Oil refining planning under price and demand uncertainties: case of Algeria

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Abstract
This paper aims to analyze the Algerian refining industry development in the presence of uncertainties, both on the domestic products demand and the international markets, with a linear dynamic model. The Algerian refining industry is a simple type, or hydroskimming, which is designed to treat a sweet light crude oil. Currently, this industry has to be adapted to meet demand progress both in terms of volume and also in terms of specifications, in a general context marked by a strong volatility of the oil markets.

Generally, refining operations planning models are based on a deterministic linear programming. However, because of the volatility of the raw materials prices, demand fluctuation, and other conditions for the market, many parameters should be considered as uncertain. In our study, we propose a dynamic long-term linear model to analyze the development of the Algerian refining sector by 2030. We treat particular uncertainties on demand and prices of oil and petroleum products. Considering multiple uncertainties on demand and oil price, the model provides the production levels, the rate units running and the foreign trade of products by the year 2030. This model assesses the impact of the prices and the demand volatilities on the development of this industry.

The paper is organized as follows: After the introduction and the overview about the methodology, the second section gives a brief overview about the refining modeling with linear dynamic programming. The third section deals with the model with uncertainties in which we focus on the particularities of an oil producer and exporter country. In section four, we present the main data used in the model. In the final section, before the conclusion, the main results will be presented and discussed.

1. Introduction
The demand for petroleum products from refining crude oil has changed drastically, in both quantitative and qualitative terms. Energy concerns in Algeria are those of a developing country that must satisfy not only the growing energy needs, but also intend to finance its economy from oil export revenues, this requires the development of production infrastructure, processing and trade. At the time when Algeria is responsive to the economy of the market, rational management of this industry has become an obvious necessity.

The linear programming is the most widespread mathematical tool in the field of optimization. It is to optimize plant operations, taking into account various constraints of products qualities and their applications and the operating constraints of the refinery. The constraints also affect the quality of the feedstock and its availability (Refining Model of IFP).

An approach to optimize the refinery operations in the medium and long term would be possible by adopting the techniques of dynamic linear programming: it consists in taking into account...
developments in the technology matrix, the second members of the constraints and the coefficients of the objective function, from a period to another. This allows the planning of investment in each period until the end of the planning horizon. In addition, it offers flexibility to determine the limits of variation of the optimal solution by applying the techniques of sensitivity analysis on either the coefficients of the objective function or the right hand sides of the constraints or by simulating the model on several scenarios. By adopting these techniques, the model generates very different results from one scenario to another, this puts the decision maker faced to difficulties in choosing the best scenario for the planning of the investments.

This leads us to reflect for other optimization methods that take into account uncertainties on both the coefficients of the objective function and the second members of the constraints. The aim of this paper is to propose a stochastic programming approach to optimize the Algerian refinery and propose the most likely scheme of the industry development by 2030.

2. Problem description

The Algerian refining industry, hydroskimming with a capacity of 22 million tons per year, was designed to process light sweet crude. Given its simplicity, the latter has to be adapted as a result of two major concerns:

- The first, together with the growing of the demand in the domestic market, is the increasing evolution of the required quality at the international markets, leading to a tightness of fuel specifications, particularly the content of lead, sulfur, aromatics, benzene and oxygen ... etc...
- The second is the valorization of the Algerian crude oil on the international market (maximizing the revenues of oil and petroleum products exports).

There are numerous optimization techniques and investment planning, linear and linear dynamics programming are the most common techniques in the refining industry. A drawback of these approaches, called deterministic, is associated to the data used (demand of petroleum products, crude and products prices, production costs etc ....), which are considered to be known with certainty. However, in reality, this is not the case: refining environment is exposed to multitude of internal and external uncertainties.

The uncertainty in the refining begins first form the fluctuating of the oil price, which is the capital charge of the refiner. Furthermore, the cost of the different operating conditions and the uncertainty about the sale prices of finished products, which represent the turnover of the refiner, either on domestic or international markets. Prices of different products from refining multiplied by their yields are the turnover of the refiner, a small fluctuation in the selling prices leads straightforwardly to a negative margin.

In our case, which concerns the long-term development of the refining industry in Algeria, the problem might be somewhat different, given the particularity of this industry. The first feature concerns the organizational aspect: all refineries in Algeria are the property of the national company (Sonatrach), which is also the only national operator in the oilfields with a current production capacity of approximately 70 million tons per year; this will ensure a continuous supply of oil to refineries throughout the country. Thus, the first uncertainty about the feedstock supplied to the refineries will be eliminated at least in the medium term. Nonetheless, this creates a dilemma of the crude oil valorization that begs the question: is it going to Algeria to export oil in its raw state or to export refined products that require the investment in the development of a more sophisticated tool for refining?
On the other hand, the uncertainty on the domestic market persists for different petroleum products, particularly fuels. Thus, the second uncertainty concerns the Algerian refining industry is the level of the national future demand for petroleum products, a question arises: what is the trend of development of the national demand for petroleum products and what are the necessary investments in refining to meet them?

These two issues (prices and demand uncertainties) that we will study in the following sections

3. Methodology

The refining models are considered among the first models developed in linear programming in the fifties that seek to optimize the blending obtained by processing crude oil into finished products. The reader can find at Favennec and Babusiaux (1998, p.167) a presentation of this type of modeling.

Thus, the constraints of the problem come from the refinery process (balance of intermediate and finished products, balance of the refinery gas), the quality control of products, the demand for finished products, the availability of resources (capacity of processing units and supply of crude oil) and the pollutant emissions. The objective function consists in minimizing the discounted annual cost of refining, including the economic depreciation of investments.

The model that we develop is based on research works done in the modeling area of refining, in particular their latest work C.khor et al., 2008, it differs from a standard model because it is a long term model and we consider various levels of demand and prices for petroleum products, which are associated with a probability distribution. We introduce in the linear model as much equations of the demand as the uncertainties. The objective function could be formulated as the minimization of the certain refining costs increased with the expectation of the recourse variables costs. Moreover, we use the Markowitz model to analyze the impact of price fluctuations on the investment decisions that we detail in the following sections.

We present first the dynamic linear model and then we explain how to introduce uncertainties.

4. Deterministic model: refining modeling by a linear dynamic model

The overall model contains all variables and dynamic constraints for the periods of the investment planning, from 2005 to 2030. We have taken 2005 as reference for planning because we built our first model and calibrated based on the data of the refining and the market of that year.

For our aggregate model, we consider in the first and the second period, until 2015, the new investments already planned by Sonatrach and the extensions and the rehabilitations of existing refineries. Subsequently, based on the growing of the domestic demand and the specifications tightness and new environmental standards required for petroleum products, the following extensions and the new production units will be determined by our optimization model for each period. A detailed description of linear programming models is beyond the scope of this section. We can see in Saint-Antonin (1998) for more details on the short term models and in McDonald and Karimi (1997) and Ierapetritou Pistikopoulos (1994) for medium-term models.

4.1 Variables

This model contains the flow variables of intermediate and finished products, production, supply, exports, imports and capacity expansion. These variables will be indexed by an index $t$ expressing
their evolution from one period to another. Obviously, these variables take different values at the optimum from a period to another.

4.2 Constraints

Like in the case of the variables, the constraints are duplicated for each period: there are as many equations for demand of gasoline or diesel, for instance, as the planning periods. These constraints vary from one period to another by changing market conditions (changes in demand, specifications, technology ... etc). We explain briefly in the following paragraphs the different constraints and the objective function of the model and we point out the particularities of Algeria.

4.2.1 Balance of intermediate and finished products

The balance equations of intermediate and finished products equilibrate the quantities of each product with their different affectations. To obtain linear constraints and as products yields depend on the operating conditions of each refinery unit, some units have been modeled by considering different severities representing different steps of processing.

The material balance of an intermediate product expresses that its output is equal to its internal use. The production is represented by the product of yields and the quantity of the charge. The units processing yields are different for each severity and also depend on the type of feedstock and the crude processed. Total internal uses are the sum of all transfers to the finished products, as a feedstock of another unit or as a refinery fuel. For the balance of a finished product, the sum of the components of a pool is equal to the quantity of finished products. In both types of the balances, the variables are defined by weight. In some cases, they are defined in volume and we have to modify the equations by introducing their gravities.

The refinery fuel balance is a special case. In this equation, the demand for the refinery fuel may be satisfied by intermediate or finished products. Each product has a distinct calorific value. Thus, from the supply side, refinery fuels are weighted according to their calorific values. From the demand side, we consider that the need for refinery fuels is proportional to the inputs of the processing units.

4.2.2 Demand equations and exportation variables

Supply (production and possible imports) must satisfy domestic demand plus exports. These are the variables of the problem. Several export variables which are associated with different prices are set. These are deducted from the CIF (Cost Insurance Freight) in importing countries.

We will then be tasked to build econometric models of some petroleum products consumption in Algeria. For the rest, we take the demand scenarios developed by specialized agencies in Algeria or from international agencies (IEA and OPEC). These models will be based on a classical approach with an income effect (GDP or other), a price effect and possibly effect of park equipment or lagged endogenous variables.

Based on the main level of consumption of various petroleum products, we can estimate the potential development of the Algerian exports.

4.2.3 Equations of product’s specifications

The finished products must comply with certain properties (specifications) legal and technical. These include density, vapor pressure, aromatics content, olefins content, octane etc.... for gasoline, the sulfur content, cetane number ... etc.. for diesel. Thus, a linear constraint is obtained
by multiplying the quantity of intermediate products (in volume for octane and in weight for sulfur for example) by their qualities and assigning a minimum or a maximum specification to the finished product. When the relationship is not linear, this feature is replaced by an index that can be used in a linear constraint.

4.2.4 Capacity constraints

Capacity constraints of the processing units are designed to limit the amount of the charge that can be treated. The capacity expansions are represented through investment. In the case of a refinery, the investment cost is often a nonlinear function of the capacity, which leads to model capacities expansion through several variables associated with distinct costs.

In the case of an aggregate model, we assume that investments are made for standard size units which can overcome this problem. However in the case of Algeria, where the number of refineries is limited, we may need to take into account these phenomena of non-linearity in the cost. We detail in the following section how to plan capacity expansion through the introduction of integer variables and standard sizes of units, while taking into account the effect of economies of scale.

As the demand for petroleum products will have a significant change in terms of the quantity and a structural change of the quality required by the motorists and the conditions imposed by the standards of environmental protection, capacity constraints include both processing units (atmospheric and vacuum distillations), the quality improvement units (catalytic reforming, isomerization, alkylation, HDS ... etc.), the conversion units (FCC, hydrocracking etc. ...) and finishing units and environmental protection (hydroprocessing, softening ... etc.). These constraints are to limit the inputs of each unit by its current installed capacity increased with a variable representing its possible expansion. Thus, for the variables that represent the expansion of production capacity, Ierapetritou and Pistikoupolos (1994) suggested in their paper a term of the capacity expansion, which is expressed as follows.

\[
CAP_{j,t} = CAP_{j,t-1} + EC_{j,t} \quad (1)
\]

\[
EC_{j,t}^L \leq EC_{j,t} \leq EC_{j,t}^U \quad (2)
\]

The capacity of the production \( CAP_{j,t} \) of the processing unit \( j \) at the period \( t \) is equal to its precedent capacity \( CAP_{j,t-1} \) augmented with its possible expansion \( EC_{j,t} \). The limits \( EC_{j,t}^L \) and \( EC_{j,t}^U \) are respectively the minimum and the maximum capacities of the variable \( EC_{j,t} \).

4.2.4.1 Planning of the refining capacity using integer variables

One of a linear programming problem with mixed variables is the fact that this latter shows continuous variables (such as the flow of petroleum products) and variable constrained to take only integer values (number of processing units).

In general, for a linear program, capacity expansions for different units are continuous variables that take values from zero to the maximum capacity, the optimal solution for the column could vary from zero and go up to the maximum possible, for example 20 million tons per year. This is not so evident in the fact that there is no column of atmospheric distillation or catalytic cracking of low tonnage per year, and the same for 20 million tons per year (usually for this latter capacity, two or three columns will be set). To remedy this, we have proposed to introduce in our model various standard production units. We have proposed three levels for each processing unit, while taking into account the effect of the scale economy. In fact, we chose a large capacity, a medium
capacity and a small capacity. Each size is associated with an investment cost as we show in the following table. (See table 1).

<table>
<thead>
<tr>
<th>Processing unit j</th>
<th>Size 1</th>
<th>Cost 1</th>
<th>Size 1</th>
<th>Cost 1</th>
<th>Size 1</th>
<th>Cost 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric distillation</td>
<td>SS11</td>
<td>SC11</td>
<td>SS12</td>
<td>SC12</td>
<td>SS13</td>
<td>SC13</td>
</tr>
<tr>
<td>Vacuum distillation</td>
<td>SS21</td>
<td>SC21</td>
<td>SS22</td>
<td>SC22</td>
<td>SS23</td>
<td>SC23</td>
</tr>
<tr>
<td>Catalytic reforming</td>
<td>SS31</td>
<td>SC31</td>
<td>SS32</td>
<td>SC32</td>
<td>SS33</td>
<td>SC33</td>
</tr>
</tbody>
</table>

The investment cost increases with the size of the column to install. Therewith, if the capacity of the column size 2 is twice the size 1, for example, the investment cost will be less than double, hence the effect of economies of scale.

A comprehensive way to adapt the information available on processing unit’s capacities is to apply the method of ”extrapolation factor”. This procedure is based on the statistical processing and smoothing of historical data, different quantities are linked by an empirical expression takes the following form:

$$\frac{I'_1}{I_1} = \left( \frac{C'_1}{C_1} \right)^\delta \quad (3)$$

$I_1$ and $I'_1$ represent the investment costs. $C_1$ and $C'_1$ are the capacities of the processing units related to these assets. $\delta$ is called "extrapolation factor" (Chauvel et al. 2001, page 200). This factor varies, for the refining industry between 0.5 and 0.8, the most often used factor for the production units is 0.65 with the exception of the hydrogen plant is 0.5 (source ifp).

Thus, constraints on capacity expansion will be presented differently to take into account the effect of standard capacities:

$$EC_{j,t} = \sum_e \sum_j \sum_t f_{e,j,t} \cdot SS_{e,j,t} \quad (4)$$

The integer variables $E_{e,j,t}$ ($e = 1,2,3$ et $j=1,2,...,nj$) could take only positive integer values. The indexes $e$ and $j$ are respectively incremented index of the numbers of production units to be installed; it is simply the number of standard distillation or processing columns, $j$ denotes the index of the processing units (atmospheric distillation, catalytic reforming...etc) and $t$ is the index of the time period.

4.2.5 Supply constraints

These constraints are limiting the supply of various grades of crude oil. The supplies may be subject to a number of constraints limiting the availability of crude for refineries. These constraints differ from one country to another and depend on the availability of this or that quality of oil that can be bounded by a constraint (supply constraint). In general case, the limitation of the total supply of crude oil is equal to the sum of the quantities of each crude feeding distillation units.
Algeria, like other OPEC countries, exports around 70% of the oil production in its crude state. The FOB prices are indexed to the international markets on Brent for Europe or on WTI for America or other benchmark crudes for other regions of the world. This price is fluctuating from day to day, see extremely variable from one year to another. This variation may be justified the enhancement of the Algerian crude for processing into petroleum products at the refinery. The Algerian oil produced in southern countries is routed through a transport network to the loading ports or refining.

Thus, all the refineries are fed by the same type of crude which is now widely abundant to meet the installed refining capacity. Refineries in Algeria are located at the same ports of oil exports (Arzew and Skikda). A question arises: is it worthwhile for Algeria to continue to export oil in its raw state or to invest in refining and exports oil products? Figure 1 below illustrates this problem.

**Figure 1: Integrated system: oil production-refining-exports**

In the case of the Algerian refining, since the refineries treat only the Algerian oil "Sahara Blend" with the exception of some quantities of imported residues (imported reduced crude) for making bitumen, we proposed to restrict the supply, for a producer and exporter country, the following constraint:

\[ \text{APPRO}_{b,t} + \text{CRUDEXP}_{b,t} \leq \text{CAPCRUDE}_{b,t} \]  \hspace{1cm} (5)

\(\text{APPRO}_{b,t}\): Quantities of crude oil \(b\) treated by the refineries in Algeria during the period \(t\),

\(\text{CRUDEXP}_{b,t}\): Quantities of crude oil \(b\) exported during the period \(t\),

\(\text{CAPCRUDE}_{b,t}\): Production capacity of the crude oil \(b\) from oilfields during the period \(t\).
This constraint is to limit the supply of crude oil to the refineries with the capacity of the oil fields of Sonatrach; the excess (the crude not refined) is to be exported. The aim is twofold for adding a such equation, first to limit the expansion of refining capacity to the capacity of the oilfields (which currently is three times the capacity of the refining) and at what level it is profitable for Algeria to export oil in its raw state or invest in refining to export petroleum products, on the other side.

### 4.2.6 Pollutant emissions

The constraints on emissions of pollutants that are introduced in the modeling related to air pollution. They result in the definition of specifications of maximum pollutant concentrations in the fumes emitted by the refinery. Emissions of sulfur dioxide and carbon dioxide can be so limited. Moreover, the rights to pollute can be introduced into the objective function (for carbon dioxide).

Till present, there is no limitation on emissions of pollutants in Algeria, but to meet environmental concerns in the medium term; this can become a major constraint. For the short-term model, we do not consider this constraint; contrawise for the planning model in the medium and long term, this constraint will be taken into account.

### 4.2.7 Budget constraint

Generally, each company looks forward to optimize its investment portfolio, taking into account the availability of budget. Sonatrach, as several national companies, carries on business from oil exploration to the trading of hydrocarbons through the transformation process. Its investment budget is allocated among different activities. Certainly, the budget allocated to the refining industry will be limited. In this context and in one of the scenarios that we will discuss, we will limit future investments by different budget levels. Budget constraint is expressed as well as the total investment costs of all processing units lower or equal to the budget allocated to refining industry.

### 4.3 Objective function

The objective function incorporates the discounted costs of the refining industry throughout the period 2005-2030 i.e. treatment costs and pollution, procurement and investment, import of petroleum products and export revenues. We have opted to the objective function of minimizing the operating cost of refining activity in the long term than maximization the profit, as sales prices of petroleum products on the Algerian domestic market are given (fixed and subsidized by the state) and do not follow neither evolution nor trend of the fluctuations of the international spot prices, this makes it very difficult to determine the actual profit of the refining business to maximize profit. The objective function is the minimization of the discounted sum of:

- Cost of supply
- Operating cost
- Cost of imports
- Cost associated with pollutant emissions
- Investment cost

Decreased of:
- The values of exports.
If the units are standard sizes, the total investment cost per processing unit is as follows:

\[ INV_{j,t} = \sum_e \sum_j \sum_t f_{e,j,t} \cdot SS_{e,j,t} \cdot SC_{e,j,t} \quad (6) \]

Where \( e \) is, as we mentioned earlier, the ranges of costs for each standard size for the production unit \( j \).

This approach, called deterministic, based on assumptions of future development of different parameters, namely, the demand for petroleum products, the prices of oil and petroleum products... etc. For this purpose, we can use techniques of sensitivity analysis which is to vary a parameter and know its impact on the optimal solution. However, decisions will differ from one scenario to another, this leads us to another problem of decision making in presence of uncertainty, which is the subject of the following sections.

5. General formulation of a stochastic model of refinery operations planning

The classical two-stage stochastic linear program (SLP) with recourse variables is originally proposed in the work of Dantzig (1955) and Beale (1955) has the general form:

**first-stage**

\[ \text{Min } C^T x + E[Q(x, \xi(\omega))] \]

With constraints

\[ Ax = b \]
\[ x \in \mathbb{X} \geq 0 \quad (7) \]

**Second-stage**

\[ Q(x, \xi(\omega)) = \text{Min } q^T(\omega)y(\omega) \]

With constraints

\[ W(\omega)y(\omega) + T(\omega)x = h(\omega) \]
\[ y(\omega) \geq 0 \quad (8) \]

\( x \): vector of decision variables of the first period of size \((n \times 1)\)

C, A and b are respectively matrices of data in the first period of sizes \((nx1)\), \((mxn)\) and \((mx1)\), \( \omega \) : represents the random event, and \( T(\omega), h(\omega) \) and \( q(\omega) \): matrices of data for the second period with the sizes \((k \times 1)\), \((1 \times k)\) and \((k \times 1)\) respectively and \( y \) is the vector of decision variables of the second period. \( W(\omega) \): The coefficients of the recourse random variables matrix.

This model could be simplified and treated as an equivalent deterministic. The readers can see Kall and Wallace (1994) for more detail on the treatment of equivalent models and the constraints of nonanticipativity.
5.1 Stochastic parameters in our refining model

The uncertainties persist throughout the oil chain, uncertainty about the form of the oil geological reservoir to the fuel prices at the pump stations. The stochastic programming models can be classified into three categories according to Ponnambalam (2005): (a) Uncertainties on the coefficients of the objective function, (2) Uncertainties about the second members of constraints (RHS), and (3) uncertainties of the technological coefficients of the constraints (LHS). In our case, we look to the first and to the second categories. For the third, one we consider many severities levels. The first one includes the uncertainties about oil and products prices. The second category includes the uncertainties about petroleum products demand.

5.2 Methodology of scenarios generation in the presence of uncertainties on the demand

The uncertainty about market demand introduces randomness in the constraints of the production demand, which is actually the sum of the quantities of intermediate products necessary for their manufacture, as we have described in equations of the balance of intermediate products and finished products. The methodology used to generate scenarios for the recourse model with uncertainty based on the dispersion (standard deviation) compared to the mean. We can apply demand $d_{i,s}$, where $i$ represents the type of product demanded and $s$ indicates the scenario considered, as a random variable by the following equation:

$$d_{i,s} = z\sigma_d + \mu_d \quad (9)$$

Where $z$ represents the variable of the standard normal distribution with a mean of 0 and a standard deviation of 1. $\mu$ and $\sigma$ are, respectively, the mean and the standard deviation of the probability distribution of the demand.

5.3 Modeling the uncertainty on the demand by the penalty functions and the slack variables

We develop in this section the methodology of taking into account the uncertainty about demand constraints with recourse variables and how to introduce them into the objective function.

5.3.1 Penalty functions

As has been mentioned in previous sections, one of the main consequences of the uncertainty in the context of decision making is the possibility of infeasibility in the future. The stochastic two-stage or more than two stages provide more choices to address this issue by delaying some decisions in the second stage, but this comes at the expense of using the corresponding penalties in the objective function.

To design appropriate penalties functions, we must resort to the introduction of some slack variables in the probabilistic constraints to eliminate the possibility of infeasibility of the second stage. Moreover, the philosophy of models that based on the recourse variables requires to decision makers to attribute a price (or cost) in the objective function as penalty to adjust the activities considered as random. For some applications, such as in models of production planning and inventory, these costs are standard. However, in some other situations, it seems more appropriate to accept the possibility of infeasibility in some circumstances, if the probability of this event is restricted below a given threshold, as was treated by Sen and Higle (1999).
The slack variables, explain the deficit or the surplus in the production as were stated by Clay and Grossmann (1997), are presented in the stochastic constraints as follows:

- The inequality constraints are replaced by equality constraints;
- The feasibility of stochastic constraints can be ensured for all events;
- The penalty costs can be added to the objective function.

In the case of production planning, an example would be to add a slack variable in the case where production is less than the demand for a product and then penalize this variable, based on the cost of buying this product in an outdoor market, as well, in the case of the production surplus comparing to the market demand, the slack variable is entrenched from the constraint and penalized by a cost-based storage that allows to take into the stock the surplus of the production.

The use of penalty functions in the objective function for the stochastic models has been initiated by Evers (1967) as a technique to explain the losses due to infeasibility. In our context of production planning of petroleum refineries in the presence of exogenous uncertainty of product demand, a penalty term is used in the objective function to measure the effect of lost profits and loss of customer confidence. It typically takes the following form:

\[ -\gamma \sum_{s=1}^{S} \beta_s \max[0, (\beta_s - Q_s)] \]  

(10)

\( \beta_s \): Vector of nominal values of uncertain parameters, \( Q_s \): vector of real values of uncertain parameters and \( P_s \): probability of scenario \( s \).

Where \( \gamma \) is the penalty coefficient value which determines the relative weight given to production shortfalls as a fraction of profit margins (Ierapetritou and Pistikopoulos, 1996).

### 5.3.2 Slack variables

C. S Khor, A. Elkamel and PL Douglas, 2008, have treated the problem of making short-term decision in presence of uncertainties on the demand for petroleum products. We base on the approach used in this work of short-term to develop a model for long-term planning of the refining sector in Algeria.

Based on the concepts presented above, penalty coefficients in the stochastic model are expected to be proportional to the deficit or surplus of products. These penalties are interpreted and valued accordingly, per unit of product unmet or overproduced, as follows:

\( c_{s,L}^+ \): The fixed penalty costs paid per unit of \( d_{is} \) that cannot be satisfied or provided by the refinery, and thus considered as a cost for lost demand, or if it could be obtained from another sources, then it represents the cost of the purchase or the importation into the open market to meet the shortfall in production.

\( c_{s,L}^- \): The fixed penalty costs paid per unit of product over the demand \( d_{is} \). It is typically, in the short term, the cost of storage of the production surplus exceeding the demand. It should be noted that the cost of storage should always be less than the cost of product purchase on the open market, otherwise it would be comparatively cheaper for refineries to outsource its production, and this avoids the installation of stockyard. In the long term, this represents the export price.
The penalty costs incurred, as determined by Kall and Wallace (1994), are actually determined after observation of the random data, so they represent the recourse costs that are imposed to the variables of the second stage. Based on statistical concepts, it is appropriate to apply a criterion of mathematical expectation. Specifically, the objective of the model is to minimize the present value of the costs resulting from the first stage while minimizing the expectation of the cost of the variables used in the second stage. Thus, the nonnegative recourse variables used in the second stage are presented and defined as follows:

\( y_{i,s}^+ \): The amount of the unmet demand for the finished product \( i \), which is due to a deficiency in production of the refinery (or lack in production) for the scenario \( s \). The satisfaction of this deficit is through either the import or the investment in processing units.

\( y_{i,s}^- \): The amount of extra product \( i \), which is due to the excess in production by completion of the scenario \( s \). In our case of Algeria, the excess will be exported.

Where \( y_{i,s}^+ \equiv \max(0, y) \) is the positive part of \( y \) and \( y_{i,s}^- \equiv \max(0, -y) \) is the negative part of \( y \).

Thus, the penalty cost of the recourse for the second stage, which is due to uncertainties on the demand for product \( i \) for all scenarios generated, is represented by the following formula:

\[
E_{s,\text{demand}} = \sum_{i \in I} \sum_{s \in S} P_s (c_i^+ y_{i,s}^+ - c_i^- y_{i,s}^-) \quad (11)
\]

Where \( P_s \) is the probability of scenario \( s \). To ensure that the original structure of the model, related to the order of decision-making process is followed for each product whose demand is uncertain, if new constraints are added to the stochastic model instead of the original deterministic constraint to model \( s \) scenarios generated for each product. In general, the new constraints are expressed as follows:

\[
x_i + y_{i,s}^+ - y_{i,s}^- = d_{i,s} \quad i \in I \text{ and } s \in S \quad (12)
\]

\( x_i \) is the amount of the product \( i \).

We can present graphically these scenarios as follows:

\[
x_i \leq d_i
\]

\[x_i + y_{i,1}^+ - y_{i,1}^- = d_{i,1} \text{ , Scenario 1} \]

\[x_i + y_{i,2}^+ - y_{i,2}^- = d_{i,2} \text{ , Scenario 2} \]

\[\cdots\]

\[x_i + y_{i,s}^+ - y_{i,s}^- = d_{i,s} \text{ , Scenario S} \]

In the long term this demand will be formalized as a scenarios tree as shown in the figure2 below.

In this decision tree, there are two levels of demand after each node with a step of 10 years, from 2010 to 2030. In our model, we will take into account the intermediate periods, namely the periods 2010-2015 and 2020-2025.
The objective function could be formulated as the minimization of certain operating costs of the activities of the refining industry increased by the expectation cost of the recourse variables of the other stages.

5.4 Modeling the uncertainty on the objective function coefficients

The uncertainties are inherent to all objective function coefficients. One approach of the stochastic modeling, variance is used as a measure of variability or dispersion. The approach most widely adopted is that of Markowitz (1952, 1959) which refers to the model Mean -Variance portfolio optimization. The objective of the Markowitz model is to optimize two criteria namely, to maximize the expected profit or margin while minimizing the second criterion, which is the risk inherent to the price volatility, measured through its variance. In our case of refining model, the objective function et the minimization of both the refining cost and the variance of the price volatilities.

The refining optimization model taking into account the uncertainty on the coefficients of the objective function can be presented as follow:

\[
	ext{Min } (E[z] + \lambda \text{Var}[z]) \quad (13)
\]

Where \(z\) is the objective function of the deterministic model. \(E[z]\) presents the mathematical expectation of \(z\) and \(\text{Var}[z]\) is the variance of the oil and products prices. \(\lambda\) is a parameter of risk aversion.
To obtain a term that is dimensionally consistent with the terms of the expected value, standard deviation of $z$ can be considered, instead of variance, as a measure of risk to reflect the dispersion of the random objective function.

An approach is proposed by some authors Terwiesch et al., 1994, and Ponnambalam, 2005, is to minimize the variance or standard deviation adding a new constraint, in addition to the constraints of the model. This one consist on limiting the expectation $z$ with a target value.

6. Data analysis

We present in the following paragraphs the evolution of the key data used in the model. We begin with the program launched by the company for the development and the rehabilitation of the refineries in order to take them into account in the modeling.

6.1 Rehabilitation and development program of the existing refineries

Given the age of the refineries and the environmental constraints and specification of petroleum products, the company has launched and planned a program of rehabilitation and capacity expansion of existing refineries. Beginning with the largest refinery, Skikda or RA1K, the company plans to increase its capacity by about 10% for the two topping units, with the renewal and modernization of the obsolete equipment.

For the second refinery of Arzew, Sonatrach intends to increase the capacity of topping by 50% and the installation of new units: HDS and isomerization. Finally, for the refinery of Algiers, the rehabilitation program includes: Increase the capacity of topping by around 35%, replacement of existing reforming, installing a new unit of isomerization of light naphtha, installation of a new catalytic cracking unit (RFCC). This will increase the existing capacity of 22 million tons per year to approximately 27 million tons per year.

In parallel, the company is launching a major national project to build a new complex refinery of 15 million tons per year.

All these treatment units will be taken into account in our planning model. They are regarded as lower bounds of capacities expansion variables $E_{j,t}$.

6.2 Prospects for the economic aggregates

In this section we will fix these different assumptions in the horizon 2020-2030. Referring to the specialized agencies, the International Energy Agency, we present in the following table the different outlook of GDP, household consumption, equipments and population in Algeria.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>1.6%</td>
<td>1.3%</td>
<td>1.3%</td>
<td>0.9%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Equipments</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>GDP</td>
<td>4.4%</td>
<td>2.7%</td>
<td>2.7%</td>
<td>2.3%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Household consumption</td>
<td>2.7%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>1.4%</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

Source: AIE, World Energy Outlook

6.3 Future prices of crude oil

Oil prices are very volatile, rising from less than 10 dollars per barrel in December 1998 to over 140 dollars per barrel in July 2008 (Platts). In 2009, with the financial crisis, oil prices have been declining considerably. For the first semester of 2010, the prices stabilized between 70 and 80
dollars per barrel. Therefore, an accurate short-term forecasting of the oil price is very delicate and seems impossible in the long term. Thus, in the case of our model, we take the baseline scenario estimates of the specialized agencies, the International Energy Agency. Table 3 below shows the outlook for oil prices till 2030.

### Table 3: Future Oil Prices

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude price $/barrel</td>
<td>62</td>
<td>65</td>
<td>71</td>
<td>80</td>
<td>90</td>
<td>108</td>
</tr>
<tr>
<td>Crude price $/ton</td>
<td>452</td>
<td>476</td>
<td>518</td>
<td>586</td>
<td>660</td>
<td>791</td>
</tr>
</tbody>
</table>

Source: AIE, World Energy Outlook

The prices on the above table are expressed in current dollars. For the prices of petroleum products, instead of estimating their evolution, we estimate co-integration relations linking them to the crude price.

After a series of statistical tests on samples of price change between 1999 and 2008, the equivalence relations adopted for the prices of major petroleum products and crude oil prices on European and American markets are:

\[
P_{\text{premium gasoline}} = 1.33 \times P_{\text{crude oil}} \tag{14}
\]

\[
P_{\text{diesel 50ppm}} = 1.21 \times P_{\text{crude oil}} \tag{15}
\]

\[
P_{\text{fuel oil 1%}} = 0.52 \times P_{\text{petrole brut}} \tag{16}
\]

For the stochastic model we use probabilized scenarios for the oil prices, as shown in the following table.

### Table 4: Scenarios of Oil Prices Exports, Dollars/ton

<table>
<thead>
<tr>
<th>Percentage of variation</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>524</td>
<td>476</td>
<td>428</td>
</tr>
<tr>
<td>2015</td>
<td>570</td>
<td>518</td>
<td>466</td>
</tr>
<tr>
<td>2020</td>
<td>645</td>
<td>586</td>
<td>527</td>
</tr>
<tr>
<td>2025</td>
<td>726</td>
<td>660</td>
<td>594</td>
</tr>
<tr>
<td>2030</td>
<td>870</td>
<td>791</td>
<td>712</td>
</tr>
</tbody>
</table>

| Probability          | 0.35  | 0.45   | 0.20 |

The three columns of the table above show the evolution scenarios of oil prices. The medium scenario or the base, the high scenario represents a positive deviation of 10% compared to the medium. While the low scenario presents a negative deviation of -10% compared to medium.

Likewise the prices of exportation of petroleum products, we have considered three price scenarios, high, medium and low weighted with the same probabilities as those in oil prices.

### 6.4 Forecasting of the Petroleum Products Demand

Combining data on the prospects of several data sources, the Ministry of Energy and Mines, Sonatrach, International Energy Agency and OPEC, we summarize in the following table the future evolution of demand for various petroleum products on the Algerian domestic market. It
should be noted that we have developed our econometric model for forecasting demand for diesel and gasoline, taking into account the evolution of GDP, population, vehicle fleet and price. The details of these models are not presented in this section.

| Table 5: Prospects for domestic demand of various petroleum products |
|------------------|--------|--------|--------|--------|
|                  | 2005   | 2010   | 2020   | 2030   |
| LPG - residential| 1,602  | 1,684  | 1,860  | 2,054  |
| LPG-V             | 0,305  | 0,321  | 0,354  | 0,283  |
| Naphtha and other databases | 0,030  | 0,032  | 0,035  | 0,056  |
| Gasoline          | 1,954  | 2,169  | 1,843  | 1,816  |
| Diesel and heating oil | 5,617  | 7,596  | 8,323  | 10,949 |
| Jet A1 fuel       | 0,386  | 0,406  | 0,448  | 0,716  |
| Heavy fuel oil and bunker | 0,425  | 0,683  | 0,755  | 0,617  |
| Bitumen           | 0,521  | 0,575  | 0,701  | 0,600  |
| Total             | 10,840 | 13,465 | 14,319 | 17,091 |

Unit: million tons per year

7. Results and discussion

We will present and analyze in this section a sample of the main results of deterministic and stochastic models. Since the results are numerous, we focus our analysis on the capacity development aspect for different processing units, both for the extension of the processing units already installed and for the proposed new plants.

7.1 Deterministic model analysis

In the following paragraphs we will present the desirable processing capacities by 2030 for different levels of investment budget. For 2005, the capacities shown on the table represent the actual available capacities.

We note that a limited budget to $3 billion, the investments are directed in particular to the conversion processes of the heavy products and improving quality of light products such as gasoline, jet A1 diesel, rather than atmospheric distillation to increase the capacity. The processes that involve more gasoline are isomerization, MTBE unit and catalytic cracking. Hydrocracking and hydrodesulfurization units are for the jet A1 and diesel. The hydrogen production unit is designed to produce hydrogen to feed the hydrocracking and hydrodesulfurization.

We note that there is no investment in 2010 (see table 6) since, in reality, Sonatrach has not scheduled the investments by 2010, but they will all be after this year. In our model of planning, we introduced a binary parameter, 0 or 1, which allows the authorization of investment or not; for the year 2010 this parameter equals zero.

We have simulated the model for a larger budget than the first; the results are summarized in the table below.
Table 6: desirable Algerian refining capacities from 2005 to 2030 for a capital budget of seven billion dollars:

Results of the deterministic model

<table>
<thead>
<tr>
<th></th>
<th>2005 Installed capacities</th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topping</td>
<td>22 000</td>
<td>11 600</td>
<td>2693</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacuum distillation</td>
<td>550</td>
<td>191</td>
<td>164</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catalytic reforming</td>
<td>4400</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isomerization</td>
<td>0</td>
<td>3600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catalytic cracking</td>
<td>0</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrocracking</td>
<td>0</td>
<td>7600</td>
<td></td>
<td></td>
<td></td>
<td>703</td>
</tr>
<tr>
<td>Hydrodesulfurization</td>
<td>0</td>
<td>88</td>
<td>92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSA Hydrogen</td>
<td>0</td>
<td>10</td>
<td>51</td>
<td>61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

Unit: 1000 tons per year

In this case, for a larger budget, the model chooses, in addition to the conversion units and improving quality, expanding capacity for the atmospheric distillation of about 14 million tons per year. This is to satisfy domestic demand for light products and export the surplus of these products to international standards.

For the first two simulations, we have not taken into account the investment planned by Sonatrach. We will take into account with the assumption that the new refinery is commissioned in the period 2015-2020. In the model, we limit the lower bounds of the various processing units by the planned capacities. In the following table, we present the model results with consideration of these investments.

Table 7: desirable Algerian refining capacities from 2005 to 2030, taking into account the planned investments by the company.

<table>
<thead>
<tr>
<th></th>
<th>2005 Installed capacities</th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2025</th>
<th>2030</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topping</td>
<td>22 000</td>
<td>6976</td>
<td>15000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacuum distillation</td>
<td>550</td>
<td>1000</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catalytic Reforming</td>
<td>4400</td>
<td>700</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isomerization</td>
<td>0</td>
<td>1638</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Catalytic Cracking</td>
<td>0</td>
<td>500</td>
<td>500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrocracking</td>
<td>0</td>
<td>5138</td>
<td></td>
<td>4000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrodesulfurization</td>
<td>0</td>
<td>1000</td>
<td></td>
<td>2000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MTBE plant</td>
<td>0</td>
<td>7</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSA Hydrogen</td>
<td>0</td>
<td>37</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0</td>
<td>53</td>
<td>62</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unit: 1000 tons per year

We notice an increase in distillation capacity of 22 million tons per year; this allows doubling the capacity, compared to the existing, to 44 million tons per year. The budget needed to double that capacity is estimated by the model to about $12 billion.

7.2 Stochastic model analysis

For our stochastic model, we have established the scenarios tree for each product required for 2010-2030. We limit our analysis of the results, alike for the deterministic model, to the
development of the scheme of refining capacity through the expansion of production facilities or the proposition of the new units to satisfy demand and meet new requirements in terms of petroleum products qualities. The results of model simulation for a budget of seven billion dollars are summarized in Table 7 below.

Table 8: desirable Algerian refining capacities from 2005 to 2030 for a capital budget of seven billion dollars

<table>
<thead>
<tr>
<th>Results of the stochastic model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Topping</td>
</tr>
<tr>
<td>Isomerization</td>
</tr>
<tr>
<td>Hydrocracking</td>
</tr>
<tr>
<td>PSA Hydrogen</td>
</tr>
<tr>
<td>Hydrogen</td>
</tr>
</tbody>
</table>

Unit: 1000 tons per year

These results, for the same budget of seven billion dollars, show that the capacities expansion is bit low compared to those of the deterministic model. This is mainly due to the choice of penalty costs for the recourse variables. Nevertheless, both models give almost the same pattern of development of the refining industry in Algeria. In fact, the stochastic model shows the most likely outcome. Thus, we have performed several levels of simulations to determine the budget for different patterns of development according to budget allocated to refining. For the simulation model with consideration of investment planned by Sonatrach, the model is limited to the proposed lower bounds.

After presenting a sample of results for the impact of the variation of product demand, we present in the following paragraphs some results for the impact of price variation on the investment during all the planning period. Note that since this is a cost minimization problem, larger values of the risk factor $\lambda$ correspond to higher costs. Different ranges for $\lambda$, 0 to 1, 1 to 10 and 10 to100 have been set to plot the following graph.

Figure 3: The efficient frontier of the total processing units' investment versus risk imposed by variation in petroleum products prices

This graph shows the impact of the variation of the risk parameter, which is multiplied by the variance of the petroleum products exportation prices as we noticed in equation 13, on the investment amount for all processing units during the period 2010-2030. We notice that when the
risk is increasingly higher, that means the variance of the price increases, the investment will be curbed.

8. Conclusion

We have developed a linear dynamic programming model for the development of the refining industry in Algeria, taking into account the various constraints of qualities, demand for petroleum products and capacity. Subsequently, we have improved our modeling approach by introducing uncertainties in demand and prices of oil and petroleum products, we relied mainly on Kall and Wallace, 1994 and the work of Khor et al., 2008. Both models perform all the variables in the refining industry in Algeria from 2005 to 2030 (production, level of product specifications, exports, imports, product compositions etc ....). We have been limited in this paper to analyze the pattern of development through processing capacities. The pattern of results of both models is almost similar, but the results of stochastic models are more probable, since it takes into account the change in demand. We have also found that large fluctuations in export prices of oil products slow down investment in expanding and refining process units. These models are thus a tool for decision support for the company to develop the refining industry.

References