Plain vanilla options for price-quantity hedging in the Colombian wholesale electricity market

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Abstract
This paper applies the conceptual framework of Oum, Oren and Deng (2006) to hedge against price and quantity fluctuations in spot electricity markets. Both price and quantities are usually correlated and price hedging is insufficient to cover the risks faced by load-serving firms supplying regulated customers. By maximizing a static expected utility problem subject to plausible constraints, an infinite collection of derivatives (“exotic option”) emerges as the solution of both price and quantity hedging. This exotic option is approximated and calibrated with real data for the Colombian spot market. The approximation is designed as a portfolio composed by risk free bond, futures/forward contracts, 2 put and 3 call options with optimal strike prices, and it is applied to month-ahead and quarterly-ahead hedging during the peak hours. The approximation and time framework is carefully discussed from a market maker’s perspective. The proposal addresses major problems such as lack of liquidity and anonymity of the current bilateral trading scheme. The analysis could be easily extended to non-regulated customers and to power generators.

Key words: Competitive Electricity Markets, Colombia Power Market, Energy Derivatives, Incomplete Markets, Volumetric Hedging.

1. Introduction
Over the last two decades the electricity industry worldwide has undergone a profound restructuring process. Particularly in Colombia, laws 142 and 143 of 1994 in addition to later reforms, opened up an intensive reorganization of the electricity market, like the creation of the wholesale electricity market (MEM1) in 1995 simultaneously to the vertical unbundling of the generation, transmission, distribution and retail activities, seeking to improve the efficiency and quality of the electricity industry. Under this new framework, the power generation and retail businesses were allowed to be competitive deregulated markets, whereas the remaining two, transmission and distribution, were established as regulated activities.

As a result, nowadays there are three major markets at which generation and retail agents can trade energy: (i) a day-ahead spot market which determines the efficient dispatch of power generation resources, considering the suppliers offers submitted a day ahead of actual dispatch; (ii) a firm energy market created in 2007 by the power regulatory agency of Colombia (CREG2).

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1Spanish acronym for Mercado de Energía Mayorista.
2Spanish acronym for Comisión de Regulación de Energía y Gas (www.creg.gov.co).
This recent market is based on a firm energy product\footnote{The ability to supply electricity in a dry period, to consider the fact that Colombia has a high share of hydropower generation.} which is assigned through an annual auction mechanism and includes both a financial call option executed at a strike price—scarcity price—and the physical capability to supply firm energy. Thus the firm energy market coordinates investment in new generation assets to assure that in the future there will be enough firm energy available; (iii) a bilateral contract market with a long trajectory but today, under strong scrutiny given some problems that have come to light after these years of functioning \footnote{Spanish acronym for Mercado Organizado Regulado.}.

To overcome bilateral contracting issues, CREG has proposed the Organized Regulated Market (MOR\textsuperscript{4}) (see \footnote{Spanish acronym for Expertos en Mercados (www.xm.com.co)}.\footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).})\footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).} to address them, looking ahead for an efficient price formation through an energy forward contract market for regulated customers. This paper implements an alternative approach (eventually complementary) to MOR, through a financial derivative market to deal with these problems, supported on the theoretical work developed in \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).}.\footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).} Different financial derivative markets for the Colombian power sector has been studied and analyzed in \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).}, \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).} and \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).}. Those works concluded that one of the main problems to implement this market was the guarantees scheme for the transactions, since it was not possible to create a clearing house to handle credit risk due to financial policy weakness. However, the current situation allows the development of a derivative energy market, since law 964 of 2005 and decree 2893 of 2007 consent the trading of financial derivatives and the creation of clearing houses respectively. Furthermore, the Colombian power market administrator (XM\footnote{Spanish acronym for Expertos en Mercados (www.xm.com.co).}) and the stock market manager (BVC\footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).}) joined together to create the first clearing house in Colombia and to implement a derivative market with electric energy as underlying asset.

The remaining of this paper is organized as follows: Section II describes the current deficits in the Colombian bilateral contract market, Section III gets into a review of the literature about risk management and international experiences power derivative markets, Section IV assesses the suitability for a derivative power market in Colombia, Section V describes the methodology followed to undertake the work here presented which is supported on \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).}. Sections VI analyzes the power market information required, and Section VII displays results of a particular example. Finally a discussion section, some conclusions and further work ideas are presented in Sections VIII to X.

2. Bilateral contracting issues within Colombian power market

According to \footnote{Spanish acronym for Mercado Organizado Regulado.}, \footnote{Spanish acronym for Expertos en Mercados (www.xm.com.co).} and \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).} the ongoing framework to long-term bilateral contracts between the generation utilities and the load serving entities (LSE), suffers four major problems:

- Bilateral contract trading does not allow an efficient future price formation, therefore there is not a proper market estimation of the future power price, implying a greater price risk.

- Castro in \footnote{Spanish acronym for Bolsa de Valores de Colombia (www.bvc.com.co).} states that there are forty-two different kinds of bilateral contracts being traded in the market (they vary in quantity, price, maturity and other conditions). This absence of standard contracts reduce the liquidity of the market and also results in higher transaction costs in addition to weaker competition.
The lack of anonymity in these contracts might be encouraging discriminatory treatment toward certain LSEs, who are getting higher prices in the contracts than the others. Indeed, this fact goes against the achievement of a competitive market.

Financial guaranties are agreed between the parties. Nevertheless, the negative impact that one party would face as a consequence of this risk, is not fully hedged by this guaranties, since they are not properly set up or due to weak financial backup in some firms. This increases the credit risk that firms have to assume in each transaction.

Several alternatives have been suggested to address the problems above, like the Electronic Contracts System (SEC) documented in [5]. SEC was a bilateral contract system based on financial derivatives instruments over a computational platform. This proposal was not approved because it was required to its implementation an institution to manage the guaranties scheme (a clearing house) and by then there was not a legal framework to regulate this kind of entities. Additionally, according to [4] and [6] SEC had problems about the valuation of the financial instruments, since it proposed the Black-Scholes model to calculate the price of the option premium.

After that, CREG proposed MOR in [1], which main purpose was to implement a forward contract market for the regulated demand which is roughly 70% of the country’s aggregated demand. The proposal included an auction mechanism with an unique clearing price to assign the forward contracts for the regulated costumers. Later CREG hired professor Peter Cramton to design the auction mechanism. In [7], Cramton extends the initial MOR approach to an integrated forward energy market for both regulated and non-regulated customers, and suggests a simultaneous auction for both products. So far the Cramton’s suggestion has been studied intensively by the academy, the regulatory agency and the firms in the market, since several agents have found it as inconvenient and hard to implement. The last document published by CREG ([2]) presents an updated MOR’s version which is still under discussion, but whose product is a base-load forward contract with annual duration that is quite complementary with the on-peak financial derivatives options recommended trough this paper. This issue will be discussed later.

3. State of the art and international experience in derivative power markets

Electricity have proven to be the most volatile commodity, and it is not the exception in the Colombian competitive electricity market. It makes it compulsory to develop an appropriate risk management in order to maximize agents’ benefits and minimize the corresponding uncertainty upon them. Among the risks that firms have to handle with, are: macroeconomic risk, price risk, market risk, credit risk, regulatory risk, country risk, and quantity risk. The last one is a non-tradable risk, and an implicit feature of electricity, which has to do with the amount of energy that will be demanded in the future, which follows a stochastic process as well as the spot price. This situation directly impacts the firm’s revenues, and make it necessary to include it when designing the hedging portfolio.

Financial theory has developed research studies to find how to address this problem. In [8] electricity risk management is handled by a multi-market trading approach, while other references like [9] get focus on best risk management trough forward on-peak and off-peak contracts. Nevertheless, the electricity derivatives are increasingly studied and used around the world to manage the
financial risks and resource adequacy of power markets, like it is advocated in [10]. Particularly [11] proposed a financial call option to hedge against critic hydrologic scenarios in the Colombian electricity system by ensuring generation adequacy[8].

Further, derivative instruments has been developed to handle also the quantity risk, like swing options, weather derivatives, interruptible contracts, among other instruments named in [12]. The features of these derivative products, make them be usually traded over the counter (OTC), therefore they are low liquidity instruments, which is why they are not regarded in this paper. However, references [3] and [13] study quantity risk. In the former, Oum, Oren and Deng present a helpful approach to manage both quantity and price risk supported on positive price-quantity correlation, trough plain vanilla derivative instruments. Their model is briefly explained forward, along with the justification of why it was found suitable to design financial options for the Colombian wholesale electricity market.

To understand the way electricity is traded around the world, five different financial energy markets were studied: the nordic power market Nordpool, the biggest European market European Energy Exchange (EEX), the Great Britain market ELEXON, a market in the United States of America New York Market Exchange (NYMEX) and an Australian market d-cypha SFE Electricity Futures & Options. References [14], [15], [16], [17], [18], [19] and [20], provide an overview about the main characteristics of each of these electricity markets. Table 1 summarizes them.

### 4. Suitability of a financial derivative power market for Colombia

Three out of the four major problems of the current bilateral contract market in Colombia can be solved to a grate extend with a derivative power market. On one hand, through the development of a futures contract market it is possible to achieve an efficient future price formation for electricity, that is, the resultant price from all agents’ expectations about how the market will move in the short, medium and long run. This future price would be visible to all market players, so the arbitrage opportunities would be considerably reduced and in turn the bilateral market would benefit by having a forward curve for the energy price, as a bench mark to set contracts’ prices.

Moreover, the liquidity problem of the bilateral market may be solved by standardizing the derivative contracts that would be traded in an exchange market. This, encourages competition in the market, given the fact that a standard contract gives to its owner the possibility to close his position at any time before maturity. Furthermore, transaction costs can be lessen as under this derivative market institution, all transactions are carried out over a computational platform or via telephone to a financial desk.

Another achievement accomplished with a derivative market arises from the fact that each transaction in the market is undertaken trough a clearing house, which main purpose is to give, to each party, the financial guaranties needed to reduce its credit risk. Besides, thanks to the clearing house’s function, each party does not know each other, so the transaction is completely anonymous, avoiding the discriminatory treatment among agents.

Finally, it is worth noting that derivatives are used increasingly around the world to trade electricity, and that there is international experience in energy markets classified as efficient, which, of course, could work as a reference point to facilitate the implementation of the energy derivatives market in Colombia.

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*This work led to the beginning of current Colombian firm energy market, described above.*
Table 1: Comparison of financial electricity markets reviewed.

<table>
<thead>
<tr>
<th></th>
<th>Nordpool</th>
<th>EEX</th>
<th>ELEXON</th>
<th>NYMEX</th>
<th>d-cypha SFE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>How does it work?</strong></td>
<td>Under market rules</td>
<td>Under market rules</td>
<td>Based on a code called BSC and partially following market rules</td>
<td>Under market rules</td>
<td>Under market rules</td>
</tr>
<tr>
<td><strong>How the spot price is realized?</strong></td>
<td>Marginal cost of the latest plant dispatched</td>
<td>Marginal cost of the latest plant dispatched</td>
<td>It is a weighted average of bids submitted</td>
<td>Marginal cost of the latest plant dispatched</td>
<td>Marginal cost of the latest plant dispatched</td>
</tr>
<tr>
<td><strong>Are there derivative instruments?</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Forward Contracts</strong></td>
<td>Monthly, quarterly and annually base-load and peak-load contracts. Maximum maturity: 5 yr.</td>
<td>No</td>
<td>Only Bilateral Contracts.</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Put and Call options</strong></td>
<td>With the forward contracts as underlying asset. European type.</td>
<td>With monthly, quarterly and annually Phelix index as underlying asset. European type.</td>
<td>No</td>
<td>With daily spot price as underlying asset. European type.</td>
<td>With the futures contract as underlying asset. European type.</td>
</tr>
<tr>
<td><strong>Contracts of difference</strong></td>
<td>There are six different area prices</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>There are, but they are supervised by NEMMCO</td>
</tr>
<tr>
<td><strong>Contract Settlement Type</strong></td>
<td>Cash</td>
<td>Cash and physical</td>
<td>-</td>
<td>Cash</td>
<td>Cash and physical</td>
</tr>
<tr>
<td><strong>Markets Type</strong></td>
<td>OTC and Exchange</td>
<td>OTC and Exchange</td>
<td>OTC and Exchange</td>
<td>OTC and Exchange</td>
<td>OTC and Exchange</td>
</tr>
<tr>
<td><strong>Credit risk management</strong></td>
<td>Trough a clearing house.</td>
<td>Trough a clearing house. There are two of them.</td>
<td>Trough a regulatory framework.</td>
<td>Trough a clearing house.</td>
<td>Trough a clearing house.</td>
</tr>
<tr>
<td><strong>Computational platform</strong></td>
<td>PowerClick</td>
<td>Xetra</td>
<td>-</td>
<td>CME Globex</td>
<td>SYCOM</td>
</tr>
</tbody>
</table>

It is crucial to guarantee that the financial derivative markets with any commodity and specially with a complex commodity as it is electricity, should be designed in the clearest and simplest way, so as to facilitate the understanding of its trading and not give place to market failures like the experienced during the last year all around the world.
5. Methodology

As it was made clear at the introduction of this document, this paper implements, an alternative proposal (and eventually complementary) to MOR, a financial derivative market seeking to minimize the effects of the four major problems identified in the bilateral contract market nowadays functioning in Colombia. This is done supported on the theoretical work developed in [3], and putting the ideas of Oum, Oren and Deng to work in a practical context with real market information from Colombia’s power market, backed by the fact that in Colombia, spot price and regulated load served by retailers are positively correlated ($\approx 0.6$).

5.1. The model

A short description of the model developed in [3] is here illustrated. More details can be found in the original reference. Oum, Deng and Oren deal with the static hedging problem of a LSE who has to serve an uncertain electricity demand $q$ at a regulated fixed price $r$ in a single period from 0 to 1. Besides, the LSE procures the electricity to serve his customers, from the wholesale market at a spot price $p$. Hence the profit of the LSE would be:

$$y(p, q) = (r - p) \cdot q$$

(1)

To protect himself against price risk, the LSE can take a long position in $\bar{q}$ forward contracts at a fixed forward price $F$. However the LSE will face another risk that arises from the fact that demand $\bar{q}$ may vary from the expected value at time 0 to the actually realized value at time 1. Then, if the actual demand realized by the retailer agent is $\bar{q} + \Delta q$, then, the share of the profit in (1), that is at risk is: $(r - p) \cdot \Delta q$, where $\Delta q$ could be different to zero (gains/losses).

To deal with this hedging problem, the authors derive the optimal hedging portfolio as a function of spot price $p$. That is:

$$Y(p, q) = (r - p)q + x(p)$$

(2)

where $x(p)$ is the optimal hedging portfolio as function of the spot price $p$.

Regarding the LSE’s preferences and risk aversion profile, it is necessary to identify its utility function $U$ over total profit $Y$. In turn, because of the positive correlation between price and demand, which will be evident later in the paper for the colombian electricity market, there exists a joint probability function $f(p, q)$ defined on the probability measure $P$, which characterizes the behavior of $p$ and $q$ at time 1. On the other side, $Q$ is a risk-neutral probability measure by which the hedging instruments are priced and $g(p)$ is the probability density function of $p$ under $Q$. Keeping this in mind, in [3] the optimization problem is formulated as follows:

$$\max_{x(p)} E[U(Y(p, q))]$$

s.t. $E^Q[x(p)] = 0$

(3)

where $E[\cdot]$ and $E^Q[\cdot]$ denote expectations under probability measures $P$ and $Q$, respectively. In [3] the constraint implies that the hedge portfolio $x(p)$ is self-financing, that is, the LSE can borrow funds in the money market, in order to purchase the derivative instruments needed to obtain the

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9Expected demand in time 1, at time 0.
maximum expected utility over the total profit $Y(p,q)$. This constraint also means that there are not arbitrage opportunities through this hedging portfolio, under a constant risk-free rate. The optimization process (detailed in [3]) yields as a result the optimal payoff function $x^*(p)$.

In [21], through an extension of the fundamental calculus theorem, it is demonstrated that any twice continuously differentiable function can be written as follows, for fixed value $F$:

$$x(p) = x(F) \cdot 1 + x'(F) \cdot (p - F) + \int_0^F x''(K) \cdot (K - p)^+ dK + \int_F^\infty x''(K) \cdot (p - K)^+ dK \quad (4)$$

The expression $(\cdot)^+$ in the above equation, is equivalent to the function $\max(\cdot, 0)$. It is important to note that in the expressions above, $\max(K - p, 0)$, $\max(p - K, 0)$ correspond to the payoff profile of a bond, forward contract, put option and call option, respectively. In this sense, and remaining the LEGO© approach theory presented in [22], with $x(F)$ units of bonds, $x'(F)$ units of forward contracts, $x''(K)dK$ units of put options with strike price $K$ ($K < F$) and $x''(K)dK$ units of call options with strike price $K'$ ($K' > F$), it is possible to replicate the resultant optimal hedging portfolio $x^*(p)$ from the optimization process. This financial derivatives have as underlying asset the electricity spot price.

Viewed from this angle, to replicate the optimal function $x^*(p)$, the equation (4) implies that it is necessary to have a set of continuum strike prices for both put and call options. Since markets are incomplete, there are no markets with that amount of strike prices on board, and assuming that there is only $n$ put options and $m$ call options available in the market, Oum, Deng and Oren proposed in [3] a portfolio compounded by $x(F)$ units of bonds, $x'(F)$ units of forward contracts, $x''(K_i)dK$ units of put options with strike prices $K_i$, $i = 1, \ldots, n$, and $x''(K'_j)dK$ units of call options with strike prices $K'_j$, $j = 1, \ldots, m$.

The errors of this replicating strategy are calculated in [23] depending on the range in which spot price is actually realized at time 1.

5.2 Goodness of the model for the Colombian electricity market

The non-storability feature of electricity along with the steeply rising supply and the inelastic demand curve, both schematically represented in Figure 1, makes the electricity price $p$ and the electricity demand $q$ to be positively correlated. It actually happens in this way in the Colombian electricity market, since the dispatch is carried out in order of merit, so, when demand raises during on-peak hours, it forces the system to put in operation a more expensive generation resource, increasing the spot price as well.

Additional to the fact explained above, throughout summer seasons, the hydropower plants\footnote{According to XM, on December 2007, the hydropower capacity represented the 64% of the total installed capacity, corresponding to 87% of the electricity generated.} which are technologies with lower variable costs, reduce their power production due to natural water inflows shortages and of course to a diminishment in water reserves into reservoirs. This turns out in a higher marginal system cost, so an increment in demand translates into an increase in the spot price. This increment in the spot price reaches larger values during summer seasons than during winter seasons. Therefore, the correlation between $p$ and $q$ is bigger during dry periods in the Colombian wholesale electricity market.

The earlier reasons justify the usage of the price-quantity hedging strategy presented in [3], in order to design suitable derivative instruments to be used by the agents of the electricity market.
It is clear that the advantage offered by the financial derivative products, arises from the fact that it is likely to achieve the maximum expected value of total profit $Y(p,q)$, together with the minimum deviation of this profit (which means less uncertainty over LSE’s total profit), after building a hedging portfolio with those instruments. To do so, a utility function to represent the risk aversion preferences of the agents like a mean-variance utility function is a good candidate to accomplish both goals at the same time. Here it is utilized the same mean-variance utility function used in [3], which has been used in financial hedging literature to deal with non-tradable risk:

$$U(Y) = Y - \frac{1}{2}a(Y^2 - E[Y]^2)$$

(5)

where $a$ represents the agent’s risk aversion coefficient. Certainly, maximizing the expected value of the utility function in (5) ($E[U(Y)] = E[Y] - \frac{1}{2}aVar(Y)$), is equivalent to maximize the expected value of $Y(p,q)$ and also to minimize the variance of (2), that is the advantage provided by a price-quantity hedging portfolio.

In equation (6) it is presented the optimal hedging function constrained to the mean-variance utility function. The arithmetic procedure to find this expression is clearly developed in [3].

$$x^*(p) = \frac{1}{a} \left( 1 - \frac{g(p)/f_p(p)}{E^Q[g(p)/f_p(p)]} \right) - E[y(p,q)p] + E^Q[E[y(p,q)p]] \frac{g(p)/f_p(p)}{E^Q[g(p)/f_p(p)]}$$

where $f_p(p)$ is the marginal density function of $p$ under the probability measure $P$.

From the perspective of a clearing house or a market maker, who is willing to design the most adequate hedging instruments for all the participants in the market, the goodness of this model, is that equation (6) can be used to calculate the optimal payoff function for each retailer, which will determine the optimal hedging instruments needed by each particular retailer. Once this task is undertaken, and seeking to find the optimal hedging instruments for all of them jointly interacting in the market, here it is proposed to estimate a weighted average market payoff function $\bar{x}(p)$ as presented in equation (7), which will represent the joint needs of all the agents and could be used to determine general hedging instruments -suitable for all-, by no favoring big or small agents and giving to each of them the same hedging opportunities.

$$\bar{x}(p) = \sum_{i=1}^{N} w_i x^*_i(p)$$

(7)

Where,
The variable $S$ corresponds to a price-cap value defined under the market maker or clearing house criteria. Within the Colombian framework, this variable could be interpreted as the scarcity price associated to the firm energy market described above, since hedging above this price already exists given the call options related to that market.

After calculating $\bar{x}(p)$, the replicating methodology suggested in [3] is used to find the right number of bonds, forward contracts, put and call options needed to best describe the behavior of $\bar{x}(p)$.

Some of these financial instruments are currently available in the Colombian market, except the financial options. The simplest of them, bonds, can be found available in the stock market; forward contracts needed to replicate the function $x^*(p)$, could be either the current bilateral contracts (not suggested), the forward contracts that are planned to be included in MOR proposal \[11\] or even better, future contracts which are quite possibly to be launched the next year by a central chamber of counter-party risk (CRC\[12\]) to be established soon in Colombia. Since there are no financial options to replicate the optimal hedging function on the current electricity market, the present work just attempts to design suitable plan vanilla options to be offered by the clearing house to the market agents.

As circumstances may require from a market maker perspective, in order to design a simple market with enough instruments to offer the proper hedging for each agent and minimize transaction costs related to the trading activities, it is necessary to provide the least amount of financial options sufficient to best replicate the $\bar{x}(p)$ function. In this paper, it was decided to propose two put options with strike prices $K_1$ and $K_2$, next to three call options with strike prices $K'_1$, $K'_2$ and $K'_3$. An additional call option compared to those put options proposed, is justified by the fact that even though the LSE’s net profit is lower when the spot price goes down (its demand also goes down as a consequence of the positive correlation between these variables), the potential profit losses when the spot price goes up are wide higher. So, it is very important that the derivative market offer to retail agents, a good hedge against this upside of the spot price.

After some analysis about the Colombian hydrologic cycle, electricity spot price behavior throughout the year, and the technologic composition of the power generation park within the country as well as the resources availability for power generation in the future, it was decided to set up financial options with two different maturities: (i) monthly options, which are suggested in order to build a liquid market for these instruments and considering that it is likely that futures market will have a monthly basis; add to that, (ii) quarterly options, which are suggested to match the annual hydrologic cycle in Colombia where there are two winters and two summers alternating along the annual cycle, and which, as it will be explained in the discussion section of this paper, could also match with the product that is planned to be included under the MOR proposal.

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\[ w_i = \frac{\int_0^S x_i^*(p) dp}{\sum_{i=1}^N \int_0^S x_i^*(p) dp} \] (8)

\[ 11 \] This last issue is addressed at the discussion section of this paper.

\[ 12 \] Spanish acronym for Camara Central de Riesgo de Contraparte.
5.3. Programmed Algorithm

After defining the right number of financial options to replicate the market optimal hedging strategy $\bar{x}(p)$, an algorithm was programmed in a mathematical programming language\textsuperscript{13} to find the optimal strike prices for the options. This optimization is carried out to find the strike prices for both kind of options which minimize the total error associated to the replicating strategy of the market optimal hedging function $\bar{x}(p)$. The algorithm follows the flux diagram exhibited in Figure 2.

Inputs in the first block of the diagram are time series of electricity spot price and regulated demand served by different LSEs, as well as information about the regulated tariff $r$ and the forward price $F$; the last two must correspond to a particular time interval $T$ which may be either the month or the quarter associated to the hedging strategy to be estimated. Time series information is used in the second block to calculate the correlation coefficient $\rho$ between spot price and demand for each LSE, and the parameters of probability distribution function (pdf) fitted for the period $T$, that is, if $T$ corresponds to a time period of a month, let’s say May, the prices and demands of all Mays are fitted to pdfs separately. The parameters of these pdfs are required at the next block, where the optimal payoff function $x^*_i(p)$ is calculated using equation (6), for each LSE.

At the fourth box, the algorithm uses equation (7) to calculate the market optimal hedging function $\bar{x}(p)$, which is replicated in the fifth block, using the strategy presented in [3].

The minimization process for the total error of the replicating strategy is performed at the sixth box. The formulation of the optimization problem is presented as follows:

\footnotesize\textsuperscript{13}MATLAB R2008a was used as programming tool.
\[
\min_{K_1, K_2, K'_1, K'_2, K'_3} \sum_{p=0}^{S} (\tilde{x}(p) - \hat{x}(p))^2
\]  

(9)

\[
\begin{align*}
\hat{x}(p) &= \tilde{x}(F) + \tilde{x}'(F)(p - F) + N_{p1}(K_1 - p)^+ + N_{p2}(K_2 - p)^+ \\
&\quad + N_{c1}(p - K'_1)^+ + N_{c2}(p - K'_2)^+ + N_{c3}(p - K'_3)^+
\end{align*}
\]

s.t

\[
\begin{align*}
N_{p_i} &= \frac{1}{2}(\tilde{x}'(K_{i+1}) - \tilde{x}'(K_{i-1})), i = 1, 2 \\
N_{c_j} &= \frac{1}{2}(\tilde{x}'(K'_{i+1}) - \tilde{x}'(K'_{i-1})), j = 1, 2, 3 \\
0 &< K_1 < K_2 < F < K'_1 < K'_2 < K'_3 < S
\end{align*}
\]

where \(F\) denotes the future price for time interval \(T\). In turn, the error function \(E_p\) (derived in [23]) depends on the range where the spot price \(p\) is realized. The variable \(S\) will constraint the distribution of strike prices for call options in the minimization problem.

The Newton Raphson approach is used, given its convenience over quadratic objective functions, to find optimal strike prices \(K_1, K_2, K'_1, K'_2\) and \(K'_3\), of put and call options, that minimize the total replicating error.

Once the strike prices are optimized, a Monte Carlo simulation (10,000 iterations) is carried out to estimate net profits and utility measures for each LSE, as a result of several scenarios of price and quantity, to assess the impact of the market replicating strategy on each agent.

Finally, the procedure can either stops or starts again from the second block, to find the strike prices for a different time interval \(T\).

6. Analysis of available information to conduct the assessment

As it was mentioned above, the input information required is:

- Hourly spot price (historic time series).
- Hourly regulated energy demands served by each LSE (historic time series).
- Retailers’ risk aversion coefficient \(a\) (estimated).
- Regulated tariff \(r\), at which each LSE serve its customers.
- Correlation coefficient \(\rho_{p,q}\) for each LSE.

6.1. Spot price

On December 2006, it came into effect Resolution 071 of 2006 ([24]) written by CREG, with the purpose to establish a new firm energy market for capacity adequacy, changing the way generators bid in the spot market, so the behavior of spot energy price throughout the day changed as well. Since it was necessary to look for a time interval in which hourly spot prices had the same impact due to policy intervention, the data analysis was restricted to the business days information available from December 2006 up to October 2008 (time of the writing of this paper).
However, the price-quantity uncertainty that market agents has to face, is not uniform along the day or the year. Figures (a) and (b) show mean spot price behavior and standard deviation spot price variation, respectively.\footnote{Spot prices are in constant units (Colombian pesos) to October 2008, and were put into dollars at the exchange rate of the same month.}

In Figure (a) it is noted that higher prices occur from hours 18 to 20 (usually, hour 19th corresponds to day’s peak-hour). Statistical analysis support this. Backed by this fact, from now on, hours 18, 19 and 20 will be defined as on-peak hours for this application. It is also seen that August, September and October show the higher prices along the year, which could be explained because these months match a summer season and also during these months, industry works full time to produce commodities for Christmas season, causing an increment on energy prices.

On the other hand, Figure (b) exhibits a similar hourly and monthly behavior for the spot price standard deviation, which means that suppliers and retailers face higher price uncertainty (as well as demand uncertainty as it will be argued in the next section) over the on-peak hours and over the months with the higher prices (Aug-Sep-Oct). The annualized volatility of spot price, calculated as the standard deviation of logarithm yields for a daily time window, is roughly 130%. This huge volatility forces agents to hedge against bad financial scenarios.

Regarding the above analysis, it was found worthwhile developing the hedging strategy with financial options just for the on-peak hours, given that it is considered that any agent could accomplish a good hedge for the off-peak hours, just by holding long positions on forward or future base-load contracts, which are available in the OTC market, and eventually at MOR as it is stated in the proposal, or even better, at a potential exchange market to be implemented.

Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) goodness of fit tests were carried out to spot prices for each one of the on-peak hours independently. It comes up by applying the tests for monthly or quarter time intervals, that spot prices fit to log-normal pdf with a significance level of 95%.

After undertaking mean difference statistical tests among the on-peak hours spot prices, it is
possible to say that there is no significant difference (i.e., differences range between 1.0 and 1.2
US$/MWh approximately with a significance level of 95%), or that it can be neglected; same thing
happens with standard deviations. Thus, it was decided to set the pdf of spot prices on 19 hour,
as the distribution of the whole on-peak period.

6.2. LSEs’ demands

Input data analysis allows to state that correlation coefficient between spot price and regulated
demand is much greater than the correlation between spot price and non-regulated demand, partly
explained because regulated demand represents roughly the 67% of the total system demand,
and the non-regulated demand has a nearly flat path throughout the day (see Figure 4). Non-
regulated demand does not contribute as much as the regulated demand to on-peak prices, that is
way the analysis focuses on LSEs’ hedging, which serve regulated demand.

To make this analysis representative for the market, five different electricity retailers were
chosen and named $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$. They aggregate the 72% of the total regulated
demand and the 48% of total power system demand.

Just as in the case of prices, higher regulated demands happen to occur during on-peak hours.
This can be argued by the high correlation existent between price and regulated load. Even so,
variations of the regulated demand along the year are nor as strong as the prices’ variations, since
customers in Colombia are not directly exposed to spot system price making them to be an inelastic
demand. This situation gives place to a regulated demand with a uniform profile throughout the
year. Even tough, energy demand slightly raises when Christmas is coming.

Similarly to spot prices, KS and AD tests were applied to business days on-peak demands data
for each LSE. It turns out that the monthly and quarterly on-peak energy demand served by each
retailer, fits to normal pdf, with 95% significance level.

6.3. Risk aversion coefficient

To estimate the risk aversion coefficient $a$ of each each retailer $E_i$, it was considered the per-
centage of energy purchases trough bilateral contracts over the total demand served by each of
them. Defining the risk aversion coefficient space as the interval $[0, 1]$, each LSE was placed within
this space following the criteria mentioned above. In this sense, $a = 1$ means that the firm bought
all its energy demand trough bilateral contracts, so the higher the value of $a$, the greater the risk
aversion coefficient. Table 2 shows the coefficient $a$ for each retailer.

This parameter estimation is just a practical approx to face the problem. Improvements can
be done by different techniques.

6.4. Regulated tariff

In [25] CREG establishes the pricing formula for regulated customers. Since the regulated tariff
$r$ is going to be compared with the spot price $p$, it must be determined regarding only the sum
of generation and retail components of the regulated tariff established by the regulator, otherwise
the remaining transmission and distribution components should be considered into spot price $p$, as
a cost for the retailer.

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15This curves correspond to average behavior along the time interval selected as relevant for the analysis.
6.5. Correlation coefficient

The positive correlation between price and demand for the Colombian wholesale electricity market, has been argued and highlighted throughout the paper. This important fact, does not allow a full hedge for retailers by using just forward or futures contracts (i.e., just price hedging).

In this paper, this parameter was calculated by correlating the daily mean log-prices behavior in each month of the year (24h times 12 months) with the corresponding daily mean regulated demand for each LSE. Thus, to estimate the correlation coefficient for each LSE there are 288 pair of values available. Table 2 shows the correlation coefficient of all of them.

<table>
<thead>
<tr>
<th>Retailer</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>0.925</td>
</tr>
<tr>
<td>$E_2$</td>
<td>1.000</td>
</tr>
<tr>
<td>$E_3$</td>
<td>0.967</td>
</tr>
<tr>
<td>$E_4$</td>
<td>0.922</td>
</tr>
<tr>
<td>$E_5$</td>
<td>0.914</td>
</tr>
</tbody>
</table>

7. Results

Results are going to be presented as the estimation of optimal strike prices for 2 put and 3 call monthly options for January 2008. This procedure intends to illustrate a practical approach by which a market maker or a clearing house could design the instruments to offer appropriate hedging for market participants.

\[\text{As prices obey log-normal pdf.}\]
It is assumed that the hedging strategy is built up by each LSE at the beginning of the month, and it remains constant throughout this time interval\textsuperscript{17}. Specific parameters required to follow the flux diagram illustrated in Figure 2 are listed below:

- Spot price, which is log-normally distributed  
  \[ \log(p) \sim N(4.11, 0.13^2) \]  
  under the real and risk-neutral probability measures \( P \) and \( Q \).

- January’s future energy price is assumed to be the expected value of the log-normal pdf fitted to spot price, \( F = 61.88 \) US$/MWh.

- Regulated tariffs for each LSE and their corresponding parameters for the normal pdf fitted, are listed in Table 3:

<table>
<thead>
<tr>
<th>Retailer</th>
<th>( \mu ) [MWh]</th>
<th>( \sigma ) [MWh]</th>
<th>( r ) [US$/MWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 )</td>
<td>1281.4</td>
<td>64.0</td>
<td>51.40</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>260.5</td>
<td>12.7</td>
<td>44.53</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>907.7</td>
<td>26.1</td>
<td>53.27</td>
</tr>
<tr>
<td>( E_4 )</td>
<td>1364</td>
<td>70.1</td>
<td>45.18</td>
</tr>
<tr>
<td>( E_5 )</td>
<td>265.4</td>
<td>11.7</td>
<td>55.44</td>
</tr>
</tbody>
</table>

- January’s scarcity price \( S = 166 \) US$/MWh.

- Risk aversion and correlation coefficients are those presented in Table 2.

In the third block of the proposed algorithm, equation (6) is used to calculate retailers’ optimal payoff \( x^*_i(p) \) by replacing the parameters of the pdf for price and demands, correspondingly. The development of this equation for a bivariate log-normal-normal joint probability distribution, is illustrated and explained in [23] (See Appendix for details). Figure 5-a illustrates this curves for each LSE, as well as the market optimal hedging function \( \bar{x}(p) \) calculated with equation (7), painted with a continuous line.

Figure 5-b illustrates next to market hedging function \( \bar{x}(p) \) also painted in 5-a, the replicating strategy after found optimal strike prices (sixth block), which define the horizontal grid in the picture (\( K_1 = 33.49, K_2 = 56.87, K_1' = 65.94, K_2' = 92.66, K_3' = 125.41, \) US$/MWh). As it can be seen, both curves seem to merge in one, since the replicating strategy closely follows the \( \bar{x}(p) \) function. Right below this one, Figure 5-c illustrates the absolute replicating error. This optimal strikes were calculated for \( \bar{x}(p) \) function; although, it is necessary to estimate how appropriate they are for each LSE. Calculating a relative replicating error for each retailer, it was found that these are: \( \delta_{E_1} = 0.66\% \), \( \delta_{E_2} = 0.57\% \), \( \delta_{E_3} = 0.17\% \), \( \delta_{E_4} = 0.62\% \) and \( \delta_{E_5} = 0.59\% \), which with no doubt, are considerably small and evidence no favoring any kind of retailer (big or small).

\textsuperscript{17}Hence, retailers have the chance to exercise the options one day ahead of each month’s business day, taking as reference the spot price from the day-ahead daily auction. This comment is an initial guessing of how the options could be exercised that could be subject to changes as the institution for financial derivatives in the electricity market is designed.
Just to mention, after a sensitivity analysis on the future energy price variable $F$ to the error minimization problem, it was noticed that strike prices for second put option (Put$_2$) and first call option (Call$_1$) are systematically close to the value of this $F$ variable, implying that these two options are more likely to be at the money options. Another implication of a sensitivity analysis, this time carried out on $S$ variable to the error minimization problem, showed that strike prices are strongly affected by changes in this variable $S$.

Finally, Figure 5-d illustrates profit distribution for $E_1$ after price-quantity hedging (similar results not painted in the paper are exhibited by remaining LSEs), compared to just price hedging and no hedging at all. It is easy to identify that with price-quantity hedging it is simultaneously achievable a reduction in the dispersion of the retailer’s profit, and an increase in the expected value of the profit. This information is summarized in Table 4. Estimating the utility function for each agent, there are found the same results as in the case of profit in the mean-variance sense.

Table 4: Profit distribution comparison. January 2008.

<table>
<thead>
<tr>
<th>LSE</th>
<th>No hedge</th>
<th>Price Hedge</th>
<th>P-Q Hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>$E_1$</td>
<td>-13,611</td>
<td>10,814</td>
<td>-13,712</td>
</tr>
<tr>
<td>$E_2$</td>
<td>-4,553</td>
<td>2,252</td>
<td>-4,575</td>
</tr>
<tr>
<td>$E_3$</td>
<td>-7,777</td>
<td>7,379</td>
<td>-7,866</td>
</tr>
<tr>
<td>$E_4$</td>
<td>-22,962</td>
<td>11,799</td>
<td>-23,088</td>
</tr>
<tr>
<td>$E_5$</td>
<td>-1,736</td>
<td>2,195</td>
<td>-1,758</td>
</tr>
</tbody>
</table>

It is important to highlight that negative values for expected profit in this exercise, result because during on-peak hours the expected spot price is always higher than the regulated tariff $r$. This means that nowadays, retailers are subject to financial losses during on-peak period which balance with prices during off-peak hours.

Table 5 lists the optimal portfolio for each LSE, identifying the number of instruments: bonds, forwards/futures, puts and call options. This means, following the analysis for $E_1$, that it must loan money to hold long positions on 1,277 bonds, 1,132 forward/futures contracts, 296 put options with strike 33.49, 148 put options with strike 56.87, 160 call options with strike 65.94, 310 call options with strike 92.66 and 382 call options with strike 125.41. By holding this positions, $E_1$ can replicate with an acceptable error, its optimal hedging function showed in Figure 5-a.

Table 5: Optimal number of instruments for each LSE. January 2008.

<table>
<thead>
<tr>
<th>LSE</th>
<th>Bonds</th>
<th>Forwards</th>
<th>Put$_1$</th>
<th>Put$_2$</th>
<th>Call$_1$</th>
<th>Call$_2$</th>
<th>Call$_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>1,277</td>
<td>1,132</td>
<td>296</td>
<td>148</td>
<td>160</td>
<td>310</td>
<td>382</td>
</tr>
<tr>
<td>$E_2$</td>
<td>230</td>
<td>242</td>
<td>54</td>
<td>27</td>
<td>29</td>
<td>56</td>
<td>69</td>
</tr>
<tr>
<td>$E_3$</td>
<td>228</td>
<td>879</td>
<td>53</td>
<td>26</td>
<td>28</td>
<td>55</td>
<td>68</td>
</tr>
<tr>
<td>$E_4$</td>
<td>1,306</td>
<td>1,253</td>
<td>305</td>
<td>153</td>
<td>165</td>
<td>319</td>
<td>394</td>
</tr>
<tr>
<td>$E_5$</td>
<td>232</td>
<td>234</td>
<td>53</td>
<td>28</td>
<td>29</td>
<td>56</td>
<td>69</td>
</tr>
</tbody>
</table>
8. Discussion

Although, results presented in the preceding section seem to be appropriate and the goodness of the model has been evidenced, there are certain issues to be considered if a serious, practical and applicable proposal for the Colombian power market is to be made.

Some readers could find it inconvenient to use as underlying asset the electricity spot price, since, as it is explained in [26], electricity is a non-storable and non-tradable asset, while forward contracts are; this implies difficulties at the time of pricing the instruments, because the methodology of Black-Scholes-Merton cannot be directly applied.

Moreover, Colombia’s power market does not have a well developed forward market today, and the existent forward market does not have either price signals strong enough to drive a derivative market. Actually, today the forward market is just under development, and for an initial proposal to be applied to the local market, the spot price seems to be convenient as underlying asset. References like [27], [28] among others, have dealt with derivatives’ pricing problem when spot prices are underlying. This issue is proposed in the further work section.

Some of plenty advantages included with the approach here developed, is that it could be used not only as hedging instruments for the regulated, but also to the non-regulated market, guaranteeing complementarity between markets. Not to mention, future advantages from a market perspective if Colombia effectively gets interconnected with Panama, Central America, and other Latin-American countries. Plus to the fact that derivatives suggested for the market correspond to plain vanilla options, which add value to the market by offering different hedging strategies with
simple and understandable instruments, adding no complexity to the current market.

One additional issue that should be considered at the time to implement this proposal, if this comes to happen, is that it will be necessary to clean up the signal sent by the underlying (spot price), considering that today, the electricity spot price includes taxes and some other charges oriented to non-interconnected zones, as fuzzy information about the international transactions.

9. Conclusions

Certainly, as it has been shown along the paper, the current major problems of the Colombian bilateral contract market, could be handled by the implementation of a financial derivatives market, either if it is designed to work independently to CREG’s proposal, or in the case it fits in as a complement to MOR. This can be done, taking advantage of the fact that nowadays there exists the legal and regulatory framework required to put in operation such market in Colombia, and of the international experience on electricity derivatives.

Moreover, the electricity risk management theory has developed sophisticated instruments and methods to handle the spot price risk, regarding the quantity risk as well, which is a non-traded risk and impacts directly the agents’ incomes. Particularly Oum and Oren in [4], perform a method that exploits the high correlation between spot price and energy demand, in order to mitigate price risk as well as volumetric risk, by maximizing the expected utility of the LSE’s hedged profit through the construction of an optimal portfolio with electricity standard derivatives such as forward contracts, call and put options with different strike prices. These plain vanilla instruments, make authors’ proposal worth, since add no complexity to the market, offering good quality hedging.

The present paper has applied the single period hedging strategy model of [3], considering the positive correlation of spot price and energy demand (≈ 0.6) in the Colombian power market, to develop a practical method by which a market maker or a clearing house could design financial options to be offered in the exchange market. Monthly and quarterly on-peak options are suggested. The analysis is done from an LSE’s perspective but it may be extended to power generator as well.

Results demonstrate that the options’ strike prices, calculated through a programmed algorithm which employs Newton-Raphson optimization method, are suitable for all retailers, since each of them can accomplish a better hedge during on-peak hours by holding a portfolio with bonds, futures/forwards and these financial options, than just having base-load forward/futures contracts. This obeys to the high and positive correlation between price and demand, and its implications on LSE’s net profits.

It can be said, that derivative instruments are suitable and viable to overcome the current Colombian power market problems, and also to improve and encourage competition among agents. Furthermore, it is proposed that, if CREG finally decides to implement its proposal, MOR should be a non-obligatory market for regulated customers, this on behalf of the retail market competition. Besides, if derivatives are planned to fit in MOR, it is suggested that MOR’s product should be a base-load forward contract, in order to make this market liquid and in addition, to allow each agent to handle price risk during on-peak hours on its own, either with other forward/futures contracts or with the financial options here presented. On the other hand, if MOR’s proposal is dismissed, the derivatives’ market will have to be supported on a base-load forward/futures contract, which could be launched by CCRC.
10. Further Work

Seeking to improve the purpose of this paper to implement a derivative market in Colombia under the methodology proposed in [3], it is necessary to further develop some ideas:

- More attention should be payed to the risk aversion coefficient estimation. This should be a detailed and deep research seeking to adequately represent agent’s risk profile.

- It would be also necessary, in order to obtain accurate conclusions, to extend the analysis developed in this paper for LSE, to the case of power generators within the trading activities. As a preliminary conclusion, after some analysis, a few formulas and variables should be changed to extend the analysis in this direction.

- Looking forward to continue developing the idea to offer financial hedging for the agents interacting in the Colombian power market, it will be necessary to rigorously develop fields like: (i) to design methodologies considering the dynamic feature of financial hedging, regarding that prices follow stochastic processes as well as quantity; (ii) to implement practical methodologies to price derivatives that underly on electricity spot price, overcoming this pricing issue; and finally, (iii) to create and design an appropriate institution that allows trading activities of the derivative products here proposed.

References

[7] P. Cranton, Colombia’s forward energy market.
Appendix

The optimal payoff function under a bivariate lognormal-normal distribution for price and quantity, assuming $\log(p) \sim N(m_1, s^2)$ under $P \equiv Q$, $q \sim N(m, u^2)$ and $\text{Corr}(\log(p), q) = \rho$:

$$x^*(p) = E^Q[E[y(p, q)|p]] - E[y(p, q)|p]$$

where

$$E[y(p, q)|p] = (r - p)(m + \rho \frac{u}{s}(\log(p) - m_1))$$

$$E^Q[E[y(p, q)|p]] = (r - e^{m_1+\frac{1}{2}s^2})(m - \rho \frac{u}{s}m_1) + \rho \frac{u}{s}(rm_1 - (m_1 + s^2)e^{m_1+\frac{1}{2}s^2})$$