

# A Dynamic Oligopolistic Electricity Market Model with Interdependent Segments

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- Traditional electricity sectors: vertically integrated  $\implies$  overcapacity
- Deregulation: competition between generators (i.e. profit maximization)
- Each generator
  - has two types of decision: production (short-run), investment (long-run);
  - can invest in different types of technology: base-load capacity, peak-load capacity;
  - faces interdependent demand segments: base-load demand, peak-load demand.

# Review of selected papers

References	Market model	Information structure	Main feature	Solution method	Intended market
Lise and Krusemann (2008)	Stackelberg	Feedback	Model with a static part (trade, capacity and environmental constraints) and a dynamic part (investment decisions).	Recursive dynamic formulation	EU8
Genc and Sen (2008)	Cournot equilibrium	S-adapted open loop	Capacity investments dynamic and pricing behavior of suppliers as uncertain demand evolves over time.	Games with probabilistic scenarios formulation	Ontario
Genc et al. (2007)	Cournot equilibrium	S-adapted open loop	Several stochastic programming formulations of dynamic oligopolistic games under uncertainty.	Stochastic programming	Ontario
Pineau and Zaccour (2007)	Cournot equilibrium	Complete information	Two-load-period electricity market model with interdependent demand, where oligopolistic generators make investments in peak- and base-load capacities.	Analytic with numerical illustration	Ontario

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References	Market model	Information structure	Main feature	Solution method	Intended market
Murphy and Smeers (2005)	Cournot equilibrium	Open loop; closed loop	Conversion of the inverse of the load duration curve to a probability density function (a way to cope with overall uncertainty about the future demand for electricity).	Variational inequality; Equilibrium subject to equilibrium constraints	
Pineau and Murto (2003)	Cournot equilibrium	S-adapted Open loop	Dynamic stochastic oligopoly model to describe the production and investment in deregulated electricity markets, where the demand growth rate is modeled as a stochastic variable.	Variational inequality; Non-linear programming	Finland
Ventosa et al. (2002)	Cournot equilibrium; Stackelberg	Open loop; closed loop	Cournot-based model: mixed linear complementarity structure; Stackelberg-based model: mathematical program with equilibrium constraints structure.	Cournot-based model: directly solved taking advantage of its complementarity structure; Stackelberg-based model: solved by general-purpose non-linear solvers.	
Chuang and Varaiya (2001)	Cournot equilibrium	Open loop	Generation expansion planning model that may characterize expansion planning in pool-dominated generation supply industries, where separate markets for energy and capacity reserves are modeled.	Iterative search procedure	California ISO/PX system

# Our contribution:

- 1 Investment in both base- and peak-load capacity;
- 2 Dynamic framework;
- 3 Interdependent segments;
- 4 Numerical analysis with Ontario data;
- 5 More realistic costs function.

Inverse demand laws:

$$P_b^t(Q_b^t, Q_p^t) = A_b^t - B_b \cdot Q_b^t - C_b \cdot Q_p^t \quad t = 0, \dots, T \quad (1)$$

$$P_p^t(Q_b^t, Q_p^t) = A_p^t - B_p \cdot Q_p^t - C_p \cdot Q_b^t \quad t = 0, \dots, T \quad (2)$$

Investment cost functions:

$$\Gamma_b(I_{ib}, k_{ib}) = \kappa_i \cdot I_{ib} \cdot k_{ib} + \frac{1}{2} \psi_i \cdot I_{ib}^2 \quad (3)$$

$$\Gamma_p(I_{ip}) = \frac{1}{2} \varphi \cdot I_{ip}^2 \quad (4)$$

Production cost functions:

$$G_{ib}(q_{ib}, k_{ib}) \triangleq G_{ib}(q_{ib}) = \alpha_b \cdot q_{ib} \quad (5)$$

$$G_{ip}(q_{ip}, k_{ip}) = \alpha_p \cdot q_{ip} + v_p \left( \frac{h_p \cdot k_{ip}}{\phi + 1} \right) \left( \frac{q_{ip}}{h_p \cdot k_{ip}} \right)^{\phi+1} \quad (6)$$

# Model

$$\max \pi_i = \sum_{t=0}^T \beta^t \left\{ \sum_{l \in \{b,p\}} [q_{il}^t P_l^t (Q_b^t, Q_p^t) - G_l (q_{il}^t, k_{il}^t) - \Gamma_l (I_{il}^t, k_{il}^t)] \right\} \\ + \beta^T \sum_{l \in \{b,p\}} V_l (k_{il}^T)$$

subject to :

*Expansion of capacity :*

$$k_{il}^{t+1} = (1 - \delta) k_{il}^t + I_{il}^t, l \in \{b, p\}, k_{il}^0 \text{ given}, t = 0, \dots, T - 1$$

*Capacity of base load :*

$$q_{ib}^t \leq h_b \cdot k_{ib}^t, t = 0, \dots, T$$

*Capacity of peak load :*

$$q_{ip}^t \leq h_p \cdot k_{ip}^t, t = 0, \dots, T$$

*Non-negativity constraints :*

$$I_{il}^t \geq 0, q_{il}^t \geq 0, l \in \{b, p\}, t = 0, \dots, T$$

**Table 1. Generation Capacities in Ontario, by Fuel type**

<b>Ownership</b>	<b>Fuel Type</b>	<b>Capacity (MW)</b>
OPG	Nuclear	6,600.0
Bruce Power	Nuclear	4,223.0
OPG & affiliates	Hydro	7,894.9
OPG	Coal	7,205.0
NUGA	Natural Gas	2,674.4
OPG	Oil/Gas	2,100.0
NUGA	Wood Waste	224.6
<b>Total</b>		<b>30,921.9</b>



**Table 2. Generation Capacities in Ontario, by Fuel type and by Ownership**

Technology	Player	Capacity (MW)	Capacity for Residential and Commercial consumers (MW)
Base	OPG	21,699.9	9,208.8
	Bruce	4,223.0	2,156.2
	NUGA	0	0
	<b>Total</b>	<b>25,922.9</b>	<b>11,365.0</b>
Peak	OPG	2,100.0	1,333.3
	Bruce	0	0
	NUGA	2,899.0	1,840.7
	<b>Total</b>	<b>4,999.0</b>	<b>3,174.0</b>

**Table 3. Production Costs**

<b>Technology</b>	<b>Marginal production cost (\$/MWh)</b>	<b>Duration of use</b>
Base-load	20	8,760 hours
Peak-load ( $\alpha_p$ )	60	3,845 hours
Peak-load ( $\alpha_p + v_p$ )	110	3,845 hours

**Table 4. Investment Costs**

<b>Technology</b>	<b>Investment cost (\$/MW)</b>	<b>Duration of use</b>
Base-load	4,000,000	8,760 hours
Peak-load	800,000	3,845 hours

- **Table 5. Direct- and Cross-Price Elasticities**

Period	Short-run		Long-run	
	Elasticity	Cross elasticity	Elasticity	Cross elasticity
Base	-0.3	0.02	-0.6	0.04
Peak	-0.1	0.01	-0.8	0.02

- To determine the parameters of the demand laws in (1) and (2), we use the above elasticity values, the quantities purchased in Ontario in 2005 during peak- and base- load periods, and the uniform price of \$92.20/MWh paid by residential and commercial customers in both periods.
- Finally, we assume that energy demand is increasing at a constant growth rate  $g = 0.013$  each year. This value corresponds to the highest growth rate that IMO expects for the future per year (see Genc and Sen (2008)).

- Here, our goal is to predict investment and production equilibrium strategies over a ten-year period for the three players forming the Ontario electricity industry, i.e., OPG, Bruce and NUGA.

- Here, our goal is to predict investment and production equilibrium strategies over a ten-year period for the three players forming the Ontario electricity industry, i.e., OPG, Bruce and NUGA.
- We also investigate the impact on these strategies of varying three key parameters of our model, namely, the number of players, the salvage value of production capacities, and price elasticities.

# NUGA's capacity trajectories

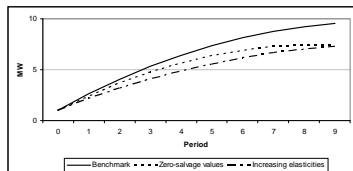


Fig.9 NUGA's base-load capacity ( $n = 3$ )

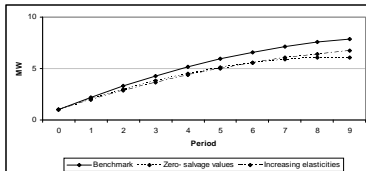


Fig.10 NUGA's base-load capacity ( $n = 5$ )

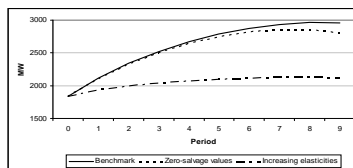


Fig.11 NUGA's peak-load capacity ( $n = 3$ )

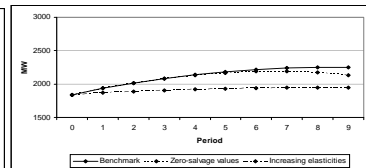


Fig.12 NUGA's peak-load capacity ( $n = 5$ )

# Bruce's capacity trajectories

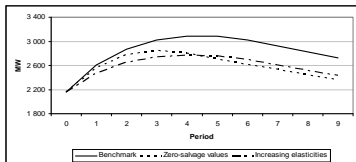


Fig.13 Bruce's base-load capacity ( $n = 3$ )

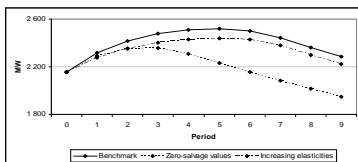


Fig.14 Bruce's base-load capacity ( $n = 5$ )

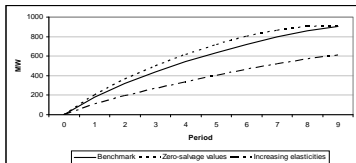


Fig.15 Bruce's peak-load capacity ( $n = 3$ )

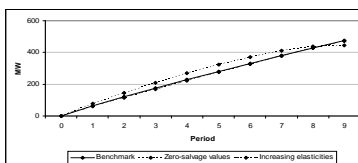


Fig.16 Bruce's peak-load capacity ( $n = 5$ )

# OPG's capacity trajectories

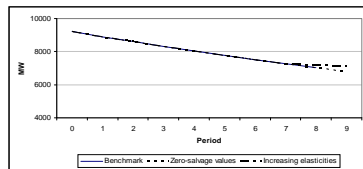


Fig.17 OPG's base-load capacity ( $n = 3$ )

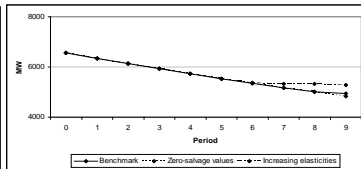


Fig.18 OPG's base-load capacity ( $n = 5$ )

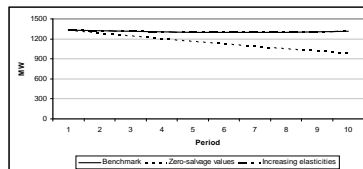


Fig.19 OPG's peak-load capacity ( $n = 3$ )

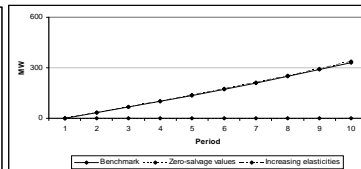


Fig.20 OPG's peak-load capacity ( $n = 5$ )



# Conclusion

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- Our numerical results show that when elasticity changes overtime, then prices decrease more in the peak-load market segment than in the base-load one, especially toward the end of the planning horizon.
- The numerical results also suggest that the distribution of initial capacity is generally determinant.

- As in any modeling effort, we made here some simplifying assumptions. The two main ones being that demand is known with certainty and that the players adopt an open-loop information structure to determine their strategies. It is clearly of interest to relax both assumptions in future investigations.

Thank you for your attention!