Nonlinear Structure in Time Series of the Energy Markets

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Abstract

This paper applies statistical techniques to assess the existence of nonlinear structures in the time series data generating mechanism of the energy sector of the aggregate economy. The statistical techniques incorporate the most well-known univariate tests for nonlinearity, with distinct power functions over alternatives, as well as different null hypotheses. This study utilizes monthly observations on the U.S. field production of crude oil for over 90 years, as well as daily spot prices on five major products in the energy market for over 16 years. Incorporating the production side of the energy sector, which is the variable that responds to the price, provides a more inclusive study of the energy market and distinguishes the approach of this paper from the existing methods of analyzing the energy markets’ structures. All the tests detect strong evidence of nonlinear structure in the time series data, indicating that the employed series in both quantity and prices are generated by a nonlinear mechanism. The results show that each individual series exhibits general nonlinear serial dependence, as well as nonlinearity in the mean, variance and skewness functions. The findings imply that nonlinear time series modeling that is more agreeable with the data generating process will provide more plausible empirical results.

Key words: Nonlinearity, Energy Market, Time Series Analysis

JEL classification: C22, Q43, C46

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1 Introduction

The energy sector, in particular, the petroleum market, has played a key role in the aggregate economy. Historically, this sector has been influenced by political disturbances. Over the last decades, the price of petroleum has dramatically increased in response to a series of major events. For instance, during the recent political unrest in the Middle East, the price of petroleum accelerated to nearly $120 per barrel per day, after being relatively stable at around $80 per barrel per day since the 2008 credit crisis. As a result of various shocks, a large number of studies have centered their attention on the correlation between the energy sector disruption and the aggregate economic activity, such as Hamilton (1983), Hamilton (2003) and Rotemberg & Woodford (1996) among many others.

However, to attain a more precise relation between the energy sector and the economy, it is crucial to employ appropriate specifications, which are reasonably close to the data generating mechanism, and to examine whether the time series observations in the market are generated by a linear process or a nonlinear dynamic mechanism. As illustrated by Brockett, Hinich & Patterson (1988), given the nature of confounding linear and quadratic coefficients in the estimation of time series models, it is important to test for significant nonlinearity in the observed time series and to determine which time series are not amenable to linear time series modeling. Moreover, usual linear model coefficients can be shown to be biased in the face of nonlinear time series structure – [See Brockett et al. (1988) for more details]. If the nonlinearity is present in the data, choosing a nonlinear time series can provide more plausible post sample forecasting ability (Ashley & Patterson (2006)). Furthermore, investigating the sector’s data generating process helps to resolve whether or not the market’s fluctuations are exogenous, as noted Kyrtsou, Malliaris & Serletis (2009).

The aim of this paper is to uncover the data generating mechanism of observed time series of the energy market and assess the existence of a nonlinear deterministic structure in the market’s fundamentals by employing statistical methods and econometrics techniques. This study incorporates the most well-known univariate tests for nonlinearity with distinct power functions over alternatives and tests different null hypotheses. It utilizes the monthly observations on the U.S. field production of crude oil between January 1920 to June 2011, and daily spot price observations between January 1995 to August 2011 on five major products in the energy market – crude oil (West Texas Intermediate (WTI) and Europe Brent), heating oil, gasoline and natural gas. All the time series data are obtained from the Energy Information Administration (EIA). The results suggest that both quantity and price in the energy market product are highly nonlinear in their nature. They demonstrate apparent evi-
dence of general nonlinear serial dependence in each individual series, as well as nonlinearity in the first, second and third moment of the series. The findings imply that it is important to check the existence of the nonlinearity in the time series data and employ a time series modeling that is consistent with the data generating mechanism to achieve more plausible empirical results.

This paper is organized as follows. The next section discusses the role of the energy market in the global economy. Section 3 reviews the related literature. Section 4 describes the data and related different unit root analyses. Section 5 discusses the inference methods as well as the results of performing the nonlinearity tests to examine the markets’ data generating mechanism. A brief summary and conclusion are offered in section 6.

2 The Role of the Energy Market

A large body of literature reveals that U.S. economy is negatively influenced by major disruptions in the supply of oil, as well as the escalation in the petroleum price. Oil price shocks are often seen as a trigger for the aggregate economic activities fluctuations through different channels. Among all the potential explanations for how aggregate economy can respond to changes in oil price, monetary policy has a key role in the literature (Bernanke, Gertler & Watson 1997). Classic supply shocks associated with real business cycles are another scope of study for examining the effect of oil price shocks on the aggregate economic activity. Moreover, oil price volatility plays a major role in the financial market dynamics as well as infrastructure costs and adjustment costs.

2.1 The Petroleum Consumption

Energy components’ demand, particularly petroleum, has increased over time. Figure 1 describes the changes in total petroleum consumption from 1990 to 2010 of the Organization for Economic Cooperation and Development (OECD) countries, non-OECD countries and also the WTI price levels. It is noticeable that rising oil prices held down the growth of oil consumption growth in OECD countries in 2008 and 2009, in contrast with non-OECD countries. This is partially because of a relatively slower economic growth rate and more efficient transportation sectors, so the impact of the rising prices has been more apparent in OECD countries. However, in 2010, the OECD organization consist of 34 countries, accounts for 53 percent of worldwide oil demand, and 41 percent of this number belongs to U.S. The United States stands as the first ranked consumer of the petroleum and almost all other
energy components in the world by reaching nearly 19180 thousands barrels per day for petroleum consumption in 2010.

Figure 1: OECD and Non OECD Petroleum Consumption, WTI Crude Cil Price

![Graph](image)

**Data Source:** Energy Information Administration (EIA)

Therefore, the industrialized countries consume oil more intensively than developing countries, and North America, dominated by the United States, is the second largest consuming area in the world, as it is demonstrated in figure 2. For instance, oil consumption in North America (the United States and Canada) is nearly 3 gallons per day per capita while in the rest of the OECD countries is equal to 1.4 gallons per day per capita, and outside the OECD the is almost 0.2 gallons per day per capita.

Although the United States economy may not be as energy dependent as in the previous decades, the market for petroleum has been characterized by strong demand growth over time, in particular in from 2003 to 2007 as a result of global recovery. As also demonstrated by figure 3, the United States has the first spot in consuming the petroleum product among the North American countries.

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1. The difference is the transportation system in these countries and the dependence of this sector of the vehicles to cover the long distances.
2. Oil Market Basics, Energy Information Administration (EIA).
The escalation of global oil consumption, particularly by China and India, and also the declining output from oil-producing countries (such as Libya) can potentially result in high prices for petroleum yet again and suppress the global economy. The petroleum price and its related issues are to be discussed in the following section.

2.2 The Petroleum Price

As the consumption of petroleum has grown in the last three decades, the price of this strategic commodity has also fluctuated dramatically over time. In the early 1970s to early 1980s, the price of oil increased considerably in response to the major conflicts in the Middle East, which reduced the world supply of oil. The first fall in supply in that decade was experienced in late 1973 as a result of tightening the oil embargo by the Organization of the Petroleum Exporting Countries (OPEC). Oil production was cut by 5 million barrels per day and the price of oil increased 400 percent in six months (Sill 2007). The crude oil prices reaction to a variety of global geopolitical events is shown in figure 4. The next dramatic increase in oil price occurred as a result of the Iranian Revolution, which begun in late 1978
and resulted in a drop of 3.9 million barrels per day of the Iran’s crude oil production until 1981. In 1980, the Iran-Iraq war began and by 1981 OPEC production declined by 7 million barrels per day from its level in 1978. The world oil price jumped from $14 per barrel in 1979 to more than $35 in 1981. The subsequent event was the Persian Gulf Crisis in 1990, when Iraq invaded Kuwait and resulted in another sudden increase in crude oil price. The price of crude oil, which was relatively stable, escalated from $16 per barrels per day in July to more than $36 per barrel per day in September 1990.

After 1990, world oil demand had a dramatic increase during the global recovery period of 2003-2007 until the global financial collapse in 2008, when the oil price escalated much further, reaching about $134 barrel per day in July 2008. Once again, the energy market encountered another dramatic increase in oil prices as a result of unrest in the Middle East in 2011. The WTI spot price accelerated to nearly $120 per barrel per day in April 2011. Those rises and falls in the energy market and the oil price shocks have influenced U.S. economy through different channels. As Hamilton (1983) has noted in his, paper seven out of eight postwar U.S. recessions were proceeded by a significant increase in price of petroleum. In another paper, Hamilton(2011) states that the count today stands at ten out of eleven. High oil prices and energy supply disruptions may lead to economic downturns due to the variations in the business cycle because of the supply shocks. Moreover, oil price shocks may also influence the aggregate economic activity through monetary policies. If a rise in oil price is related to general price inflation, monetary authorities may adopt restrictive
monetary policies, which could slow the economy’s growth. Bernanke et al. (1997) argue that the effect of oil price shocks on the economy results in changes in monetary policies, i.e., increase in the interest rates, which causes the downturn in economy.

Therefore, historically the energy market has always had a crucial role in the economy and has a substantial impact on different sectors. The aggregate economy has tended to perform poorly after major disruptions in supply, which corresponds to the increase in the price of oil. Hence, it is critical to understand the nature of the energy market and discover the structure of the market and its dynamic, which is essentially the aim of this study.

3 Literature Review

There is an extensive body of literature about the energy market and its impacts on economic activities. Also, there are studies that focus on the structure of the energy market, the interaction of the energy market with other markets, and the energy related policies.

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3Robert Pirog (2005), CRS report for congress
3.1 Energy Market and its Impact on the Aggregate Economy

Hamilton (1983) uses Sim’s (1980) six-variable quarterly vector autoregressive (VAR) model and shows that all but one of the U.S. recessions since World War II have been preceded by a dramatic increase in the price of crude petroleum. He discusses the evidence that is presented even over the period of 1948-72, which shows the significant and nonspurious correlation, supporting the fact that oil shocks were a contributing factor in at least some of the U.S. recessions prior to 1972. The same VAR model is employed by Mork (1989) to investigate whether Hamilton’s results hold when the sample is extended to include the oil market collapse. The asymmetric response to oil price increase and decrease is under particular investigation in Mork’s paper. The results confirm the negative correlation with the oil price increase and the behavior of GNP growth.

In an extensive review, Hamilton (2003) analyzes the existing literature that relates the oil price shocks to economic activity, and he states that oil price increases are much more important than oil price decreases. Also, increases have significantly less predictive content if they correct earlier decreases. According to Hamilton’s findings, the recent increase in the oil price is because of an increase in demand, which differs from past observations.

Oil price fluctuations have also affected the monetary policies. In a seminal study Bernanke et al. (1997) investigate the responses of monetary policy to economic disturbances by focusing on oil price shocks and using VAR approach. They argue that the effect of oil price shocks on the economy results from the monetary policies i.e. increase in the interest rates responding to oil price shocks, which causes the downturn in the economy. Their view has been challenged by Hamilton and Herrera (2004). They conclude that the monetary policies designed to offset the tightening consequences of oil price shocks are not as influential as stated by Bernanke et al. (1997). Since the oil shocks have more impact on the economy than Bernanke et. al (1997) argue, the feasibility of the monetary policy to offset even the small shock is unpersuaded. Hamilton (2011) reviews some of the literature on the macroeconomic effect of the oil shocks with a particular focus on possible nonlinearities in the relation. He includes both supply and demand shocks and states that the relation between GDP growth and oil prices is nonlinear.

3.2 Empirical Time Series Analysis of the Energy Market

Serletis (1992) examines the evidence for random walk behavior in energy future prices by employing the daily observations for crude oil, heating oil, and unleaded gasoline and also
performing the test for unit roots. The findings indicate that the unit root hypothesis can be rejected if the possibilities of a one-time break in the intercept and the slope of the trend function at an unknown point of time are allowed. An extension to Serletis (1992) is another study by Elder & Serletis (2007) that re-examines the empirical evidence for random walk behavior in energy future prices. The paper employs a newly developed semi-parametric estimator called the Wavelet OLS Estimator. Their finding with this new estimator suggests that each energy return series displays unambiguous evidence of long memory, with no evidence of infinite unconditional variance. As they state:

“The particular form of long memory is anti-persistence, characterized by the variance of each series being dominated by high frequency (low wavelet scale) components.” (Elder & Serletis (2007))

Serletis and Herbert (1999) explore the degree of shared trends across the North America energy market. They test for unit root in univariate time series representations of six natural gas prices as well as of power and fuel prices. Based on the augmented Dicky-Fuller (ADF) unit root testing procedure, one of the paper’s findings shows that the random-walk hypothesis cannot be rejected for the natural gas and fuel oil prices. The power price series, however, appears to be stationary. Also, Serletis and Rangel-Ruiz (2004) discuss the strength of shared trends and shared cycles between North American natural gas and crude oil markets. Their results show that there has been ’decoupling’ of the prices of these two sources of energy as a result of oil and gas deregulation in the United States. In other work for analyzing the energy price behavior, Serletis & Kemp (1998) investigate the basic stylized fact of energy price movements. The results are robust compare to alternative measures of the cycle and indicate that the crude oil and heating oil prices are synchronous and procyclical whereas unleaded gasoline and natural gas prices are lagging procyclically. Moreover, they find that energy prices are positively and contemporaneously correlated with consumer prices and their cycles lead the cycle of consumer prices, suggesting a possible role for energy prices in the conduct of monetary policy.

3.3 Nonlinearities and Chaos in Economic Data

Identifying nonlinearities and chaos in economic data has attracted considerable attention in the literature. Barnett, Gallant, Hinich, Jungeilges, Kaplan & Jensen (1995) apply nonlinear tests to detect nonlinear behavior or chaos in various monetary aggregate data series, and discuss the controversy that has arisen about the available results. They use five inference
methods to test for nonlinearity and chaos: the Hinich bispectrum test, the BDS test, the Lyapunov exponent estimator of Nychka, the White’s test, and the Kaplan’s test. The findings provide a possible explanation for the controversies that exist regarding empirical evidence of chaos in economic data. They also state that the source of controversies can be found in the lack of robustness of the inference. In another influential study, Barnett, Gallant, Hinich, Jungeilges, Kaplan & Jensen (1997) explore the reasons for empirical difficulties with the interpretations of nonlinear and chaos tests’ results that have increased over time. They design and run a single-blind controlled competition among the aforementioned five highly regarded tests for nonlinearity or chaos with 10 simulated data series. The results shows that although there are some clear differences among the power functions of the tests, there exists some consistency in their inferences across the method of inference. They also discuss different issues that need to be taken into consideration in interpreting the results. As they state

“One consideration is the difference in the power functions over alternative, for fixed null. The other consideration is the differences in null hypotheses of each test. The latter consideration produces a degree of noncomparability of the tests and the possibility that some of the tests could be used jointly”. (Barnett et al. (1997))

Barnett, Jones & Nesmith (2004) test the existence of nonlinearity in the cointegration relations of a system containing money demand variables, by applying the Hinich bispectrum test. The findings have some evidence of nonlinearity, and therefore they find that the issue is empirically relevant.

The detection of chaos in economic data is also examined by Barnett & Hinich (1993) using Divisia monetary aggregate and applying the Hinich bispectrum test. They produce a very strong rejection of linearity with the Divisia $M_1$ data and state that these data are deeply nonlinear.

Kyrtou & Serletis (2006) discuss univariate tests for independence and hidden nonlinear deterministic structure in economic and financial time series. They apply the tests to Canadian exchange rate, using daily data over a 30-year period and they identify an interesting relationship between high-dimensional nonlinearity and shocks.

Furthermore, interest in studying the behavior of the energy market and applying the existing tests to detect the nonlinearities and chaos in this market has been growing over time. Kyrtou et al. (2009) discuss number of widely used univariate test from dynamical system theory and apply them to the energy market. They apply these tests to daily observations
of the energy market for nearly 15 years. They find indications consistent with nonlinear dependencies in each of the markets. They also suggest that an effective nonlinear model of energy prices would produce a deeper perception of the energy market fluctuations than existing linear models. Sertletis and Gogas (1999) test for deterministic chaos in the North American Natural Gas Liquids Market. They use the Lyapunov exponent estimator and they find that there is evidence consistent with a chaotic nonlinear generation process in natural gas liquid markets. Serletis and Andreadis (2004) use daily observations on West Texas Intermediate crude oil prices, and Henry Hub natural gas prices and various tests from dynamical theory to support a random fractal structure for North American energy markets. The result is consistent with the reported result by Serletis and Gogas (1999) as they find evidence of nonlinear chaotic dynamics in North American natural gas liquids markets but not in the crude oil and natural gas markets.

As discussed above, some studies in the literature have investigated the energy market’s fundamentals by applying univariate nonlinearity tests to detect the nonlinear structure in the energy market. However, existing literature’s focus is mainly on the prices of the energy product, and there is little mention of the production of the petroleum, which is the variable that responds to the price. Therefore, in order to provide a more inclusive study of the energy market structure, this paper will include the quantity of the crude oil for over 90 years to address the gap, and will use different sample data series for the prices of the major products in the energy market. Moreover, it includes one main series for daily prices of an important crude oil product, which has not been considered in the past studies.

4 Data Description and Unit Root Analysis

The study uses the monthly observations data on the U.S. field production of crude oil and the daily prices data on five energy products obtained from Energy Information Administration (EIA). The descriptions of the employed data are as follows:

- Monthly observation on the U.S. field production of crude oil. The sample period is from January 1920 to June 2011, a total of 1098 observations.
- Daily spot price on crude oil, West Texas Intermediate (WTI-Cushing)\textsuperscript{4} and Europe

\textsuperscript{4}WTI-Cushing: A crude stream produced in Texas and southern Oklahoma, which serves as a reference or “marker” for pricing a number of other crude streams, is traded in the domestic spot market at Cushing, Oklahoma. (Energy Information Administration (EIA))
Table 1: Summary Statistics of Differenced Log Series

<table>
<thead>
<tr>
<th>Series</th>
<th>Sample Mean</th>
<th>Sample Median</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Field Crude Oil Production</td>
<td>0.0006</td>
<td>0.0004</td>
<td>0.0239</td>
<td>0.0500</td>
<td>5.9722</td>
<td>0.0000</td>
</tr>
<tr>
<td>WTI (Crude Oil)</td>
<td>0.0001</td>
<td>0.0004</td>
<td>0.0110</td>
<td>-0.1924</td>
<td>7.6120</td>
<td>0.0000</td>
</tr>
<tr>
<td>Europe Brent</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0103</td>
<td>-0.1111</td>
<td>7.8295</td>
<td>0.0000</td>
</tr>
<tr>
<td>Heating Oil</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0116</td>
<td>-1.4674</td>
<td>39.0782</td>
<td>0.0000</td>
</tr>
<tr>
<td>Gasoline</td>
<td>0.0001</td>
<td>0.0006</td>
<td>0.0122</td>
<td>0.0218</td>
<td>6.6622</td>
<td>0.0000</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0201</td>
<td>0.4861</td>
<td>22.6005</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Brent\(^5\). The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations for each series.

- Daily spot price on the New York Harbor heating oil\(^6\). The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations.
- Daily spot price on New York conventional gasoline regular\(^7\). The sample period of 01/03/1995 to 08/16/2011 consists of 4174 observations.
- Daily spot price on Henry Hub golf coast natural gas\(^8\). The sample period of 01/07/1997 to 08/16/2011 consists of 3654 observations for each series.

The descriptive statistics of the first difference of the log levels production and prices are reported in table 1. Moreover, figure 5 to 16 depict the individual series, the log levels, and the first differenced log levels for each series.

Before conducting nonlinear and chaos analysis, the first step is to test for stochastic trend (unit root) in each individual series. It is important to test for unit root to render the data stationary and avoid any possible spurious regression. In doing so, the study employs three alternative tests for unit root to discover whether or not the series’ behavior follow the random walk.

\(^5\)Brent: A blended crude stream produced in the North Sea region, which serves as a reference or “marker” for pricing a number of other crude streams. (Energy Information Administration (EIA))

\(^6\)The location specified in either spot or futures contracts for delivery of a product in New York Harbor. (Energy Information Administration (EIA))

\(^7\)Finished motor gasoline not included in the oxygenated or reformulated gasoline categories. Excludes reformulated gasoline blendstock for oxygenate blending (RBOB) as well as other blendstock. (Energy Information Administration (EIA))

\(^8\)A gaseous mixture of hydrocarbon compounds, the primary one being methane delivered at the Henry Hub in Louisiana. (Energy Information Administration (EIA))
Table 2: Augmented Dickey-Fuller Unit Root Tests
Null Hypothesis: The log levels and the differenced log of the series have unit root
Lag length: Automatic Selection Based on SIC.

<table>
<thead>
<tr>
<th>Log Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil</th>
<th>Brent</th>
<th>Heating Oil</th>
<th>Gasoline</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF Test Statistic ($t(\hat{\beta})$)</td>
<td>-1.833</td>
<td>-3.183</td>
<td>-2.875</td>
<td>-2.953</td>
<td>-3.502</td>
<td>-2.894</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.687</td>
<td>0.087</td>
<td>0.170</td>
<td>0.145</td>
<td>0.039</td>
<td>0.164</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DLog Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil</th>
<th>Brent</th>
<th>Heating Oil</th>
<th>Gasoline</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF Test Statistic ($t(\hat{\beta})$)</td>
<td>-7.693</td>
<td>-48.015</td>
<td>-63.855</td>
<td>-35.131</td>
<td>-61.932</td>
<td>-50.560</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>


Notes: The sample period for U.S. field production of crude oil is 1920:01 to 2002:12.
The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

4.1 Unit Root Analysis

I employ three alternative conventional test procedures to deal with the behavior of the data, the Augmented Dickey-Fuller test (ADF), the Philips and Perron test (PP), and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test.

The first test is the augmented Dickey-Fuller (ADF) test to check the existence of a unit root in an $AR(p)$ process. The unit root test is carried out under the null hypothesis $H_0: \beta = 0$ versus the alternative hypothesis $H_a: \beta < 0$ using the regression

$$\Delta y_t = c_t + \beta y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + e_t \quad (1)$$

where $c_t$ is a deterministic function of the time index $t$ and $\Delta y_j = y_j - y_{j-1}$ is the differenced series of $y_t$. The $t$-ratio of the statistic is computed by

$$ADF - test = \frac{\hat{\beta}}{std(\hat{\beta})} \quad (2)$$

where $\hat{\beta}$ denotes the least squares estimates of $\beta$, and the $t$-ratio is known as the augmented Dickey-Fuller (ADF) unit root test – [See Dickey and Fuller (1981) for details]. The error term is assumed to be homoscedastic and also the value of $p$ is set such that the error is serially uncorrelated.

Furthermore, the Philips and Perron (1988) known as (PP) unit root test is employed to test whether or not the log level of the series exhibit a random walk behavior. The PP test
Table 3: Philips-Perron Unit Root Test

Null Hypothesis: The log levels and the differenced log of the series have unit root
Bandwidth: (Newey-West automatic) using Bartlett Kernel

<table>
<thead>
<tr>
<th>Log Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil</th>
<th>Brent</th>
<th>Heating</th>
<th>Gasoline</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP Test Statistic ($Z_{t(π)}$)</td>
<td>-3.232</td>
<td>-2.970</td>
<td>-2.905</td>
<td>-3.036</td>
<td>-3.565</td>
<td>-3.094</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.018</td>
<td>0.140</td>
<td>0.160</td>
<td>0.122</td>
<td>0.032</td>
<td>0.107</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DLog Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil</th>
<th>Brent</th>
<th>Heating</th>
<th>Gasoline</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP Test Statistic ($Z_{t(π)}$)</td>
<td>-64.110</td>
<td>-65.323</td>
<td>-63.855</td>
<td>-64.445</td>
<td>-61.879</td>
<td>-59.182</td>
</tr>
<tr>
<td>p-value*</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>


Notes: The sample period for U.S. field production of crude oil is 1920:01 to 2002:12. The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

differs from the ADF test in handling the serial correlation and heteroscedasticity in the errors, and it allows for errors not to be independently and identically distributed (iid). The PP unit root test is essentially based on equation 1 but without the lag differences. While the ADF test correct for the higher-order serial correlation by adding lagged difference terms to the right-hand side, the PP unit root test makes a non-parametric correction to account for residual serial correlation (Maslyuk & Smyth 2008). Therefore, the PP test statistic is robust to a variety of serial correlation and time-dependent heteroscedasticity. The test regression for PP test is

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t$$

where $u_t$ is $I(0)$ and can be heteroscedastic. The PP test corrects for any serial correlations and heteroscedasticity in the error $u_t$ of the test regression by modifying the test statistics $t_{π=0}$ and $T_{π}$.

Under the null hypothesis that $π = 0$, the PP statistic have the same asymptotic distribution as the ADF t-statistic and normalized bias statistic – [See Philips and Perron (1988) for more details].

The t-statistics for the ADF and PP tests ($t_{(β)} and Z_{(π)}$) as well as the p- values for the log levels of the series are reported in table 2 and table 3.

In the specification of the unit root regressions for the ADF and the PP test in log level of the individual series, I included the constant term as well as the time trend. As the results show in tables 2 and table 3, I fail to reject the null hypotheses of a unit root for the ADF.
Table 4: Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) Unit Root Test

Null Hypothesis: The log levels and the differenced log of the series are stationary

Bandwidth: (Newey-West automatic) using Bartlett kernel

<table>
<thead>
<tr>
<th></th>
<th>U.S. Production of Crude Oil</th>
<th>Crude Oil WTI</th>
<th>Brent Heating Oil Europe</th>
<th>Gasoline Natural Oil Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPSS test statistic (LS)</td>
<td>2.659</td>
<td>7.123</td>
<td>7.081</td>
<td>7.024</td>
</tr>
<tr>
<td>KPSS test statistic (TS)</td>
<td>1.048</td>
<td>0.441</td>
<td>0.452</td>
<td>0.453</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>U.S. Production of Crude Oil</th>
<th>Crude Oil WTI</th>
<th>Brent Heating Oil Europe</th>
<th>Gasoline Natural Oil Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>KPSS test statistic (LS)</td>
<td>0.933</td>
<td>0.038</td>
<td>0.042</td>
<td>0.044</td>
</tr>
<tr>
<td>KPSS test statistic (TS)</td>
<td>0.063</td>
<td>0.036</td>
<td>0.029</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Notes: The 1%, 5% and 10% critical values for KPSS test statistics (LS) [given in Kwiatkowski, Phillips, Schmidt & Shin (1992)] are 0.739, 0.463 and 0.347, respectively.
The 1%, 5% and 10% critical values for KPSS test statistics (TS) [given in Kwiatkowski et al. (1992)] are 0.216, 0.146 and 0.119, respectively.
The sample period for U.S. field production of crude oil is 1920:01 to 2002:12.
The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 7, 1997 to August 16, 2011.

and PP tests for each of the variables in log levels at 1% significant level.

Hence, I employ another test to verify the results of the ADF and PP tests, and to identify the random walk behavior in the series. The test is the Kwiatkowski, Phillips, Schmidt, and Shin (1992) test known as (KPSS) test. The ADF and PP unit root tests are carried out under the null hypothesis of whether a time series is $I(1)$. The KPSS test, on the other hand, is known as a stationary test and will test the null hypothesis that the series is $I(0)$, that is to say $H_0 : y_t \sim I(0)$. The test is conducted under the null of either level stationary or trend stationary to investigate whether a series is $I(0)$, $I(1)$ or are not in fact informative about whether they are stationary or follow random walk behavior. Table 4 shows the results for the KPSS tests in which the first test statistic (LS) has the null hypothesis of level stationary and the second test statistic (TS) tests the null hypothesis of trend stationary. Both $t$-statistics exceed the 1%, 5% and 10% critical values given in Kwiatkowski et al. (1992). Therefore, I can reject the null hypotheses of the stationarity of the log levels at the 1%, 5% and 10% significant levels.

It is to be noted that in all the regressions for unit root tests in log levels, the trend terms have been included to distinguish whether or not the series are “trend stationary” (TS) model, where a stationary component is added to a deterministic trend term.

The decision to deal with the random walk behavior is to transform the log levels into the first differenced of the logs. The ADF and PP unit root test results, after performing
them on the first differenced log levels, indicate that I can reject the null hypotheses of unit root in first differenced levels. Moreover, the null hypotheses of the KPSS test, level and trend stationary, cannot be rejected in the first differenced log levels. Hence, I use the first differenced of the log levels for each individual series throughout the rest of the paper unless otherwise noted.

5 The Inference Methods

In this section, I introduce the inference methods for detecting nonlinearities in this study: The BDS test, the Hinich bicovariance test, the Hinich bispectrum test, the Engle LM test, the McLeod-Li test, and the Tsay test. All the above tests, except Hinich bispectrum test, require to remove any serial dependence from the data via a prewhitening model. Any other serial dependence is the result of a nonlinear data generating mechanism. The Hinich bispectrum test directly tests the data generating mechanism and it is invariant to filtering of the data (Patterson & Ashley (2000a)).

5.1 The BDS Test: A Test for Serial Independence

The well known Brock, Dechert, Scheinkman and LeBaron(1996) test, also known as the BDS test, is one form of portmanteau tests for independence. Portmanteau tests are residual-based tests in which the null hypothesis is well stated, but they do not have a specific alternative hypothesis. The BDS test (Brock, Dechert & Scheinkman 1986) is a popular test to detect the serial independence in time series data. The BDS test introduces a test of independence that can be applied to the estimated residuals of any time series model, if the model can be transformed into a form with independent and identically distributed errors. The test employs the correlation function (correlation integral) to calculate the test statistics. The correlation function was introduced as a method of measuring the fractal dimension of deterministic data. The correlation function (integral) measures of the sequential pattern’s frequency that exist in the data – [See Brock et al. (1986) for more details]. It is to be mentioned here that the correlation function is different than the correlation dimension, which is the method used in testing for chaos introduced by Grassberger and Procaccia (1983). Barnett et al. (1995) state that correlation dimension is potentially helpful in testing for chaos, however modeling for high-dimensional chaos needs a large number of variables. Moreover, the sampling properties as well as the derived distribution of the correlation
dimension are unknown, therefore the BDS test uses the correlation function as a test statistic (Barnett et al. 1995). They explain that

“Since the derived distribution of the correlation dimension is unknown, the BDS test uses the correlation function as the test statistic. The asymptotic distribution of the correlation function is known under the null hypothesis of whiteness (independent and identically distributed observations). As a result, the BDS test can be used to produce a formal statistical test of whiteness against general dependence. However, the sampling distribution of the BDS test statistic is not known under the null of chaos. When testing for chaos by this means, we are left with the uncomfortable choice between the correlation dimension, which produces a direct test for chaos, but only when no substantial stochastic shocks exist within the model, or the correlation function, which does have known sampling properties when there are stochastic shocks within the model, but only under a different null hypothesis (i.e. whiteness).” (Barnett et al. 1995)

The BDS test is used to test the null of linearity against a variety of possible deviation from independence in the series including nonlinearity and chaos. The test is applied to a series of estimated residual after removing any linear structure. Under the null hypothesis of independent and identically distributed (i.i.d) or whiteness, the BDS statistic is

$$\frac{\sqrt{n}C_{m,n}(\epsilon) - C_{1}(\epsilon)^m}{\sigma_m(\epsilon)}$$

where $C_{m,n}(\epsilon)$ is the correlation integral, $\sigma_m(\epsilon)$ is the asymptotic standard deviation of the numerator and $m$ is the embedding dimension. The test converges to $N(0,1)$ under the null hypothesis of whiteness [The details for the test statistic and the formula can be found in Brock et al. (1986)]. The BDS test statistic is a transformation of the correlation function, which asymptotically becomes a standard normal $Z$ statistics under the null hypothesis of whiteness (Barnett et al. 1995). I apply the BDS test to the differenced log of the individual time series of the energy data. To carry out the BDS test, the data are prefiltered by fitting the linear ARMA model, and the BDS test is applied against the remaining nonlinear structure in residuals. The choice of the values of $\epsilon$ and $m$ can be a difficulty in using the BDS test. The results with BDS are reported in tables 5 for dimension 2-8 and the chosen $\epsilon$ equals to 1 and 2 standard deviation of the data.

$^9\epsilon$ is calculated as a multiple of the standard deviation of the series.
5.1.1 Results with the BDS Test

I produce the BDS test statistic for all the embedding dimension from 2 to 8, and the inferences are always the same and robust at each embedding dimension. As can be observed in the tables, the results indicate the significance at the 1%, 5% and 10% level based on the asymptotic distribution. Therefore, the BDS test rejects the null hypothesis of independent and identically distributed observations and detect the nonlinearity in all the series. The BDS test has high power against a numerous nonlinear alternatives. Therefore, accepting the null hypothesis in BDS test indicates that there are strong evidence for the null. In that sense, the BDS test can be the first test to run. In the case of this study in which the linearity is rejected with the BDS test, the results reflect little information to distinguish the existing forms of nonlinearity in the data. Hence, I utilize the more focused tests, to identify the other possible forms of nonlinearity in the data – [See Barnett et al. (1997) for more details].

5.2 Tests for Nonlinearity

5.2.1 The Hinich Bicovariance Test

As noted by Patterson & Ashley (2000a) this test assumes $x_t$ is a realization from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The $(r, s)$ sample bicovariance is defined as

$$C_3(r, s) = (N - s)^{-1} \sum_{t=1}^{N-s} x_t x_{t+r} x_{t+s} \quad 0 \leq r \leq s. \quad (5)$$

Therefore, the sample bicovariances are a generalization of a skewness parameter. The $C_3(r, s)$ are all zero for zero mean, serially i.i.d data. One would expect non-zero values for the $C_3(r, s)$ from data in which $x_t$ depends on lagged cross-products, such as $x_{t-i}x_{t-j}$ and higher order terms. Let $G(r, s) = (N - s)^{0.5}C_3(r, s)$ and define $X_3$ as

$$X_3 = \sum_{s=2}^{\phi} \sum_{r=1}^{s-1} [G(r, s)]^2 \quad (6)$$

Under the null hypothesis that $x_t$ is a serially i.i.d process, Hinich & Patterson (1995) show that $X_3$ is asymptotically distributed as $\chi^2[\phi(\phi - 1)/2]$ for $\phi < N^{0.5}$. Based on their simulation, they recommend using $\phi = N^{0.4}$. Under the assumption that $E((x_t)^{0.5})$ exists, the $X_3$ statistic detects nonzero third-order correlations. It can be considered as generalization of
Table 5: BDS Test Z-Statistic (Dimension 2–8)

**Difference Log of U.S. Production of Crude Oil**

<table>
<thead>
<tr>
<th>m</th>
<th>$\epsilon$</th>
<th>$1\sigma$ p-values</th>
<th>$2\sigma$ p-values</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>23.3336</td>
<td>0.000</td>
<td>16.8553</td>
</tr>
<tr>
<td>3</td>
<td>25.7004</td>
<td>0.000</td>
<td>14.2563</td>
</tr>
<tr>
<td>4</td>
<td>27.6393</td>
<td>0.000</td>
<td>11.4692</td>
</tr>
<tr>
<td>5</td>
<td>31.3215</td>
<td>0.000</td>
<td>9.0335</td>
</tr>
<tr>
<td>6</td>
<td>37.8373</td>
<td>0.000</td>
<td>7.5150</td>
</tr>
<tr>
<td>7</td>
<td>49.5211</td>
<td>0.000</td>
<td>7.0203</td>
</tr>
<tr>
<td>8</td>
<td>65.3457</td>
<td>0.000</td>
<td>6.2050</td>
</tr>
</tbody>
</table>

**Difference Log of Crude Oil- WTI**

<table>
<thead>
<tr>
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<th>$\epsilon$</th>
<th>$1\sigma$ p-values</th>
<th>$2\sigma$ p-values</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>8.6502</td>
<td>0.000</td>
<td>13.1413</td>
</tr>
<tr>
<td>3</td>
<td>11.2398</td>
<td>0.000</td>
<td>16.9129</td>
</tr>
<tr>
<td>4</td>
<td>12.7804</td>
<td>0.000</td>
<td>18.5466</td>
</tr>
<tr>
<td>5</td>
<td>13.7043</td>
<td>0.000</td>
<td>19.4291</td>
</tr>
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<td>14.9684</td>
<td>0.000</td>
<td>19.9851</td>
</tr>
<tr>
<td>7</td>
<td>16.6185</td>
<td>0.000</td>
<td>20.5845</td>
</tr>
<tr>
<td>8</td>
<td>18.2287</td>
<td>0.000</td>
<td>20.6910</td>
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</table>

**Difference Log of Crude Oil- Europe Brent**

<table>
<thead>
<tr>
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<th>$1\sigma$ p-values</th>
<th>$2\sigma$ p-values</th>
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<tbody>
<tr>
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<td>7.5338</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td>10.4088</td>
<td>0.000</td>
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</tr>
<tr>
<td>5</td>
<td>12.2642</td>
<td>0.000</td>
<td>13.2835</td>
</tr>
<tr>
<td>6</td>
<td>14.2073</td>
<td>0.000</td>
<td>14.2868</td>
</tr>
<tr>
<td>7</td>
<td>16.1597</td>
<td>0.000</td>
<td>15.0526</td>
</tr>
<tr>
<td>8</td>
<td>17.9577</td>
<td>0.000</td>
<td>15.6694</td>
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**Difference Log of Heating Oil**

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<th>$2\sigma$ p-values</th>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>9.7411</td>
<td>0.000</td>
<td>12.4350</td>
</tr>
<tr>
<td>5</td>
<td>10.9942</td>
<td>0.000</td>
<td>13.0850</td>
</tr>
<tr>
<td>6</td>
<td>12.3876</td>
<td>0.000</td>
<td>13.8142</td>
</tr>
<tr>
<td>7</td>
<td>13.7737</td>
<td>0.000</td>
<td>14.4790</td>
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<tr>
<td>8</td>
<td>15.2485</td>
<td>0.000</td>
<td>15.0990</td>
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**Difference Log of Natural Gas**

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<th>$2\sigma$ p-values</th>
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</thead>
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<td>0.000</td>
<td>19.9861</td>
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<tr>
<td>3</td>
<td>18.3402</td>
<td>0.000</td>
<td>22.2270</td>
</tr>
<tr>
<td>4</td>
<td>20.6047</td>
<td>0.000</td>
<td>23.3393</td>
</tr>
<tr>
<td>5</td>
<td>22.7769</td>
<td>0.000</td>
<td>24.2032</td>
</tr>
<tr>
<td>6</td>
<td>25.5779</td>
<td>0.000</td>
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<tr>
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<tr>
<td>8</td>
<td>31.3578</td>
<td>0.000</td>
<td>26.1315</td>
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</table>
the Box-Pierce portmanteau statistics – [See Hinich & Patterson (1985) for more discussion].

5.2.2 The Hinich Bispectrum Test

A process is said to be third-order nonlinear dependence if the skewness function in the frequency domain is not flat as a function of frequency pairs. The definition of the square of the skewness function is shown in equation 8. This form of the nonlinearity is called third order, since the skewness function is a normalization of the Fourier transform of the third-order autocovariances. That Fourier transform is called the bispectrum. (Barnett et al. 1997)

The Hinich bispectrum test is a nonparametric test that examines the third-order moments (bicovariance) of the data in the frequency domain to obtain a direct test for a nonlinear generation mechanism, regardless of any linear independence that might be present in the data. Therefore, when the tests rejects the null (the skewness function is flat), there is no need to check the possibility that the linear prewhitening model has failed to remove all linear serial dependence in the data. (Ashley & Patterson 2006)

Hinich (1982) develops this test for flatness of bispectrum. He argues that the bispectrum in the frequency domain is easier to interpret than multiplicity of the third-order moments $c_{xxx}(r, s) : s \leq r, r = 0, 1, 2 \cdots$ in the domain. Barnett & Hinich (1993) explain the computation of the test statistic. For frequencies $f_1$ and $f_2$ in the principle domain

$$\Omega = (f_1, f_2) : 0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1$$

is the Hinich bispectrum of the series at frequency pair $(f_1, f_2)$, and its double Fourier transformation of the third-moments function is:

$$B_{xxx}(f_1, f_2) = \sum_{r=-\infty}^{r=\infty} \sum_{s=-\infty}^{s=\infty} c_{xxx}(r, s)exp[-2\pi(f_1 r + f_2 s)]. \quad (7)$$

The square of the skewness function $\Gamma^2(f_1, f_2)$ is defined in terms of the bispectrum as:

$$\Gamma^2(f_1, f_2) = \frac{|B_{xxx}(f_1, f_2)|^2}{S_{xx}(f_1)S_{xx}(f_2)S_{xx}(f_1 + f_2)} \quad (8)$$

where $S_{xx}(f)$ is the (ordinary power) spectrum of $x_t$ at frequency $f$. If the time series $x_t$ is linear then the squared of skewness function $\Gamma^2(f_1, f_2)$ is constant over all frequency pairs $(f_1, f_2)$ in $\Omega$, and the skewness function $\Gamma^2(f_1, f_2)$ is zero over all frequencies if $x_t$ is Gaussian. Linearity and Gaussianity can be tested using a sample estimator of the skewness function $\Gamma^2(f_1, f_2)$ – [See Barnett & Hinich (1993) for more details on computation of the test and
Hinich (1982) for more details on the test].

5.2.3 Engle LM Test

The test was proposed by Engle (1982) to examine nonlinearity in the second moment, particularly for ARCH disturbances. The test employs the Lagrangian multiplier procedure and runs the OLS regression and saves the residuals. Then the next procedures is to regress the squared residuals on a constant and $p$ lagged values of the squared residuals and test $NR^2$ as a $\chi^2_p$.

$$\hat{\varepsilon}_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \hat{\varepsilon}_{t-j}^2 + u_t \tag{9}$$

As most Lagrange multiplier tests, the test statistic is based on the $R^2$ of the regression. Under the null hypothesis of a linear generating mechanism for $x_t$, $NR^2$ for the above regression is asymptotically distributed as $\chi^2_p$.

5.2.4 The McLeod-Li Test

McLeod and Li (1983) developed a portmanteau test for nonlinear statistical dependence in the squared-residual autocorrelations of fitted ARMA models. They found numerous time series in which the squared residuals of the best fitting ARMA are significantly autocorrelated even though the usual residuals autocorrelations do not suggest any model inadequacy. The tests looks at the autocorrelation function of the squares of the prewhitened data and tests whether $corr(x_t^2, x_{t-j}^2)$ is nonzero for some $j$. The autocorrelation at the lag $j$ for the squared residuals $x_t^2$ is estimated by

$$\hat{r}(j) = \frac{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)(x_{t-j}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{N}(x_t^2 - \hat{\sigma}^2)}; \text{ where } \hat{\sigma}^2 = \frac{\sum_{t=1}^{N} x_t^2}{N} \tag{10}$$

Under the null hypothesis that $x_t$ is an i.i.d process, McLeod and Li (1983) showed that, for sufficiently large and fixed $L$,

$$Q = N(N+2) \sum_{j=1}^{L} \frac{\hat{r}(j)}{N-j} \tag{11}$$

is asymptotically $\chi^2_L$ under the null hypothesis of a linear generating mechanism for the data. They have set $L = 20$ for their small-sample simulation in their examination.
5.2.5 The Tsay Test

The Tsay test introduced by Tsay (1986) examines the nonlinearity in the mean while Engle (1982) test checks the evidence for nonlinearity in the variance. The Tsay (1986) test explicitly look for quadratic serial dependence in the data, using quadratic terms lagged up to $K$ periods. Let the $K = k(k+1)/2$ column vectors $V_1, ..., V_k$ contains all the unique cross-products of the form $x_{t-i}x_{t-j}$, where $i \in [i,k]$ and $j \in [j,k]$. Let $\hat{v}_{t,i}$ denote the projection of $v_{t,i}$ on the subspace orthogonal to $x_{t-1}, ..., x_{t-k}$, which is the residuals from a regression of $v_{t,i}$ on $x_{t-1}, ..., x_{t-k}$. The parameters $\gamma_i, ..., \gamma_k$ are estimated by applying OLS to the regression equation:

$$x_t = \gamma_0 + \sum_{i=1}^{k} \gamma_i \hat{v}_{i,t} + \eta_t$$  \hspace{1cm} (12)

Then, the Tsay test statistic is the usual $F$ statistic for testing the null hypothesis that $\gamma_1, ..., \gamma_k$ are all zero.

5.2.6 The Results for Nonlinearity Tests

The results for the Hinich bicovariance, the Hinich bispectrum, the McLeod-Li, the Engle, and the Tsay test are reported in table 7 and table 8 for the significance levels of bootstrapping simulation and asymptotical distributions. As stated by Patterson & Ashley (2000a) the described tests are only asymptotically justified similar to the most econometrics procedure. Therefore, the significance levels of all the tests are routinely bootstrapped. Also, the significance levels based on the asymptotic distributions are computed – [See Patterson & Ashley (2000a) for further details on the bootstrap simulation].

In the Hinich bicovariance test, I use $\phi = N^{0.4}$ based on the Hinich & Patterson (1985)'s simulation, where N is the sample size for each individual series. Moreover, the test is calculated using up to 15 lags and also with the number of bootstrap iterations equal to 1000. As also displayed by the results, based on the bootstrapped as well as asymptotic distributions, this test rejects the null hypothesis that $x_t$ is a serially i.i.d process in every case for 1%, 5% and 10% significance levels.

The Hinich bispectrum test, on the other hand, examines the third order moments (bicovariance) of the data in frequency domain to obtain a direct test for a nonlinear generating mechanism. More importantly, this test focuses on nonlinear serial dependence and it substantially differs the usage of the sample bicovariance data than the Hinich bicovariance test described earlier. The Hinich bisspectrum test accepts the linearity if it cannot reject the
Table 6: Utilized Nonlinearity Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Focus</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>BDS</td>
<td>General serial dependence</td>
<td>Brock et al. (1986)</td>
</tr>
<tr>
<td>Hinich bicovariate</td>
<td>Third-order moments (time domain)</td>
<td>Hinich &amp; Patterson (1995) and Hinich (1996)</td>
</tr>
<tr>
<td>Hinich bispectrum</td>
<td>Third-order moments (frequency domain)</td>
<td>Hinich (1982)</td>
</tr>
<tr>
<td>McLeod and Li</td>
<td>ARCH/GARCH</td>
<td>McLeod &amp; Li (1983)</td>
</tr>
<tr>
<td>Tsay</td>
<td>Quadratic terms (time domain)</td>
<td>Tsay (1986)</td>
</tr>
</tbody>
</table>

Source: Ashley & Patterson (2006)

flatness of bispectrum, and accepts the Gaussianity if the bispectrum is flat and also equals to zero. As can be observed in the table 8, the results of Gaussianity indicate extremely small p-values for each energy components market in the case of asymptotic distribution. As a result the null hypothesis of the Gaussianity is rejected in 1%, 5% and 10% significance level. Moreover, the null of linearity for each individual series exhibits a very significant results by very small p-values for the 80 percent fractile bispectrum linearity test for every series. Hence, in the case of asymptotic distribution, the null hypothesis of the linearity is also rejected in 1%, 5% and 10% significance level for each individual series. In other words, the rejection of linearity provides strong evidences for the presence of the third order nonlinearity in the data generating process as also noted by Barnett et al. (1997). Ashley & Patterson (2006) show that the bispectrum $B_{xxx}(f_1, f_2)$ is consistently estimated using an average of appropriate triple products of the Fourier representation of the observed time series. The average is taken over a square containing $M$ adjacent frequency pairs. Hinich (1982) showed that $M$ must be above the $N^{0.5}$ to consistently estimate $B_{xxx}(f_1, f_2)$. The results here are quoted for $M$ to the integer closet to $N^{0.6}$.

The Engle LM test (1982) examines nonlinearity in the second moments. Under the null hypothesis of a linear generating mechanism for $x_t$, $NR^2$ for the regression equation 9 is asymptotically distributed as $\chi^2_p$. The results are reported for $p$ (lagged values) equals to 5, and they exhibit substantially small $p$-values in 1%, 5% and 10% significance level in both bootstrapped and asymptotic distributions. Therefore, the null hypothesis of nonlinearity in the second moments is rejected in all cases. Following the literature, the results are quoted for $p=5$.

The null hypothesis of $x_t$ is an i.i.d process in McLeod and Li (1983) test is also rejected for up to 24 lags in bootstrapped and asymptotic distributions. As shown in the tables, the results yield very small $p$-values in 1%, 5% and 10% significance level. Here the results are quoted for $L = 24$. 

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Table 7: Significance Level for Nonlinearity Tests

Bootstrap Simulation

<table>
<thead>
<tr>
<th>Log Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil WTI</th>
<th>Brent Europe Oil</th>
<th>Heating Oil</th>
<th>Gasoline Oil</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicovariance ($\phi = N^{0.4}$)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Bispectral (Gaussianity)</td>
<td>0.834</td>
<td>0.900</td>
<td>0.573</td>
<td>0.991</td>
<td>0.475</td>
<td>0.941</td>
</tr>
<tr>
<td>($M = N^{0.6}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bispectral (Linearity)</td>
<td>0.085</td>
<td>0.730</td>
<td>0.330</td>
<td>0.517</td>
<td>0.320</td>
<td>0.384</td>
</tr>
<tr>
<td>($M = N^{0.6}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engle ($p = 5$)</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>McLeod-Li ($L = 24$)</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Tsay ($k = 5$)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: Number of bootstrap iterations = 1000
The sample period for U.S. field production of crude oil is 1920:01 to 2002:12.
The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

The Tsay test Tsay (1986) examines the nonlinearity in the mean. Following the existing literature in the subject the value of $k = 5$ is used here. The reported results based on the bootstrapped as well as asymptotic distributions, are indicating that the null hypothesis is rejected in 1%, 5% and 10% significance level.

Therefore, based on the bootstrapped and asymptotic distributions, the results for the nonlinear tests reveal that the employed data have clear evidence of nonlinearity in their structure, excluding the bootstrap simulation for Hinich bispectrum test. The time series data of the energy market in both quantity of the crude oil, and the prices of major products exhibit nonlinearity in mean, variance and skewness functions. These results are consistent with other reported findings in the literature, such as Kyrtsou et al. (2009). The evidence for significant nonlinearity in data generating mechanism in the energy market implies to model the time series data into an appropriate specifications in order to infer valid parameter estimations.

6 Summary and Conclusion

This paper employed statistical and econometric techniques to investigate the nonlinear dependence in the energy market. The techniques involve the most widely used univariate tests to detect the nonlinearity in the observed time series data. To examine whether the time series data in the energy market exhibit nonlinearity in their generating mechanism, the study utilized the monthly observations on the U.S. field production of crude oil for over
Table 8: Significance Level for Nonlinearity Tests

<table>
<thead>
<tr>
<th>Log Level</th>
<th>U.S. Production of Crude oil</th>
<th>Crude Oil of WTI</th>
<th>Brent Europe</th>
<th>Heating Oil</th>
<th>Gasoline</th>
<th>Natural Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicovariance ($\phi = N^{0.2}$)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Bispectral (Gaussianity) ($M = N^{0.6}$)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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Notes: The sample period for U.S. field production of crude oil is 1920:01 to 2002:12. The sample period for the spot prices is January 3, 1995 to August 16, 2011, except for natural gas spot prices, which is January 07, 1997 to August 16, 2011.

90 years and daily spot price observations on five major products in the energy market for over 16 years. The results indicate that monthly observation of crude oil production, as well as the daily spot prices of crude oil (West Texas Intermediate (WTI) and Europe Brent), heating oil, gasoline and natural gas exhibit deep nonlinearity in their structure. It is to be noted that none of the tests have exactly the same null hypothesis and they differ in the power against the alternative hypothesis. These tests focus on different aspects of nonlinearity and detect distinct features of nonlinear serial dependence in the data. Additionally, using the tests jointly can produce deeper perception into the nature of the nonlinearity that may exist in the data. The BDS test is a test of general nonlinearity in the process, against all other possible alternatives null of linearity, and has high power against the numerous class of alternative hypotheses. The results of the BDS tests indicate that the linearity is rejected; hence it is a compelling indication to employ more particular tests that consider the more detailed feature of nonlinearity. The Hinich bicovariance test focuses on the third-order moments (time domain) of the data and detected nonlinearity in each series. The Hinich bispectrum test examines the lack of third-order nonlinear dependence (frequency domain), and the associated Gaussianity test, is a test of a necessary and not sufficient condition for Gaussianity.\textsuperscript{10} The results of the Hinich bispectrum suggest that the observed time series data in the energy market are generated by a nonlinear, non-Gaussian process. The Engle Lagrangian multiplier (LM) test focuses on the nonlinearity in the second moment. The null hypothesis of no ARCH-type disturbances is rejected by the Engle-LM test. The McLeod-Li test also rejects the null hypothesis of linearity in the variance. Finally,

\textsuperscript{10}See Barnett et al. (1997) for more details.
the Tsay test rejects the null hypothesis of linearity in the mean of each individual series. Therefore, all the tests detect strong evidence of nonlinear structure in the time series data, indicating that the employed series in both quantity and prices are generated by a nonlinear mechanism.

As noted by Ashley & Patterson (2006), the evaluation of the time series models is based on the goodness of fit and the post sample forecasting ability. Prediction can be improved by nonlinear models, when there is evidence of nonlinearity in the data generating process (Maravall (1983), Tong (1983) and Ashley & Patterson (2006)). The main implication of nonlinearity in the observed data is that the series cannot be properly forecasted with linear models in the literature. Therefore, in view of the importance of the energy sector in the aggregate economic activity, it is essential to check the existence of nonlinearity in the time series data of the energy market. The investigation will allow us to attain more plausible empirical results by employing an efficient time series modeling that is agreeable with the data generating process.

References


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Appendix A: Data Description, Key Terms and Definitions

Figure 5 represents the U.S. field production of crude oil. The variable Crude Production is defined as follows:

*Crude Production:* The volume of crude oil produced from oil reservoirs during given periods of time. The amount of such production for a given period is measured as volumes delivered from lease storage tanks (i.e., the point of custody transfer) to pipelines, trucks, or other media for transport to refineries or terminals with adjustments for (1) net differences between opening and closing lease inventories, and (2) basic sediment and water (BS&W).\(^{11}\)

![Figure 5: Monthly U.S. Field Production of Crude Oil (Thousand Barrels)](image)

*Data Source:* Energy Information Administration (EIA)

Figure 6 represents the Cushing, OK WTI spot price FOB (Dollars per Barrel). The variable West Texas Intermediate (WTI- Cushing) is defined as follows:

- *West Texas Intermediate (WTI - Cushing):* A crude stream produced in Texas and southern Oklahoma, which serves as a reference or “marker” for pricing a number of other crude streams and which is traded in the domestic spot market at Cushing, Oklahoma.

\(^{11}\)Energy Information Administration (EIA).
• **Crude Oil:** A mixture of hydrocarbons that exists in liquid phase in natural underground reservoirs and remains liquid at atmospheric pressure after passing through surface separating facilities. Depending upon the characteristics of the crude stream, it may also include:

  - Small amounts of hydrocarbons that exist in a gaseous phase in natural underground reservoirs but are liquid at atmospheric pressure after being recovered from oil well (casinghead) gas in lease separators and are subsequently commingled with the crude stream without being separately measured. Lease condensate recovered as a liquid from natural gas wells in lease or field separation facilities and later mixed into the crude stream is also included;
  
  - Small amounts of nonhydrocarbons produced with the oil, such as sulfur and various metals;
  
  - Drip gases, and liquid hydrocarbons produced from tar sands, oil sands, gilsonite, and oil shale.

Liquids produced at natural gas processing plants are excluded. Crude oil is refined to produce a wide array of petroleum products, including heating oils; gasoline, diesel and jet fuels; lubricants; asphalt; ethane, propane, and butane; and many other products used for their energy or chemical content.\(^{12}\)

Figure 7 represents Europe Brent spot price FOB (Dollars per Barrel). The variable Europe Brent is defined as follows:

*Brent:* A blended crude stream produced in the North Sea region which serves as a reference or “marker” for pricing a number of other crude streams.\(^{13}\)

Figure 8 represents New York Harbor No. 2 Heating Oil Spot Price FOB (Dollars per Gallon). The variable New York Harbor is defined as follows:

*New York Harbor:* The location specified in either spot or futures contracts for delivery of a product in New York Harbor.\(^{14}\)

Figure 9 represents New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon). The variables Conventional Gasoline and the New York Harbor are defined as follows:

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\(^{12}\)Energy Information Administration (EIA).

\(^{13}\)Energy Information Administration (EIA).

\(^{14}\)Energy Information Administration (EIA).
**Figure 6:** Daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel)

![Chart showing daily Cushing, OK WTI Spot Price FOB (Dollars per Barrel) from January 1995 to January 2011.]

**Data Source:** Energy Information Administration (EIA)

*Conventional Gasoline:* Finished motor gasoline not included in the oxygenated or reformulated gasoline categories. Excludes reformulated gasoline blendstock for oxygenate blending (RBOB) as well as other blendstock.\(^{15}\)

*New York Harbor:* The location specified in either spot or futures contracts for delivery of a product in New York Harbor.\(^{16}\)

Figure 10 represents Henry Hub Gulf Coast Natural Gas Spot Price ($/MMBTU). The variable U.S. Gulf Coast is defined as follows:

*U.S. Gulf Coast:* The location specified in either spot or futures contracts for delivery of a product in any port city along the coastline of Texas and Louisiana.\(^{17}\)

The figures 11, 12, 13, 14, 15 and 16 show the log and the differenced log of the individual series.

\(^{15}\)Energy Information Administration (EIA).

\(^{16}\)Energy Information Administration (EIA).

\(^{17}\)Energy Information Administration (EIA).
Figure 7: Daily Europe Brent Spot Price FOB (Dollars per Barrel)

Data Source: Energy Information Administration (EIA)

Figure 8: Daily New York Harbor No. 2 Heating Oil Spot Price FOB (Dollars per Gallon)

Data Source: Energy Information Administration (EIA)
Figure 9: Daily New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon)

*Data Source:* Energy Information Administration (EIA)

Figure 10: Daily Henry Hub Gulf Coast Natural Gas Spot Price ($/MMBTU)

*Data Source:* Energy Information Administration (EIA)
Figure 11: Log and Differenced log of U.S. Field Production of Crude Oil (Thousand Barrels)

**Data Source:** Energy Information Administration (EIA)

Figure 12: Log and Differenced log of West Texas Intermediate (WTI) Spot Price (Dollars/Barrel)

**Data Source:** Energy Information Administration (EIA)
Figure 13: Log and Differenced log of Europe Brent Spot Price (Dollars/Barrel)

Data Source: Energy Information Administration (EIA)

Figure 14: Log and Differenced log of New York Harbor Heating Oil Spot Price FOB (Dollars per Gallon)

Data Source: Energy Information Administration (EIA)
Figure 15: Log and Differenced log of New York Harbor Conventional Gasoline Regular Spot Price FOB (Dollars per Gallon)

Data Source: Energy Information Administration (EIA)

Figure 16: Log and Differenced log of Henry Hub Gulf Coast Natural Gas Spot Price ($/MMBTU)

Data Source: Energy Information Administration (EIA)