Directional Pricing Theory in Electricity

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Abstract: This study is a first attempt of investigating a theory of directional pricing. Directional pricing is defined as price or rate designs that apply different prices to selling and buying the concerned goods. A typical example would be rate schedules in the feed-in-tariff (FIT) policy for electricity. This study discusses how the pricing is distinctive and shows that a new development of the theory is essential for the analysis of such emerging electricity markets.

Keywords: directional pricing, feed-in-tariff, rate design, real option, digital option, binary option.

JEL Classification Codes: D11, L94, Q28.

1. Introduction

Standard economic theory presumes that prices are non-directional: Assuming that there is no transaction cost, the price of transferring Good X from Agent A to B must be equal to that of transferring the good in the opposite direction. This is owing to a simple no-arbitrage argument: If the selling price is greater than the buying price for an agent, the agent could make a profit by buying the good only to immediately sell it back. However, for some goods in the real world, this may not be the case. A typical example is of electricity. It is non-storable and thus must be consumed at the very moment of production, which makes it impossible for any agent to take the opportunity of arbitrage on a single transmission line.

Because of recent developments in electricity production and transmission technologies, bipolar trades between electric utility companies and customers have become popular. In fact, in some countries such as Germany, Spain, and Japan,¹ the feed-in-tariff (FIT) policy has been introduced to promote renewable energy generation technologies.² Under this policy, electricity sold back to the utility company is growing in trade volume. With these recent changes in

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² Couture et al. (2010) provide a comprehensive view of the FIT policy across the world. Note that some papers listed there including Agnolucci (2007), Butler and Neuhoff (2008), Couture and Gagnon (2010), Fouquet and Johansson (2008), Lesser and Su (2008), Menanteau et al. (2003), Rader and Norgaard (1996) address topics of electricity market design. While such topics are closely related to electricity pricing policies, none of them identifies issues of our interest in this paper. For further discussion of the policy in the context of promotion of renewable energy technologies, see International Energy Agency (IEA) (2008).
technology and policy, the purpose of this study is to develop a theory for pricing policy that allows the market to set directional prices. The paper is organized as follows: The next section addresses the model that describes economic agent’s behavior under a directional pricing scheme. Section 3 investigates the mathematics. Section 4 highlights distinctive features of the model. Section 5 concludes the discussion.

2. The model
Consider a single electricity utility firm and a single, representative customer. Assume that the customer owns his generator. Electricity generated from the customer’s generator can be either consumed by the customer or sold to the utility firm. Also assume that these two parties are connected to each other with a single electric transmission line. In a traditional electric system, electricity flows from the utility firm to the customer. With the FIT policy, electricity may flow in the opposite direction under a different rate scheme. That is, consumers can sell the electricity they produce using their own generator back to the utility firm at a price different from the rates they would pay to the utility firm.

The consumer’s generator may not be a conventional thermal power generator. Rather, it may be photovoltaic (PV), wind powered, or powered by another renewable source. These generators may be more costly than conventional thermal generators; however, owing to the newer energy policy requirements, they are being widely promoted throughout the country. To cover such higher costs, the customer’s electricity selling price to the utility firm may be higher than his buying price from the firm.

Hereafter, we consider the behavior of the electricity customer who faces two electricity prices: One is the buying price that applies when electricity is purchased from the utility firm. The other is the selling price that applies when the electricity produced by the customer is sold back to the utility firm.

We introduce the following notations:
- $U : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}$ welfare function of the customer
- $C : \mathbb{R}_+ \rightarrow \mathbb{R}$ cost function of the customer’s generator
- $x$: electricity produced by the customer
- $y$: electricity purchased from the utility firm
- $z$: electricity sold to the utility firm
- $b$: buying price of electricity
- $s$: selling price of electricity
- $w$: composite goods other than electricity
- $m$: upper limit of the budget constraint.

Figure 1 depicts the situation that we consider in this study.

<Insert Figure 1 here>
Definition: Directional pricing scheme

A customer cannot simultaneously purchase and sell the concerned goods. When a customer purchases the good, the buying price is applied to the trade. On the other hand, when a customer sells the good, the selling price is applied to the trade.

For the customer who faces the directional pricing scheme in electricity, the behavior is described by the following optimization problem:

\[
\begin{align*}
\max & \quad U(x + y - z, w) \\
\text{s.t.} & \quad C(x) + w + b \cdot y \leq m + s \cdot z, \quad (2) \\
& \quad y \cdot z = 0, \quad (3) \\
& \quad x \geq 0, \quad y \geq 0, \quad z \geq 0, \text{ and} \\
& \quad x + y - z > 0. \quad (5)
\end{align*}
\]

The constraint of Equation (3) \((y \cdot z = 0)\) represents a specific feature of the model in that electricity cannot flow in two directions at any given moment, and that the directions are always exclusive. Because of this constraint, the problem needs to be treated as mixed-integer programming (MIP). To simplify the analysis, we introduce following assumptions:

Assumption 1:

The welfare function is quasi-linear, i.e.,

\[
U(X, W) = u(X) + W
\]

Also, \(u(\cdot)\) satisfies the Inada condition, i.e., \(\lim_{X\to0} u'(X) = \infty\) and \(\lim_{X\to\infty} u'(X) = 0\).

Assumption 2:

For the customer’s generator, there is no fixed cost, i.e., \(C(0) = 0\). Marginal cost is monotonously increasing from the origin, i.e., \(MC(0) = 0\).

With these assumptions, the problem of (1)–(5) reduces to the following:

\[
\begin{align*}
\max & \quad u(x + y - z) - C(x) - b \cdot y + s \cdot z + m \\
\text{s.t.} & \quad y \cdot z = 0, \\
& \quad x \geq 0, \quad y \geq 0, \quad z \geq 0, \text{ and} \\
& \quad x + y - z > 0.
\end{align*}
\]

The Lagrangian for this problem is written as follows:

\[
\mathcal{L} = u(x + y - z) - C(x) - b y + s z + m + \mu y + \lambda_1 x + \lambda_2 y + \lambda_3 z + \delta \cdot (x + y - z).
\]

The first order necessary conditions (FONCs) for the solution set \((x^*, y^*, z^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \delta^*, \mu^*)\) to satisfy are the following:

\[
u'(x^* + y^* - z^*) - MC(x^*) + \lambda_1^* + \delta^* = 0, \quad (6)
\]

There are many textbooks for mathematical programming, among which Hillier and Lieberman (1990) are popular.
\[ u\left(x^* + y^* - z^*\right) - b + \mu z^* + \lambda z^* + \delta^* = 0, \]
\[ -u\left(x^* + y^* - z^*\right) + s + \mu y^* + \lambda y^* - \delta^* = 0, \]
\[ \lambda_1^* \geq 0, \ x^* \geq 0, \ \lambda_1^* x^* = 0, \]
\[ \lambda_2^* \geq 0, \ y^* \geq 0, \ \lambda_2^* y^* = 0, \]
\[ \lambda_3^* \geq 0, \ z^* \geq 0, \ \lambda_3^* z^* = 0, \]
\[ \delta^* \geq 0, \ x^* + y^* - z^* \geq 0, \ \delta^* \left(x^* + y^* - z^*\right) = 0, \]
\[ \mu \begin{cases} \geq 0 & \text{(It can be either positive or negative).} \\ < 0 \end{cases} \]

Considering the conditions provided in Appendix 1, it turns out that possible cases for these FONCs are limited to the following:

**Case I:** \( x^* > 0, \ \lambda_1^* = 0; \ y^* = 0, \ \lambda_2^* > 0; \ z^* > 0, \ \lambda_3^* = 0; \ \delta^* = 0 \)
\[ u\left(x^* - z^*\right) = MC\left(x^*\right) \]
\[ = b - \lambda_2^* - \mu z^* \]
\[ = s. \]

**Case II:** \( x^* > 0, \ \lambda_1^* = 0; \ y^* > 0, \ \lambda_2^* = 0; \ z^* = 0, \ \lambda_3^* > 0; \ \delta^* = 0 \)
\[ u\left(x^* + y^*\right) = MC\left(x^*\right) \]
\[ = b \]
\[ = s + \lambda_3^* + \mu y^*. \]

**Case III:** \( x^* > 0, \ \lambda_1^* = 0; \ y^* = 0, \ \lambda_2^* > 0; \ z^* = 0, \ \lambda_3^* > 0; \ \delta^* = 0 \)
\[ u\left(x^*\right) = MC\left(x^*\right) \]
\[ = b - \lambda_2^* \]
\[ = s + \lambda_3^*. \]

### 3. Customer’s behavior

In addition to Assumptions 1 and 2, let us introduce specific forms for welfare and cost functions.

**Assumption 3:**
\[ u\left(\cdot\right) \ has \ the \ form \ of \ constant \ relative \ risk \ aversion \ (CRRA): \]
\[ u\left(X\right) = A \frac{X^{\alpha - 1}}{1 - \alpha} \]

**Assumption 4:**
\[ C\left(\cdot\right) \ has \ the \ form \ of \ quadratic \ function: \]
\[ C\left(X\right) = \frac{1}{2} cX^2 \]
With these functional forms, Cases I–III are further simplified.

Case I:

The case indicates that \( x^* > 0, \lambda_2^* > 0, \ z^* > 0 \). From (14), the following equation holds:

\[
A(x^* - z^*)^{-\alpha} = cx^*, \text{ or equivalently } x^* - z^* = A^{\frac{1}{\alpha}} (cx^*)^{\frac{1}{\alpha}}.
\]

By the condition (16), this is equivalent to

\[
z^* = \frac{s}{c} - A^{\frac{1}{\alpha}} s^\frac{1}{\alpha}.
\]

(23)

Also, from (16), \( cx^* = s \), and from (15), \( \lambda_2^* + \mu z^* = b - s \).

Because of the condition \( z^* > 0 \), we have:

\[
z^* = \frac{s}{c} - A^{\frac{1}{\alpha}} s^\frac{1}{\alpha} > 0, \text{ or equivalently } s > c (A/c)^{\frac{1}{\alpha}}.
\]

The welfare for the representative customer for this case is calculated as follows:

\[
U^* = U\left(x^* - z^*, w^*\right)
= u\left(x^* - z^*\right) - C\left(x^*\right) + sz^* + m
= f\left(s\right) + \left[m - \frac{A}{1 - \alpha}\right],
\]

(24)

where the function \( f(\cdot) \) is defined as follows:

\[
f(X) = \alpha \ A^{\frac{1}{\alpha}} X^{\frac{1}{\alpha} - \frac{1}{2\alpha}} + \frac{1}{2c} X^2.
\]

(25)

Case II:

The case indicates that \( x^* > 0, \ y^* > 0, \lambda_3^* > 0 \). Form (17), the following equation holds:

\[
A(x^* + y^*)^{-\alpha} = cx^*, \text{ or equivalently } x^* + y^* = A^{\frac{1}{\alpha}} (cx^*)^{\frac{1}{\alpha}}.
\]

By the condition (18), this is equivalent to

\[
y^* = A^{\frac{1}{\alpha}} b^\frac{1}{\alpha} - b/c.
\]

(26)

Also, from (18), \( cx^* = b \), and from (19), \( \lambda_3^* + \mu y^* = b - s \).

Because of the condition \( y^* > 0 \), we have

\[
y^* = A^{\frac{1}{\alpha}} b^\frac{1}{\alpha} - b/c > 0, \text{ or equivalently } b < c (A/c)^{\frac{1}{\alpha}}.
\]

The welfare for the representative customer for this case is calculated as follows:
\[
U^* = U \left( x^* + y^*, w^* \right) \\
= u \left( x^* + y^* \right) - C \left( x^* \right) - by^* + m \\
= f \left( b \right) + \left( m - \frac{A}{1-\alpha} \right), \\
\]

where the function \( f \left( \cdot \right) \) is defined by Equation (25).

Case III:

This case indicates that \( x^* > 0, \lambda_2^* > 0, \lambda_3^* > 0 \). From (20), the following equation holds:

\[ A \lambda^* = c x^* . \]

This is equivalent to

\[ x^* = \left( A/c \right)^{1/\alpha} . \]

From (20), (21), and (22), \( s < cx^* < b \), or equivalently \( s < c \left( A/c \right)^{1/\alpha} < b \).

The welfare for the representative customer for this case is calculated as follows:

\[
U^* = U \left( x^*, w^* \right) \\
= u \left( x^* \right) - C \left( x^* \right) + m \\
= f \left( c \left( A/c \right)^{1/\alpha} \right) + \left( m - \frac{A}{1-\alpha} \right), \\
\]

where the function \( f \left( \cdot \right) \) is defined by Equation (25).

It is noted that function \( f \left( X \right) \) has its minimum value at \( X = c \left( A/c \right)^{1/\alpha} \), as follows:

\[
f \left( X \right) \geq f \left( c \left( A/c \right)^{1/\alpha} \right) = \frac{1+\alpha}{2(1-\alpha)} \frac{A^\alpha}{c^{1+\alpha}} . \\
\]

This is proven as below:

\[
f' \left( X \right) = -A^\alpha X^{-\alpha} + \frac{1}{c} X , \\
\]

and thus, \( f' \left( X \right) \begin{cases} \geq 0 & \text{if and only if } X/c \begin{cases} \geq \left( A/c \right)^{1/\alpha} \\ < \end{cases} \end{cases} \).

Also, we have

\[
f^* \left( X \right) = \frac{1}{\alpha} \frac{A^\alpha}{c^{1+\alpha}} X^{1/\alpha} + \frac{1}{c} > 0 \ \forall X . \\
\]

This proves that \( f \left( X \right) \) has its minimum value at \( X = c \left( A/c \right)^{1/\alpha} \).
On the basis of the investigation of FONCs, we can construct demand or supply curves and the welfare gains for the representative customer. Because we have two prices for electricity, i.e., the buying price $b$ and the selling price $s$, the customer’s behavior is classified on the $b$-$s$ plane.

**Region 1:** $c \frac{1}{\alpha} \leq \min \{b, s\}$

In this region, among the three cases mentioned above, only Case I can apply: Only selling to the utility firm can occur and its inverse supply curve is given by Equation (23). The welfare is given by Equation (24).

**Region 2:** $b < c \frac{1}{\alpha} < s$

In this region, among the abovementioned three cases, both Cases I and II can apply. The choice is up to the customer and will be determined by the comparison of possible welfare gains as follows:

The customer’s gain is $f(b) = \left( m - \frac{A}{1 - \alpha} \right)$ if he/she chooses to buy electricity from the utility firm. In this case, the inverse demand curve is given by Equation (26).

On the other hand, the gain is $f(s) = \left( m - \frac{A}{1 - \alpha} \right)$ if he/she chooses to sell electricity to the utility firm. In this case, the inverse supply curve is given by Equation (23).

Note that there exists an indifference curve that divides the choices of selling and buying. The curve is determined by the following condition:

$$f(b) = f(s) \quad \text{where} \quad b < c \frac{1}{\alpha} < s.$$  \hspace{1cm} (30)

**Region 3:** $\max \{b, s\} \leq c \frac{1}{\alpha}$

In this region, only Case II can apply: Only buying from the utility firm can occur and its inverse demand curve is given by Equation (26). The welfare is given by Equation (27).

**Region 4:** $s < c \frac{1}{\alpha} < b$

This region is equivalent to Case III where there is neither selling nor buying. In other words, the customer chooses to use electricity in the form of “stand-alone.” The welfare is given by Equation (28).

Figure 2 summarizes these results.
Let us introduce the following functions of buying and selling prices:

\( Y(b,s) \): the net demand of electricity purchased by the customer from the utility firm, i.e., \( y^* - z^* \).

\( Z(b,s) \): the net supply of electricity sold by the customer to the utility firm, i.e., \( z^* - y^* \).

\( \nu(b,s) \): the welfare for the customer.

By combining all cases of the four regions, these functions are written as follows:

\[
Y(b,s) = \frac{1}{\alpha} A q - q / c, \tag{31}
\]

where \( q = \arg \max \{ \phi_b(b), \phi_s(s) \}, \{ 1 - J(b,s) \} + c (A/c)^{1/\alpha} \cdot J(b,s), \)

\[
Z(b,s) = -Y(b,s), \tag{32}
\]

\[
\nu(b,s) = \max \{ \phi_b(b), \phi_s(s) \} + \left( m - \frac{A}{1 - \alpha} \right), \tag{33}
\]

where \( \phi_b(X) = f(X) \cdot I(X) + f \left( c (A/c)^{1/\alpha} \cdot (1 - I(X)) \right) \) and

\[
\phi_s(X) = f(X) \cdot (1 - I(X)) + f \left( c (A/c)^{1/\alpha} \right) \cdot I(X). \tag{34}
\]

The functions \( I(\cdot) \) and \( J(\cdot, \cdot) \) are index functions as follows:

\[
I(X) = \begin{cases} 
1 & \text{if } X \leq c (A/c)^{1/\alpha}, \\
0 & \text{otherwise}
\end{cases} \tag{36}
\]

\[
J(B,S) = \begin{cases} 
1 & \text{if } \max \{ \phi_b(B), \phi_s(S) \} = f \left( c (A/c)^{1/\alpha} \right), \\
0 & \text{otherwise}
\end{cases} \tag{37}
\]

Note that functions \( \phi_b(X) \) of (34) and \( \phi_s(X) \) of (35) are depicted as in Figure 3.

4. Distinctive features

The behavior of the customer under a directional pricing scheme is very complicated in a mathematical form, as was discussed in the previous section. For understanding our feature, let us make some modifications on our original problem described by Equations (1)–(5).

Consider an alternative pricing scheme that allows the customer to make simultaneous trades of buying and selling of goods. It does not usually occur in electricity while it usually happens for other ordinary goods. Even in electricity, however, when the customer has two separate
lines, one of which is used for buying and the other is used for selling, then the situation can occur.

Figure 4 depicts that situation where two separate transmission lines are installed.

<Insert Figure 4 here>

With such two-line setting, the problem of (1)–(5) is modified as follows:

Typical bilateral trades problem:

\[
\begin{align*}
\max \ U(x + y - z, w) \\
\text{s.t.} \quad C(x) + w + b \cdot y \leq m + s \cdot z, \\
x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad \text{and} \\
x + y - z > 0.
\end{align*}
\]

(1)'

(2)'

(4)'

(5)'

Note that the only difference is the removal of Equation (3).

With Assumptions 1 and 2, FONCs for this problem are stated as follows:

FONCs for the inner solution set \((x'', y'', z'', \lambda_1'', \lambda_2'', \lambda_3'', \delta'')\) to satisfy are the following:

\[
\begin{align*}
&u''(x'' + y'' - z'') - MC(x'') + \lambda_1'' + \delta'' = 0, \\
&u''(x'' + y'' - z'') - b + \lambda_2'' + \delta'' = 0, \\
&-u''(x'' + y'' - z'') + s + \lambda_3'' - \delta'' = 0, \\
&\lambda_1'' \geq 0, \quad x'' \geq 0, \\
&\lambda_2'' \geq 0, \quad y'' \geq 0, \\
&\lambda_3'' \geq 0, \quad z'' \geq 0, \\
&\delta'' \geq 0, \quad x'' + y'' - z'' \geq 0, \\
&\delta'' \cdot (x'' + y'' - z'') = 0.
\end{align*}
\]

(6)'

(7)'

(8)'

(9)'

(10)'

(11)'

(12)'

From Equations (7)' and (8)',

\[
b - s = \lambda_2'' + \lambda_3'' \geq 0 \quad \text{or equivalently} \quad b \geq s
\]

must hold for an inner solution to exist. It can be replaced by a simple arbitrage discussion: From (2)', we have the following:

\[
C(x) + w + b \cdot \Delta \leq m + s \cdot \Delta, \quad \text{where} \quad \Delta = y - z; \quad \text{the net purchase.}
\]

If \(b < s\) holds, the customer can make the budget constraint virtually limitless by setting \(z\) and \(\Delta\) such that \(z \to \infty\) and \(0 \leq \Delta < \infty\). Thus, the price scheme must be such that \(b \geq s\). Figure 5 depicts an arbitrage opportunity in the two-line setting, which illustrates the above discussion.

<Insert Figure 5 here>

With Assumptions 3 and 4, the solution of the problem is described as follows:

\[
y'' = 0 \quad \text{and} \quad z'' = s/c - A^{-\frac{1}{c}} s^{-\frac{1}{c}} \quad \text{if} \quad b \geq s \geq c(A/c)^{-\frac{1}{c}},
\]

\[
y'' = A^{\frac{1}{c}} b^{-\frac{1}{c}} - b/c \quad \text{and} \quad z'' = 0 \quad \text{if} \quad c(A/c)^{-\frac{1}{c}} \geq b \geq s, \quad \text{and}
\]

9
\[ y^* = 0 \quad \text{and} \quad z^* = 0 \quad \text{if} \quad b > c(A/c)^{1/10} > s . \]

Figure 6 depicts the solution on the b-s plane.

<Insert Figure 6 here>

It is easy to observe that this solution is a part of the solution for our original directional pricing problem. The difference is that no solution exists for the region of \( b < s \). This fact is a critical weakness of the model when it is applied to the analysis of the FIT policies. Because the basic idea of the FIT policy is to promote renewable energy sources that have relatively higher unit costs, \( b < s \) is the most probable scheme to set. It is concluded that no typical market transaction and/or consumer behavior models studied so far in economic theory can be applied to the analysis of the FIT policies.

As is discussed above, the existence of solutions for the case of \( b < s \) makes the directional pricing scheme very distinctive. To further clarify this point, consider the case of \( b \leq c^{1/10} A^{1/10} \leq s \). From Equations (31) and (32), the net demand and supply curves are depicted as in Figures 7 and 8. Note that \( b^* (s) \) and \( s^* (b) \) in these figures are the solutions for the following equations:

\[ f(b^* (s)) = f(s) \quad \text{and} \quad f(s^* (b)) = f(b) , \]

\[ \text{where} \quad b \leq c^{1/10} A^{1/10} \leq s . \]

<Insert Figure 7 here>

<Insert Figure 8 here>

The unique feature that appears on these figures is that net demand and supply curves can be discontinuous. As we have discussed in the previous section, there exists an indifference curve that divides the choices of selling and buying in the third region of \( b < c(A/c)^{1/10} < s \) on the b-s plane. The discontinuous feature of net demand and supply curves is due to the indifference curve that is determined by (30). It is easy to see that (30) is consistent with (38) and (39).

5. Conclusion

The model’s analytical solution depicts a complicated feature of directional pricing schemes. This indicates that the rate design has a higher degree of freedom than that of traditional electric utility pricing. The analysis presented here is preliminary in a sense that it only focuses on customer behavior; however, it is sufficient to show that typical market transaction and/or consumer behavior
models studied so far in economic theory are not feasible for the analysis of emerging markets and policy issues with regard to electricity. We believe that our results contribute to a better understanding of rate design for newly defined electricity markets.

Possible future research is recommended in two directions. One is further investigation of the feed-in-tariff (FIT) policy formulation. The current study identifies the net demand (or net supply) curves for electricity customers as a function of two prices—selling and buying prices. It is suggested that using the function as a building block, one can explore market design issues of the FIT policy. It would also be interesting to consider an electricity market where n customers and m electric utility firms compete with each other.

Another direction is to explore the binary feature of directional pricing. We define directional as being synonymous with a binary choice of either buying or selling. In fact, there is a parallel instrument in the finance sector—financial derivatives called binary options or digital options. Therefore, directional pricing proposed here can be interpreted as a binary real option or a digital real option. Note that while there is a long list of studies on the concept of real options, virtually none investigate such types of real options. Obviously, more research needs to be done in this direction.

References
Appendix 1: The arrangement of FONCs

(i) Suppose \( x^* = 0 \), and thus, \( \lambda_i^* > 0 \). Then, Equations (6)–(8) are reduced to the following:

\[
u'(y^* - z^*) = MC(0) - \lambda^* - \delta^* = -\lambda_i^* - \delta^* < 0.
\]

This contradicts to the Inada condition of Assumption 1. Thus, \( x^* > 0 \) and \( \lambda_i^* = 0 \) must hold true.

(ii) With the condition of \( x^* > 0 \), suppose \( y^* = 0 \). This means \( \lambda_i^* > 0 \). Equations (6)–(8) and (12) are reduced to the following equalities:

\[
u'(x^* - y^*) = u'(0) \rightarrow \infty,
\]

which is impossible for the above equalities to hold. Thus, \( x^* > z^* \) and \( \delta^* = 0 \) must hold true.

(iii) With the condition of \( x^* > 0 \), suppose \( z^* = 0 \). This means \( \lambda_i^* > 0 \). By Equations (6)–(8) and (12), the following equalities must hold:

\[
u'(x^* + y^*) = u'(0) \rightarrow \infty,
\]

which is impossible for the above equalities to hold. Thus, \( x^* + y^* > 0 \) and \( \delta^* = 0 \) must hold true.

Further arranging these conditions, we obtain Cases I–III.
Figure 1: Analytical frame

\[ U^* \text{ increases} \]

\[ \text{"No buy, no sell" region} \]

\[ \text{"Buy" region} \]

\[ \text{"Sell" region} \]

\[ b < 0 \]

\[ \left( \frac{A}{1-\alpha} \right) \]

Figure 2: Solution on the b-s plane

\[ U^* = f(b) + \left( m - \frac{A}{1-\alpha} \right) \]

\[ U^* = f(s) + \left( m - \frac{A}{1-\alpha} \right) \]

Figure 3: Functions \( \phi_s(X) \) and \( \phi_b(X) \)
Figure 4: Two-line setting

Figure 5: Arbitrage opportunity

If $b < s$, 
$q (s - b) \to \infty$ as $q \to \infty$

Figure 6: Solution on the $b$-$s$ plane for typical bilateral trades
Figure 7: Net demand curve as a function of buying price $b$ for the case of $b \leq \frac{a}{A^{1/\alpha}} \leq s$.

Figure 8: Net supply curve as a function of selling price $s$ for the case of $b \leq \frac{a}{A^{1/\alpha}} \leq s$. 

\[ Z(b, s) = \frac{b/c - \frac{1}{\alpha} b^{1/\alpha}}{A^{\frac{1}{\alpha}}} \]

\[ A^{\frac{1}{\alpha}} s^{-\frac{1}{\alpha}} - s/c \]