	920.5	302.8	683.8	920.5	302.8	683.7
Leakage			0.8473702			0.8474			0.8474
Contract Shuffling			1.0012			1.0013			1.0013

Table 3: Summary of the results under load-based, first-seller and pure source-based "UPDATE" programs (0.25ton/MWh)

	Load-Based			First-Seller			Source-based		
Emission Cap [tons]		400				400			400
CS			107596.58			107596.58			107596.58
PS			26580.55			45128.22			49747.93
ISO			5672.6			5672.6			5672.6
LSE			23167.38			4619.71		NA	
SW			163017.1			163017.11			163017.11
Variables\Node	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
Price [$/MWh]	77.78	43.12	20.74	77.78	43.12	20.74	77.78	43.12	20.74
CO2 Emissions [tons]									
CA	312.5	0.0	0.0	312.5	0.0	0.0	312.5	0.0	0.0
NW	87.5	8.2	79.4	87.5	8.0	79.5	87.5	0.0	87.5
SW	0.0	327.8	619.5	0.0	327.9	619.3	0.0	356.4	590.9
Nodal CO2 Emissions [ton]	312.5	175.0	947.2	312.5	175.0	947.2	312.5	175.0	947.2
Total CO2 Emissions [ton]			1434.73			1434.7			1434.7
CO2 Price [$/ton]			105.59			105.59			105.6
Generating Cost [$]			43013.29			43013.29			43013.3
Allowances Rents [$]			42236			42236			42240
Electricity Sales (from\to)	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
CA	547.4	0.0	0.0	547.4	0.0	0.0	547.4	0.0	0.0
NW	375.0	16.3	158.7	375.0	16.0	159.0	375.0	0.0	175.0
SW	0.0	273.7	525.2	0.0	273.9	524.9	0.0	289.9	508.9
Total Sale [MWh]	922.4	290.0	683.9	922.4	290.0	683.9	922.4	290.0	683.9
Leakage			0.8357			0.8357			0.8357
Contract Shuffling			0.9954			0.9954			0.9954

Table 4: Summary of the resutls under load-based, first-seller and pure source-based "UPDATE" programs (0.5tons/MWh)

	Load-Based			First-Seller			Source-based		
Emission Cap [tons]		400				400			400
CS			96564.87			96519.95			96399.21
PS			28357.42			42764.71			42785.56
ISO			20251.29			20271.21			20324.79
LSE			14399.48			0			0.0
SW			159573.1			159555.9			159509.6
Variables\Node	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
Price [$/MWh]	90.26	43.12	20.74	90.31	43.12	20.74	90.46	43.12	20.74
CO2 Emissions [tons]									
CA	234.97	0.76	30.73	234.44	0.73	31.09	219.98	14.42	31.34
NW	133.54	0.48	40.99	133.73	0.46	40.81	134.26	0	40.74
SW	0	344.11	603.13	0	344.13	603.1	0	326.28	620.95
Nodal CO2 Emissions [ton]	266.5	175.0	947.2	266.3	175.0	947.2	265.7	175.0	947.2
Total CO2 Emissions [ton]			1388.7			1388.5			1388.0
CO2 Price [$/ton]			526.29			526.95			528.7
Generating Cost [$]			40019.68			40007.4			39974.4
Allowances Rents [$]			210516.0			210780.0			211492.0
Electricity Sales (from\to)	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**	**CA**	**NW**	**SW**
CA	415.22	1.35	54.21	414.31	1.29	54.85	390.68	24.85	54.04
NW	430.56	1.42	118.02	431.14	1.37	117.49	453.89	0	96.11
SW	0	287.21	511.63	0	287.31	511.52	0	265.12	533.72
Total Sale [MWh]	845.8	290.0	683.9	845.5	290.0	683.9	844.6	290.0	683.9
Leakage			0.7251			0.7250			0.7570
Contract Shuffling			0.9947			0.9947			0.9949