Effect of Suppliers' Risk Appetite on Market Performance in Electricity Markets

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1 Introduction

1.1 Objective of Our Research

The main purpose of this study is

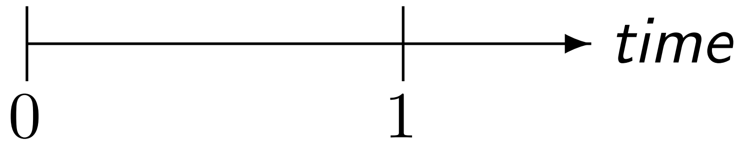
to figure out the relation among Structure, Conduct and Performance (SCP) in an electricity market under demand uncertainty,

to unveil relations between risk appetite of power generators and features of spot electricity prices,

to derive some insight on a structure which generates the electricity price features: spikes and stochastic volatility.

2 Model

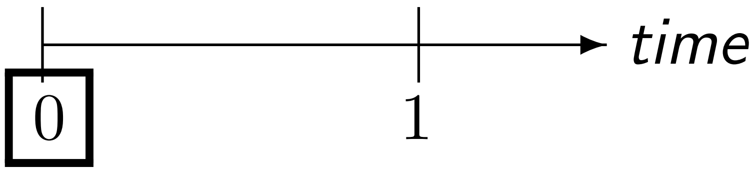
2.1 Outline



- An oligopolistic spot electricity market with a finite number of homogeneous power generating firms and many perfectly competitive retailers.
- In the market, the power generating firms are suppliers and face spot electricity demand from the retailers who distribute power to their customers.

cont.

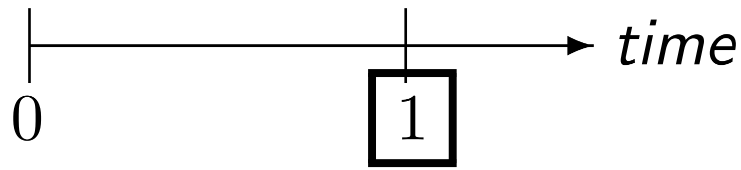
time 0



- Demand for spot electricity is not still determined or is uncertain at time 0, since it will realize at time 1.
- Each power generator knows only a probability distribution with respect to the demand, and strategically offers its supply function.
- The individual supply functions are aggregated to construct a market-wide supply function.

cont.

time 1



- A demand curve is realized.
- A spot price is determined at where the market-wide supply function equals the realized demand curve.
- Each supplier get a profit that is determined by both the spot price and the offered supply function.

2.2 Notation

- $(\Omega, \mathcal{F}, \mathrm{P})$: A probability space.
- Z : A random variable.
- c : A non-negative constant.
- $\{(x, y) \,|\, h(x, y, Z; c) = 0\}$: Demand curve.

  x denotes demand(supply), and y denotes price.

  Z is a random shock on the demand.

  c is a slope or gradient parameter of the function h

  If $c = 0$, that denotes a vertical demand curve.

cont.

- $n \in \mathrm{N}$: The number of homogeneous power generators.
- $f(x) : [0, \infty) \to [0, \infty)$ is an increasing function :
  The marginal cost function of power generator.
- $C(x) = \displaystyle\int_0^x f(u)\,\mathrm{d}u$:
  The total cost function of power generator.
  Without loss of generality, the fixed cost is assumed to be zero for each power generator.

cont.

- For each $j = 1, 2, \ldots, n$, power generator j selects a strategy $s_j \geq 0$, and offers its supply function $g(x, s_j)$ to the market at time 0.

  Here, $g(x, s_j) : [0, \infty)^2 \to [0, \infty)$ is an increasing function with respect to both x and s_j, which satisfies

  $$g(x, s_j) \geq f(x) \quad \text{and} \quad g(x, 0) = f(x)$$

  for any $(x, s_j) \in [0, \infty)^2$.

- For each $s_j \geq 0$, we can find the inverse function of $g(x, s_j)$.

  The inverse function is denoted by $g^{-1}(y, s_j)$.

cont.

For the convenience, we introduce the following symbols:

- For every strategy vector $s = (s_1, s_2, \ldots, s_n)$,

  $s_{(k)}$ is the kth smallest of $s_1, s_2, \ldots, s_n$, that is,

  $$s_{(1)} \leq s_{(2)} \leq \cdots \leq s_{(n)}.$$

- Let $r(j)$ be the rank of s_j among $s_1, s_2, \ldots, s_n$.

  i.e. $r(j) = l$ if $s_j = s_{(l)}$.

cont.

In this presentation, we consider two cases:

Linear marginal cost function (LM) case,

Exponential marginal cost function (ExpM) case.

- In LM case, we assume that

$$f(x) = ax + b,$$
$$g(x, s) = ax + b + s,$$
$$h(x, y, Z; c) = x + cy - Z.$$

  Then, the demand curve is

$$\{(x, y) \mid x + cy = Z,\ 0 \le x \le Z\}.$$

cont.

- In ExpM case, we assume that

$$f(x) = e^{ax+b},$$
$$g(x,s) = e^{ax+b+s},$$
$$h(x,y,Z;c) = x + c\log y - Z.$$

  Then, the demand curve is

$$\{(x,y)\,|\,x + c\log y = Z,\ 0 < y \leq e^{\frac{Z}{c}}\}.$$

2.3 Volume and Price

For any $y \geq 0$ and any strategy vector $s = (s_1, s_2, \ldots, s_n) \in [0, \infty)^n$, we set

$$G(y, s) := \sum_{i=1}^{n} g^{-1}(y, s_i),$$

which denotes the market-wide supply curve.

cont.

For any $z \in \mathbf{R}$ and any $s \in [0, \infty)^n$, we solve the following system of equations in two unknowns x and y:

$$\begin{cases} h(x, y, z; c) = 0 \\ x = G(y, s) \end{cases} \quad (1)$$

When system (1) has a unique solution in $[0, \infty)^2$,

the solution is written by $(\varphi_x(z, s), \varphi_y(z, s))$, and

- $\varphi_x(z, s)$ represents the amount of electricity traded,
- $\varphi_y(z, s)$ represents the spot electricity price.

cont.

$$I_0(s) = \big(-\infty, c(b + s_{(1)})\big),$$

$$I_1(s) = \left[c(b + s_{(1)}), \frac{s_{(2)} - s_{(1)}}{a} + c(b + s_{(2)})\right),$$

For $k = 2, 3, \ldots, n-1$,

$$I_k(s) = \left[\frac{\sum_{i=1}^{k-1}(s_{(k)} - s_{(i)})}{a} + c(b + s_{(k)}), \right.$$

$$\left. \frac{\sum_{i=1}^{k}(s_{(k+1)} - s_{(i)})}{a} + c(b + s_{(k+1)})\right),$$

cont.

$$I_n(s) = \left[\frac{\sum_{i=1}^{n-1} (s_{(n)} - s_{(i)})}{a} + c(b + s_{(n)}), \infty \right),$$

2.3.1 Volume and Price in LM case

The market-wide supply curve is

$$G(y,s) = \begin{cases} 0 & \text{for } y \in [0\ ,\ b + s_{(1)}], \\ \dfrac{k(y-b) - \displaystyle\sum_{i=1}^{k} s_{(i)}}{a} & \text{for } y \in (b + s_{(k)}\ ,\ b + s_{(k+1)}], \\ & \quad k = 1, 2, \ldots, n-1, \\ \dfrac{n(y-b) - \displaystyle\sum_{i=1}^{n} s_{i}}{a} & \text{for } y \in (b + s_{(n)}\ ,\ \infty). \end{cases} \tag{2}$$

cont.

System (1) has a unique solution in $[0,\infty)^2$, if and only if $z \geq c(b+s_{(1)})$.

We have

$$\varphi_x(z,s) = \frac{1}{k+ac}\left(kz - c\sum_{i=1}^{k}(b+s_{(i)})\right), \qquad (3)$$

$$\varphi_y(z,s) = \frac{1}{k+ac}\left(az + \sum_{i=1}^{k}(b+s_{(i)})\right), \qquad (4)$$

for $z \in I_k(s)$ and $k = 1,2,\ldots,n$.

cont.

Additionally, we define

$$\varphi_x(z,s) = \varphi_y(z,s) = 0 \quad \text{for } z \in I_0(s),$$

which does not lead to a loss of generality.

If event $\{Z \in I_0(s)\}$ occurs,

the demand curve $\{(x,y) \,|\, x + cy = Z,\ 0 \le x \le Z\}$ and

the market wide supply curve $G(y,s)$ do not intersect.

We refer to this situation as *untradable*.

2.3.2 Volume and Price in ExpM case

The market-wide supply curve is

$$G(y,s) = \begin{cases} 0 & \text{for } 0 \le y \le e^{b+s_{(1)}} \\ \dfrac{k\log y - \displaystyle\sum_{i=1}^{k}(b+s_{(i)})}{a} & \text{for } e^{b+s_{(k)}} < y \le e^{b+s_{(k+1)}} \\ & \quad k = 1,2,\ldots,n-1 \\ \dfrac{n\log y - \displaystyle\sum_{i=1}^{n}(b+s_i)}{a} & \text{for } e^{b+s_{(n)}} < y \end{cases} \tag{5}$$

cont.

System (1) has a unique solution in $[0, \infty)^2$, if and only if $z \geq c(b + s_{(1)})$.

For $k = 1, 2, \ldots, n$ and $z \in I_k(s)$,

$\varphi_x(z, s)$ and $\varphi_y(z, s)$ are expressed as follows:

$$\begin{cases} \varphi_x(z, s) = \dfrac{kz - c\displaystyle\sum_{i=1}^{k}(b + s_{(i)})}{k + ac}, \\ \varphi_y(z, s) = e^{\dfrac{az + \displaystyle\sum_{i=1}^{k}(b + s_{(i)})}{k + ac}}. \end{cases}$$

cont.

For the convenience, we put

$$\begin{cases} \varphi_x(z,s) = 0, \\ \varphi_y(z,s) = e^{b+s_{(1)}} \end{cases}$$

for $z \in I_0(s)$ without loss of generality.

The event $\{Z \in I_0(s)\}$ is also referred as *untradable* in ExpM case, .

2.4 Profit Function of Power Generators

$j = 1, 2, \ldots, n.$

For, $z \in \mathbf{R}$ and $s \in [0, \infty)^n$, the profit function of power generator j at time one is

$$F_j(z, s) = \varphi_y(z, s) g^{-1}(\varphi_y(z, s), s_j) - C(g^{-1}(\varphi_y(z, s), s_j)).$$

2.4.1 Profit Function in LM case

$F_j(z, s)$ can be expressed explicitly as follows:

When $s_j = s_{(l)}$ for $l = 1, 2, \ldots, n$,

$$F_j(z, s) = 0 \quad \text{for } z \in \bigcup_{i=0}^{l-1} I_i(s).$$

cont.

$$F_j(z,s) =$$

$$\frac{(k-1+ac)(k+1+ac)}{2a(k+ac)^2}\left\{-\left(s_j - \frac{a(z-bc)+\sum_{i\in N(j,k)} s_{(i)}}{(k-1+ac)(k+1+ac)}\right)^2 + \frac{(k+ac)^2\left(a(z-bc)+\sum_{i\in N(j,k)} s_{(i)}\right)^2}{(k-1+ac)^2(k+1+ac)^2}\right\}$$

for $z \in I_k(s)$ and $k = l, l+1, \dots, n$

where $N(j,k) = \{1, 2, \dots, k\} \backslash \{r(j)\}$.

2.4.2 Profit Function in ExpM case

$F_j(z, s)$ can be expressed explicitly as follows:

When $s_j = s_{(l)}$ for $l = 1, 2, \ldots, n$,

$$F_j(z, s) = 0 \quad \text{for } z \in \bigcup_{i=0}^{l-1} I_i(s).$$

cont.

$$F_j(z,s) = \frac{\exp\left\{\dfrac{az+kb+\sum_{i=1}^{k} s_{(i)}}{k+ac}\right\}}{a} \times \left\{\frac{a(z-bc)-(k-1+ac)s_j+\sum_{i\in N(j,k)} s_{(i)}}{k+ac} - e^{s_j}\right\} + \frac{e^b}{a}$$

for $z \in I_k(s)$ and $k = l, l+1, \ldots, n$

where $N(j,k) = \{1,2,\ldots,k\}\setminus\{r(j)\}$.

2.5 Non-cooperative Game

Let $\alpha \in (0,1)$ be arbitrary

Consider the following non-cooperative game:

For each $j = 1, 2, \ldots, n$,

$$\begin{cases} \text{the strategy set for player } j \text{ is } [0, \infty), \\ \text{the payoff to player } j \text{ is} \\ \qquad \inf \{u \in \mathbf{R} \mid \mathrm{P}(F_j(Z, s) \leq u) \geq \alpha\}. \end{cases} \tag{6}$$

Here, α reflects the risk appetite of power generators.

cont.

It is easy to show that

$$F_j(z_\alpha, s) = \inf\left\{u \in \mathbf{R} \mid \mathrm{P}(F_j(Z,s) \le u) \ge \alpha\right\}. \quad (7)$$

z_α is the α-quantile of Z.

2.6 Why an α-quantile is employed as the Objective Function

Reason 1

In ExpM case, for $(s_1, s_2, \ldots, s_n) = (0, 0, \ldots, 0)$, that is , all power generators bid the marginal cost function, the profit function is

$$F_j(z,0) = \begin{cases} 0 & \text{for } z < bc \\ \dfrac{e^{\frac{az+nb}{n+ac}}}{a}\left\{\dfrac{a(z-bc)}{n+ac} - 1\right\} + \dfrac{e^b}{a} & \text{for } z \geq bc \end{cases}. \tag{8}$$

cont.

When Z is distributed according to some distributions,

e.g. F, log normal, Pareto, exponential with sufficiently large expectation, and t distributions,

the expectation $\mathrm{E}(Z)$ exists,
on the other hand, the expected profit does <u>not</u> exist, i.e.

$$\mathrm{E}(F_j(Z, 0)) = \infty.$$

In these cases, maximizing expected profit is meaningless.

On the other hand, every α-quantile of $F_j(Z, s)$ exists for any distribution functions.

Reason 2

By using equation (7), the way to obtain the Nash equilibrium does not depends on the distribution of Z in this model.

The derived Nash equilibrium and the equilibrium price formula, which are explained below, can be applied to any distribution of random shock Z that generates the demand curve fluctuation.

Reason 3

In this model, we can examine relations among

- suppliers' risk appetite, which is reflected in α,
- the types of suppliers' marginal cost functions,
- the distribution of demand uncertainty Z,
- the slope or gradient of demand curve,
- the market performance, e.g. the expected price and price volatility.

In addition,

- inelastic demand case, that is, $c = 0$ (vertical demand curve) is contained in this model.

3 Nash Equilibrium and Analysis

3.1 Fundamental Results

3.1.1 Nash Equilibrium and Price in LM case

Theorem 1

In the non-cooperative game (6), we have the following:

(a) if $z_\alpha \leq bc$, then every strategy vector in $[0, \infty)^n$ is a Nash equilibrium, and all payoff functions are 0;

(b) if $z_\alpha > bc$, the unique Nash equilibrium is

$$s^*{}_j = \frac{a(z_\alpha - bc)}{(n + ac)^2 - n} \quad \text{for } j = 1, 2, \ldots, n \tag{9}$$

Hereafter, we put $s^* = (s^*{}_1, s^*{}_2, \ldots, s^*{}_n)$.

cont.

Corollary 2

Let $z_\alpha > bc$. If $Z \geq bc + \dfrac{ac(z_\alpha - bc)}{(n+ac)^2 - n}$,

$$\varphi_x(Z, s^*) = \frac{n}{n+ac}\left(Z - bc - \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}\right), \tag{10}$$

$$\varphi_y(Z, s^*) = \frac{n}{n+ac}\left(\frac{aZ}{n} + b + \frac{a(z_\alpha - bc)}{(n+ac)^2 - n}\right). \tag{11}$$

Otherwise, the spot electricity is untradable.

cont.

Corollary 3

Demand is assumed to be inelastic, that is, $c = 0$. The unique Nash equilibrium is

$$s^*{}_j = \frac{az_\alpha}{n(n-1)} \quad \text{for } j = 1, 2, \ldots, n \tag{12}$$

and the equilibrium spot price is

$$\varphi_y(Z, s^*) = \frac{aZ}{n} + b + \frac{az_\alpha}{n(n-1)}. \tag{13}$$

For the details of solving the Nash equilibrium in LM case, see Ishii and Tezuka(2014) [a].

[a] `http://www.chuo-u.ac.jp/research/institutes/economic/publication/discussion/pdf/discussno226.pdf?1509973053062`

3.1.2 Nash Equilibrium and Price in ExpM case

Let us solve the following equation with respect to u:

$$\frac{ac}{(n+ac)(n-1+ac)}u - \frac{a(z-bc)}{(n+ac)(n-1+ac)} + 1 = e^{-u}.$$

$\beta(a,b,c,n,z)$ is the unique solution.

If $z > bc$, then $\beta(a,b,c,n,z) > 0$.

Hereafter, $\beta(n,z)$ is used as an abbreviation of $\beta(a,b,c,n,z)$ for the convenience.

cont.

It is easy to show that $\beta(n,z)$ has the following properties:

(i) $\beta(n,z)$ is increasing with respect to z,

(ii) $\lim_{z\to\infty} \beta(n,z) = \infty$,

(iii) $\lim_{n\to\infty} \beta(n,z) = 0$,

Theorem 4

For $c > 0$. In the non-cooperative game (6), we have the following:

(a) if $z_\alpha \leq bc$, then every strategy vector in $[0, \infty)^n$ is a Nash equilibrium, and all of the payoff functions are 0;

(b) if $z_\alpha > bc$, the unique Nash equilibrium is

$$s^*{}_j = \beta(n, z_\alpha) \quad \text{for } j = 1, 2, \ldots, n, \tag{14}$$

Since $\lim_{z \to bc+0} \beta(n, z) = \beta(n, bc) = 0$, we put $s^*{}_j = 0$ for $z_\alpha \leq bc$ hereafter.

Corollary 5

Let $z_\alpha > bc$. If $Z > c(b + \beta(n, z_\alpha))$,

$$\varphi_x(Z, s^*) = \frac{n\{(Z - bc) - c\beta(n, z_\alpha)\}}{n + ac}, \tag{15}$$

$$\varphi_y(Z, s^*) = e^{\frac{aZ + n(b + \beta(n, z_\alpha))}{n + ac}}. \tag{16}$$

Otherwise, the spot electricity is untradable.

Corollary 6

Demand is assumed to be inelastic, that is $c = 0$.

(a) If $\dfrac{a z_\alpha}{n(n-1)} < 1$, the unique Nash equilibrium is

$$s^*{}_j = \beta(a, b, 0, n, z_\alpha) = \log\left(1 - \frac{a z_\alpha}{n(n-1)}\right), \qquad (17)$$

and the equilibrium spot electricity price is

$$\varphi_y(Z, s^*) = \frac{n(n-1)e^{\frac{aZ}{n}+b}}{n(n-1) - a z_\alpha}. \qquad (18)$$

(b) Otherwise, Nash equilibria do not exist, and the payoff function approaches to infinity.

3.2 Property of the Equilibrium Price

3.2.1 Expectation and Variance in LM case

Uniform Random Variable

For $u_0 > 0$, we assume $Z \sim \mathrm{U}(bc, bc + u_0)$.

Then, $z_\alpha = bc + \alpha u_0$ and

$$s^*{}_j = \frac{a\alpha u_0}{(n + ac)^2 - n} \quad \text{for } j = 1, 2, \ldots, n$$

cont.

The expectation of $\varphi_y(Z, s^*)$ is

$$\mathrm{E}\left(\varphi_y(Z, s^*)\,\Big|\; Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}\right)$$

$$= \frac{au_0}{2(n+ac)} + b + \frac{(2n+ac)a\alpha u_0}{2(n+ac)\{(n+ac)^2 - n\}}.$$

The variance of $\varphi_y(Z, s^*)$ is

$$\mathrm{V}\left(\varphi_y(Z, s^*)\,\Big|\; Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}\right)$$

$$= \frac{1}{12}\left(\frac{au_0\{(n+ac)^2 - n - ac\alpha\}}{(n+ac)\{(n+ac)^2 - n\}}\right)^2.$$

Exponential Random Variable

For $\lambda > 0$, we assume $Z_0 \sim \mathrm{Exp}(\lambda)$, and put $Z = bc + Z_0$.

Then, $z_\alpha = bc - \lambda \log(1-\alpha)$ and

$$s^*{}_j = \frac{-a\lambda \log(1-\alpha)}{(n+ac)^2 - n} \quad \text{for } j = 1, 2, \ldots, n$$

We have

$$\mathrm{E}\left(\varphi_y(Z, s^*) \,\middle|\, {}_{Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}}\right)$$

$$= \frac{a\lambda}{2(n+ac)} + b - \frac{(n+a)a\lambda \log(1-\alpha)}{(n+ac)\{(n+ac)^2 - n\}},$$

$$\mathrm{V}\left(\varphi_y(Z, s^*) \,\middle|\, {}_{Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}}\right) = \left(\frac{a\lambda}{n+ac}\right)^2.$$

Normal Random Variable

For positive constants μ and σ, let $Z \sim \mathrm{N}(bc+\mu, \sigma^2)$.

Then, $z_\alpha = \sigma\Phi^{-1}(\alpha) + bc + \mu$,

where Φ is the distribution function of $\mathrm{N}(0, 1^2)$, and Φ^{-1} is the inverse function of Φ.

For each $j = 1, 2, \ldots, n$,

$$s^*{}_j = \begin{cases} 0 & \text{if } \sigma\Phi^{-1}(\alpha) + \mu \leq 0, \\ \dfrac{a(\sigma\Phi^{-1}(\alpha)+\mu)}{(n+ac)^2 - n} & \text{if } \sigma\Phi^{-1}(\alpha) + \mu > 0. \end{cases} \tag{19}$$

cont.

The expectation and variance of the equilibrium price are

$$\mathrm{E}\left(\varphi_y(Z, s^*)\Big|_{\ Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}}\right)$$
$$= \frac{a\sigma}{(n+ac)R(u_\alpha)} + \frac{n}{n+ac}\left\{\frac{a(\mu+bc)}{n} + b + \frac{a(\sigma\Phi^{-1}(\alpha)+\mu)}{(n+ac)^2 - n},\right\} \tag{20}$$

$$\mathrm{V}\left(\varphi_y(Z, s^*)\Big|_{\ Z \geq bc + \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}}\right)$$
$$= \left(\frac{a\sigma}{n+ac}\right)^2\left[1 + \frac{u_\alpha}{R(u_\alpha)} - \left\{\frac{1}{R(u_\alpha)}\right\}^2\right]. \tag{21}$$

cont.

Here, u_α, ϕ, and R are defined as follows:

$$u_\alpha = \frac{1}{\sigma}\left\{\frac{ac(\sigma\Phi^{-1}(\alpha)+\mu)}{(n+ac)^2-n} - \mu\right\},$$

$$\phi(u) = \frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}},$$

$$R(u) = \frac{1-\Phi(u)}{\phi(u)}.$$

Summary of the above results

Uniform Random Variable

(i) The expectation is an increasing function of α,

(ii) For $c > 0$, the variance is a decreasing function of α,

(iii) For $c = 0$, the variance does not depend on α.

Exponential Random Variable

(i) The expectation is an increasing function of α,

(ii) The variance does not depend on α.

cont.

Normal Random Variable

(i) For $\alpha \in (0, \frac{1}{2})$, the equilibrium strategy ${s^*}_j$ decreases as σ increases, and converges to ${s^*}_j = 0$.

(ii) In case of $\alpha = \frac{1}{2}$, the value of σ does not affect ${s^*}_j$.

(iii) When $\frac{1}{2} < \alpha < 1$, the equilibrium strategy ${s^*}_j$ is an increasing function of σ.

(iv) The expectation is an increasing function of α,

(v) For $c > 0$, the variance is a decreasing function of α,

(vi) For $c = 0$, the variance does not depend on α.

cont.

Therefore,

- The market-wide supply curve is equal to the sum of the marginal costs as in the perfect competition market for such a small α and a large σ.

  That is, even in an oligopoly the performance of a perfectly competitive market is attained because of the large volatility of the demand shock when the risk appetite is small.

- An increase in the risk appetite causes averagely higher prices, but also makes prices more stable.

- The market performance depends on both demand shock uncertainty and the risk appetite of suppliers.

3.2.2 Expectation and Variance in ExpM case

Uniform Random Variable

For $u_0 > 0$, we assume $Z \sim \mathrm{U}(bc, bc + u_0)$.

Since $z_\alpha = bc + \alpha u_0$,

$$s^*{}_j = \beta(n, bc + \alpha u_0) \quad \text{for } j = 1, 2, \ldots, n$$

The expectation of the spot electricity price is

$$\mathrm{E}(\varphi_y(Z, s^*) \mid {}_{Z > c\{b + \beta(n, bc - \lambda \log(1-\alpha))\}})$$

$$= (n + ac) e^{b + \frac{\beta(n, z_\alpha)}{n+ac}} \frac{(n + ac)\left\{e^{\frac{au_0}{n+ac}} - e^{\frac{ac\beta(n, z_\alpha)}{n+ac}}\right\}}{a\{u_0 - c\beta(n, z_\alpha)\}}.$$

cont.

The variance of spot electricity price is

$$\mathrm{V}\Big(\varphi_y(Z, s^*)\,\Big|\ {}_{Z > c\{b+\beta(n,\, bc - \lambda \log(1-\alpha))\}}\Big)$$

$$= \frac{(n+ac)^2 e^{b + \frac{n+2ac}{n+ac}\beta(n,z_\alpha)}}{2\{a(u_0 - c\beta(n, z_\alpha))\}^2}$$

$$\times \left[\begin{array}{l} \left\{ \dfrac{a(u_0 - c\beta(n, z_\alpha))}{n+ac} - 2 \right\} \left\{ e^{\frac{2a(u_0 - c\beta(n,z_\alpha))}{n+ac}} - 1 \right\} \\ +4\left(e^{\frac{a(u_0 - c\beta(n,z_\alpha))}{n+ac}} - 1 \right) \end{array} \right].$$

Exponential Random Variable Case

For $\lambda > 0$, we assume $Z_0 \ \sim \ \mathrm{Exp}(\lambda)$, and put $Z = bc + Z_0$.

Then, $z_\alpha = bc - \lambda \log(1-\alpha)$ and

$$s^*{}_j = \beta(n, bc - \lambda \log(1-\alpha)) \quad \text{for } j = 1, 2, \ldots, n$$

We have

$$\mathrm{E}\big(\varphi_y(Z, s^*) \,\big|\, {}_{Z > c\{b + \beta(n,\, bc - \lambda \log(1-\alpha))\}}\big)$$

$$= \frac{n + ac}{n + ac - a\lambda} e^{b + \beta(n, bc - \lambda \log(1-\alpha))},$$

$$\mathrm{V}\big(\varphi_y(Z, s^*) \,\big|\, {}_{Z > c\{b + \beta(n,\, bc - \lambda \log(1-\alpha))\}}\big)$$

$$= \frac{(n + ac)a^2\lambda^2}{(n + ac - a\lambda)^2(n + ac - 2a\lambda)} e^{2\{b + \beta(n, bc - \lambda \log(1-\alpha))\}}.$$

Normal Random Variable Case

For positive constants μ and σ, let $Z \sim \mathrm{N}(bc + \mu, \sigma^2)$.

z_α, the α-quantile of Z, is

$$z_\alpha = \sigma\Phi^{-1}(\alpha) + bc + \mu,$$

For each $j = 1, 2, \ldots, n$,

$$s^*{}_j = \begin{cases} 0 & \text{for } 0 < \alpha \leq \Phi\left(-\frac{\mu}{\sigma}\right) \\ \beta(n, z_\alpha) & \text{for } \Phi\left(-\frac{\mu}{\sigma}\right) < \alpha < 1 \end{cases}.$$

The expectation of the equilibrium spot electricity price is

$$\begin{aligned} &\mathrm{E}\big(\varphi_y(Z, s^*) \,\big|\, {}_{Z > c\{b + \beta(n, z_\alpha)\}}\big) \\ &= e^{b + \frac{a\mu + n\beta(n, z_\alpha)}{n + ac} + \frac{1}{2}\left(\frac{a\sigma}{n+ac}\right)^2} \cdot \frac{1 - \Phi(B_1)}{1 - \Phi(B_0)}, \end{aligned} \tag{22}$$

where

$$B_0 = \frac{\beta(n, z_\alpha) - \mu}{\sigma}, \quad B_1 = \frac{\beta(n, z_\alpha) - \mu}{\sigma} - \frac{a\sigma}{n + ac}.$$

The variance of the equilibrium spot spot electricity price is

$$\mathrm{V}\Big(\varphi_y(Z, s^*) \,\Big|\, {}_{Z > c\{b + \beta(n,\, z_\alpha)\}}\Big)$$

$$= e^{2b + \frac{2\{a\mu + n\beta(n,z_\alpha)\}}{n+ac} + \left(\frac{a\sigma}{n+ac}\right)^2} \cdot \frac{1}{\{1 - \Phi(B_0)\}^2}$$

$$\times \left[e^{\left(\frac{a\sigma}{n+ac}\right)^2} \{1 - \Phi(B_0)\}\{1 - \Phi(B_2)\} - \{1 - \Phi(B_1)\}^2 \right] \tag{23}$$

where

$$B_2 = \frac{\beta(n, z_\alpha) - \mu}{\sigma} - \frac{2a\sigma}{n + ac}.$$

Summary of the above results

Uniform Random Variable

(i) The equilibrium strategy ${s^*}_j$ is increasing of α.

(ii) The expectation is an increasing function of α,

(iii) $\lim_{\alpha \to 1} \mathrm{V}(\varphi_y(Z, s^*) \mid _{Z > c\{b + \beta(n, z_\alpha)\}}) < +\infty$.

Exponential Random Variable

(i) The equilibrium strategy ${s^*}_j$ is increasing of α.

(ii) Both the expectation and variance are increasing with respect to λ and α for $c \geq 0$.

(iii) $\lim_{\alpha \to 1} \mathrm{V}(\varphi_y(Z, s^*) \mid _{Z > c\{b + \beta(n, z_\alpha)\}}) = +\infty$.

cont.

Normal Random Variable

(i) For $\alpha \in (0, \frac{1}{2})$, the equilibrium strategy ${s^*}_j$ decreases as σ increases, and converges to ${s^*}_j = 0$.

(ii) In case of $\alpha = \frac{1}{2}$, ${s^*}_j$ does not depend on the value of σ.

(iii) When $\frac{1}{2} < \alpha < 1$, the equilibrium strategy ${s^*}_j$ is an increasing function of σ.

(iv) ${s^*}_j$ is increasing with respect to μ for $\alpha \in (0, 1)$.

(v) Both the expectation (22) and the variance (23) of the equilibrium spot price are increasing functions of α.

cont.

Therefore, we can see the similarity in the case of LM:

- The market-wide supply curve is equal to the sum of the marginal costs as in the perfect competition market for such a small α and a large σ.

  That is, even in an oligopoly the performance of a perfectly competitive market is attained because of the large volatility of the demand shock when the risk appetite is small.

- An increase in the risk appetite implies averagely higher prices.

cont.

In contrast to the above LM case,

- When the marginal cost is an exponential function,

  an increase in the risk appetite makes the price more volatile, which does not depend on c.

Implication

The above results imply the followings:

- These examples uncover the relation among distribution of the random shock on demand curve, the shapes of marginal cost functions, the risk appetite, and spot electricity prices.
- Then this model can be applied to analyze the features of spot electricity price fluctuations over a period of time, especially, spikes and stochastic volatilities.
- The above results implies that these features of spot electricity prices are caused by changes of the power generators' risk appetite.

4 Conclusions and Future Research

This research sheds light on a linkage among demand uncertainty, risk appetite of power generators, and spot electricity price fluctuation:

- Different influence on the equilibrium price among three types of random demand shocks in each marginal cost curve,
- Different influence on the equilibrium price among two types of marginal cost curves in each random shock,
- An insight on spikes and stochastic volatilities of spot electricity price.

In our future research,

- It is assumed that power generators are homogeneous. However, it may be necessary to include non-homogeneous power generators.
- Based on this model, we try to do some empirical analysis to uncover the linkage among marginal cost and risk appetite of power generators, and spot electricity prices in a real market.
- We will extend our model to analyze effects of a capacity market.

Thank you for your attention!