The Impact of Outliers on Computing Conditional Risk Measures for Crude Oil and Natural Gas Commodity Futures Prices

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Why outliers?

- A common characteristic found in financial price time series is a sudden or extraordinary change in the price sequence.
- These changes may be outliers and if not addressed, they could lead to erroneous conclusions.
- Outlier detection has become a standard feature used by the United States Government and the European Union.
- Commodity price time series are no exception.

Methodology

We follow Chen and Liu (1993) and use a nonseasonal case without a constant term:

- Let Y_t be a time series following ARMA process without drift or trend:

$$Y_t = \frac{\theta(B)}{\varphi(B)\cdot\alpha(B)} a_t, t = 1, \cdots, n \quad (1)$$

- Where n is the number of observations,
- B is the back shift operator,
- $\theta(B)$ is a moving average component, roots outside of the unit circle,
- $\varphi(B)$ is an auto regressive component, roots outside of the unit circle and
- $\alpha(B)$ is a difference component, roots on the unit circle.
- a_t are Normal(0,σ_a^2) IID.

Introducing outlier effect in the model

The following model describes a time series that is influenced by a non repetitive event:

$$Y_t^* = Y_t + \omega \cdot \frac{A(B)}{G(B) \cdot H(B)} \cdot I_t(t_1), \tag{2}$$

- Y_t follows a general ARMA process defined in Equation (1).
- $I_t(t_1)$ is an indicator function equal to 1 if an outlier occurs at time t_1 and zero otherwise.
- The magnitude and dynamic impact of outliers on the process are governed by ω and $A(B)/G(B) \cdot H(B)$.

Definitions for the different types of outliers - Innovative (IO)

- By imposing a special structure on $A(B)/G(B)\cdot H(B)$, we can classify the outlier as:
- Innovative:

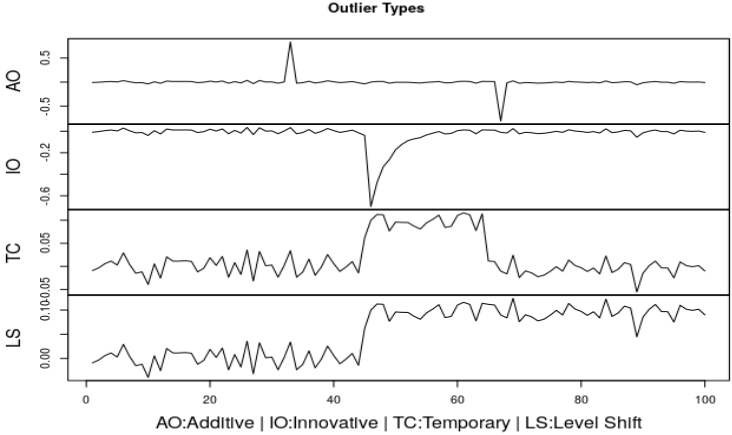
$$\frac{A(B)}{G(B)\cdot H(B)} = \frac{\theta(B)}{\alpha(B)\cdot\varphi(B)} \tag{3}$$

- This is an outlier in the innovation series a_t that occurs at time $t = t_1$ and has a dynamic effect on Y_t^*.
- The effect of this outlier on $Y_{t_1+k}^*$ for $k \geq 0$ is equal to $\omega\Psi_k$ where ω is the initial effect and Ψ_k is the k^{th} coefficient of the polynomial:
- $\Psi(B) = \dfrac{\theta(B)}{\alpha(B)\cdot\varphi(B)} = \Psi_0 + \Psi_1 B + \Psi_2 B^2 + \cdots,$
- For stationary series the IO will produce a temporary effect since Ψ_j's will decay exponentially to zero.

Definitions for the different types of outliers - Additive (AO), Temporary (TC) and Level Shift (LS)

- Additive: $\frac{A(B)}{G(B) \cdot H(B)} = 1$, the outlier only affects Y_{t_1}*.
- Temporary: $\frac{A(B)}{G(B) \cdot H(B)} = \frac{1}{(1 - \delta B)}$
- This is a disturbance that affects Y_t*, $\forall t \geq t_1$ but decays exponentially with rate $\delta, 0 < \delta < 1$ and initial impact ω.
- Level shift: $\frac{A(B)}{G(B) \cdot H(B)} = \frac{1}{(1 - B)}$
- At time t_1 there is a permanent change of size ω.

Example of outlier types



Algorithm for detecting and correcting outliers

- **Stage 1.** Outlier detection is performed estimating ARIMA models and checking for significant outliers at different times based on t-statistics from the parametric estimation.
- **Stage 2.** Filter outliers by joint estimation of ARIMA models with results from Stage 1. Outliers found to be insignificant are dropped from the initial set based on t-statistics by parameter estimation.
- **Stage 3.** Iterate over Stages 1 and 2 to determine the adjusted series and the final outlier effects.

Output from the algorithm

- The completion of the three stages on log returns of commodity prices will result in parametric specifications of the time series composed of two components if outliers are found:
- an ARIMA model specification of the log returns and functional specifications for outliers with a decay rate of $\delta = 0.7$.
- Only the ARIMA specification is returned if no outliers are found, and this is the best fit model of the time series.

Outlier Simulation Example

- We simulate a single random walk without a drift with annualized volatility of 25% for 100 days. We then introduce outliers of each type in the simulation. Our goal is to test if the algorithm will detect the outliers and recover the original DGP.

Outlier	AO	TC	LS	IO
Up Event	0.8	0.7	0.1	0.7
Down Event	-0.8			-0.7
Decay Factor		0.7		0.8
Time Period (Days) Affected	33,67	45,65	45,100	45,65

- The decay factor will exponentially decay to zero from the event over the time period.

Summary Statistics of Simulated 100 Day Returns for Base (GBM), AO, IO, TC, and LS examples

Statistic	Base	AO	TC	LS	IO
Observations	100	100	100	100	100
Minimum (%)	-3.54	-79.12	-67.80	-3.54	-69.59
Median (%)	0.01	0.01	-0.24	8.34	0.01
Arithmetic Mean (%)	0.05	0.05	-2.28	5.65	0.06
Geometric Mean (%)	0.04	-0.94	-3.05	5.52	-0.96
Maximum (%)	3.91	78.93	3.79	13.91	69.93
Standard Deviation (%)	1.59	11.34	9.69	5.29	12.00
Skewness	0.195	-0.037	-4.943	-0.233	-0.165
Kurtosis	-0.294	45.07	26.394	-1.591	19.866
Annualized Standard Deviation (%)	25.24	180.02	153.82	83.98	201.61
		Outlier Adjusted Series			
Arithmetic Mean (%)	0.052	0.054	0.048	-0.005	0.689
Standard Deviation (%)	1.584	1.594	1.591	1.590	6.430
Annualized Standard Deviation (%)	25.138	25.296	25.259	25.236	102.080

Jarque-Bera and Shapiro-Wilk test for normality

	Base	AO	TC	LS	IO
		Unadjusted Data Series			
Jarque Beta Statistic	0.995	8463.588	3309.870	11.460	1644.787
	(0.60820)	(0.00000)*	(0.00000)*	(0.00325)	(0.00000)*
Shapiro Wilk Statistic	0.992	0.261	0.397	0.869	0.455
	(0.79073)	(0.00000)*	(0.00000)*	(0.00000)*	(0.00000)*
		Outlier Adjusted Series			
Jarque Beta Statistic	0.995	0.913	0.977	1.085	6592.081
	(0.60820)	(0.63346)	(0.61359)	(0.58140)	(0.00000)*
Shapiro Wilk Statistic	0.992	0.991	0.992	0.991	0.589
	(0.79073)	(0.78066)	(0.82752)	(0.71050)	(0.00000)*

- Entries are the estimated normality test statistics and their p-values in parentheses.
- * denotes significance at 1% level.

Analysis of Crude Oil (CL) and Natural Gas (NG) – Data Description

- The data for this analysis is from the CME Group daily settlements for commodity instruments.
- The specific CME instruments are outright futures contracts for natural gas (NG) and crude oil (CL).
- The data starts on 2003-12-31 and ends on 2017-03-20.
- The contracts are monthly for each commodity.
- The CME Group lists CL future contracts 9 forward years with monthly listing for the current year and following 5 years.
- Year 6 and out are listed for June and December contract monthly.
- Additional months are added annually when the December contract expires to keep 9 years of the combination of monthly and biannual contracts listed.
- NG is listed monthly for the current year plus the following 12 calendar years with a new year added when the December contract expires for the current year.

Summary statistic comparison of log returns of the contaminated and outlier adjusted CL and NG commodity contracts

	CL				NG		
Contracts	198	Min	Max	Contracts	276	Min	Max
Observations	196,301	79	2,213	Observations	376,429	75	2,232
	Average	Range			Average	Range	
			Original Data				
Mean(%)	-0.02	-0.12	0.12	Mean(%)	-0.06	-0.29	0.09
Annualized				Annualized		-	
Mean(%)	-7.06	-44.89	45.22	Mean(%)	-22.70	104.81	32.21
Median(%)		-0.14	0.17	Median(%)		-0.09	0.07
StDev(%)	1.50	0.97	2.42	StDev(%)	1.06	0.56	2.14
Annualized				Annualized			
StDev(%)	23.78	15.42	38.43	StDev(%)	16.77	8.83	33.92
Skewness		-1.40	0.91	Skewness		-1.751	1.123
Kurtosis		-0.56	11.10	Kurtosis		1.009	22.773

Summary statistic comparison of log returns of the contaminated and outlier adjusted CL and NG commodity contracts

				Outlier Adjusted Series			
Mean(%)	0.00	-0.20	0.14	Mean(%)	-0.14	-2.32	0.14
Annualized Mean(%)	-0.68	-72.74	49.93	Annualized Mean(%)	-52.58	-847.9	52.32
Median(%)		-0.06	0.18	Median(%)		-2.28	0.12
StDev(%)	1.40	0.82	2.33	StDev(%)	0.95	0.45	1.97
Annualized StDev(%)	22.27	13.02	36.92	Annualized StDev(%)	15.13	7.19	31.19
Skewness		-1.03	0.32	Skewness		-0.58	1.02
Kurtosis		-0.56	7.97	Kurtosis		0.03	9.64

Computing modified VaR and ES risk metrics based on Cornish-Fischer approximations

- Earlier we showed that the adjustment for outliers reduced the DGP residual variation. To analyze the impact of this on the forecast error and address the cost implication, modified VaR and ES risk metrics were calculated based on a Cornish-Fischer approximations.
- CL VaR and CvaR decreased on average of 8.6% to 8.9% with NG decreasing on average of 14.4% to 16.7%.
- Some contract's Risk metrics increased.
- Each risk measure is stand alone for a long position in each contract based on 1,000,000 bbls of CL and 1 BCF (1,000,000 mmBTU) of NG.

Percentage Change in Value at Risk and Expected Shortfall metrics

Risk Metric	Average	Min		Max	
		Crude Oil			
Gaussian VaR	-8.66%	-27.19%	2022.CLF	4.23%	2008.CLM
Modified VaR	-8.91%	-40.39%	2015.CLG	7.47%	2008.CLV
Gaussian CVaR	-8.58%	-26.83%	2022.CLF	3.10%	2008.CLM
Modified CVaR	-8.66%	-26.64%	2022.CLF	12.08%	2008.CLV
		Natural Gas			
Gaussian VaR	-15.00%	-48.94%	2029.NGZ	4.80%	2007.NGN
Modified VaR	-16.85%	-65.43%	2029.NGX	6.26%	2022.NGK
Gaussian CVaR	-14.37%	-47.04%	2029.NGZ	4.37%	2007.NGN
Modified CVaR	-14.98%	-56.55%	2029.NGX	4.74%	2010.NGJ

Analysis of Risk Metrics that increased after Outlier Adjustments

Risk Metric	Risk Change > 0	Percentage Change
	Crude Oil	
Gaussian VaR	9	4.55%
Modified VaR	10	5.05%
Gaussian CVaR	15	7.58%
Modified CVaR	8	4.04%
Total Contracts	198	
	Natural Gas	
Gaussian VaR	15	5.43%
Modified VaR	17	6.16%
Gaussian CVaR	15	5.43%
Modified CVaR	15	5.43%
Total Contracts	276	

There are cases when the risk metrics increase!

- This occurred for 5% of CL contracts and 5.5% for NG contracts.
- This cases could potentially cause serious problems for a firm.
- Backtests will also suffer showing that the VaR is exceeded, instead of not, more than the predicted number of times per year. This will imply an inadequate risk metric.
- The distributional characteristics will change and the tails will be larger than originally estimated with the contaminated data. As a result, the expected loss if VaR is exceeded could be much larger than anticipated.
- This larger losses would require immediate risk capital to be deployed, such as a margin call on exchange traded instruments, posting additional capital on over the counter transactions, or being in violation of credit arrangements resulting in technical default.

Conclusion

- We show that detecting outliers is an important step in identifying the true DGP from a risk measurement point of view.
- The algorithm was able to address common issues with outliers of masking/shadowing as seen by the substantial reduction in each contacts set of final outliers from the initial set.
- The analysis demonstrated that risk could be separated between the DGP and outlier impacts.
- The analysis showed that risk metrics like VaR and ES can be inaccurately reported, which could impact hedging cost and hedging decisions from the changes in 2nd, 3rd, and 4th moments of the DGP.
- The analysis of residual variance or forecast error was similar to Tsay 1988 findings where the 95th percentile decreased by 50% in his research.

Out....liar