Welfare impacts of optimal virtual bidding in a multi-settlement electricity market with transmission line congestion

• James Hyungkwan Kim
• USAEE
• Presentation
Virtual transaction: Introduction

Virtual transactions are financial products without physical obligation that affect market outcomes

**Introduction of virtual transaction**

- Financial products to speculate or hedge price differences between forward (Day-Ahead) and spot (Real-Time) market
- Bid virtual demand or offer virtual supply at specific node in forward (Day-Ahead) market
- Do not have obligations for physical delivery of energy
- Virtual bids do not affect Real-Time physical energy consumption
- Virtual bids affect market outcomes
  - Market operation
  - Market results
- $990 million net revenue
  - 2012-2017, PJM
Two-settlement wholesale electricity market

Day-Ahead market settles based on the forecasted demand in one day ahead. Real-Time market settles based on the realized demand deviation in real time.

**Energy Offers**
- Supply side
- Energy bids
- Demand side

**Day-Ahead Market** (Every Day)
- Which generation units get committed at what levels for next 24 hours
- Settles the market at forecasted demand

**Real-Time Market** (Every 5 minutes)
- Generation levels for next 5 minutes
- Settles for demand deviations from day-ahead market, which was uncertain in day-ahead.

**Day-Ahead market settlement**
- Based on forecasted hourly quantities and day-ahead hourly prices

**Real-Time market settlement**
- Based on actual hourly deviations from Day-Ahead schedule, priced at Real-Time
Virtual bids settlement calculation

Virtual supply bids (INC) expect higher DA (forward) price than RT (spot) price and virtual demand bids (DEC) expect higher RT price than DA price

• Virtual bids are committed to buy/sell cleared quantity at the DA price and sell/buy the exact same quantity at the RT price
• Net payoff for virtual bids = Cleared bid in DA (MW) X (DA Price – RT Price)
  – Virtual supply (Positive bid): Expect higher DA price than RT price
  – Virtual demand (Negative bid): Expect higher RT price than DA price
Virtual’s price contribution in PJM market

Virtual supply and virtual demand takes almost half of the PJM Day-Ahead price component in 2016 and 2017

<table>
<thead>
<tr>
<th>Element</th>
<th>2016 Contribution to LMP</th>
<th>2017 Contribution to LMP</th>
<th>Change Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent</td>
<td>Percent</td>
<td></td>
</tr>
<tr>
<td>DEC</td>
<td>$7.93 $7.31</td>
<td>26.7% 23.7%</td>
<td>(3.0%)</td>
</tr>
<tr>
<td>INC</td>
<td>$5.43 $6.86</td>
<td>18.3% 22.2%</td>
<td>3.9%</td>
</tr>
<tr>
<td>Coal</td>
<td>$8.58 $6.51</td>
<td>28.9% 21.1%</td>
<td>(7.8%)</td>
</tr>
<tr>
<td>Gas</td>
<td>$3.14 $5.69</td>
<td>10.6% 18.4%</td>
<td>7.9%</td>
</tr>
<tr>
<td>VOM</td>
<td>$1.08 $0.91</td>
<td>3.6% 3.0%</td>
<td>(0.7%)</td>
</tr>
<tr>
<td>NO\textsubscript{2}</td>
<td>$0.23 $0.26</td>
<td>0.8% 0.9%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Dispatchable Transaction</td>
<td>$1.61 $1.00</td>
<td>5.4% 0.3%</td>
<td>(5.1%)</td>
</tr>
<tr>
<td>DASR LOC Adder</td>
<td>($0.15) ($0.06)</td>
<td>(0.5%) 0.2%</td>
<td>0.7%</td>
</tr>
<tr>
<td>Oil</td>
<td>$0.03 $0.05</td>
<td>0.1% 0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>CO\textsubscript{2}</td>
<td>$0.06 $0.05</td>
<td>0.2% 0.2%</td>
<td>(0.0%)</td>
</tr>
<tr>
<td>Ten Percent Cost Adder</td>
<td>$0.02 $0.03</td>
<td>0.1% 0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>SO\textsubscript{2}</td>
<td>$0.05 $0.03</td>
<td>0.2% 0.1%</td>
<td>(0.1%)</td>
</tr>
<tr>
<td>DASR Offer Adder</td>
<td>($0.02) ($0.01)</td>
<td>(0.1%) 0.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Other</td>
<td>$0.12 $0.01</td>
<td>0.4% 0.0%</td>
<td>(0.4%)</td>
</tr>
<tr>
<td>Constrained Off</td>
<td>$0.00 $0.00</td>
<td>0.0% 0.0%</td>
<td>(0.0%)</td>
</tr>
<tr>
<td>Uranium</td>
<td>$0.00 $0.00</td>
<td>0.0% 0.0%</td>
<td>(0.0%)</td>
</tr>
<tr>
<td>Wind</td>
<td>($0.01) ($0.00)</td>
<td>(0.0%) 0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Price Sensitive Demand</td>
<td>$0.03 $0.00</td>
<td>0.1% 0.0%</td>
<td>(0.1%)</td>
</tr>
<tr>
<td>NA</td>
<td>$0.01 $0.00</td>
<td>0.0% 0.0%</td>
<td>(0.0%)</td>
</tr>
<tr>
<td>Total</td>
<td>$29.68 $30.85</td>
<td>100.0% 100.0%</td>
<td>(0.0%)</td>
</tr>
</tbody>
</table>

- DEC bids contributed about 25.2% of LMP in recent years
- INC offers contributed about 20.0% of LMP in recent years

Note: Using identified marginal resource offers and the components of unit offers, it is possible to decompose PJM system LMP using the components of unit offers.
Virtual transaction: Conflicting opinions

Researchers disagree regarding the impact of virtual transaction in the electricity market

**Positive view of virtual transactions**

- The introduction of explicit virtual bidding significantly reduced the transactions costs associated with attempting to profit from differences between the day-ahead and real-time market prices.
- In addition, these results demonstrate economically significant benefits associated with the introduction of virtual bidding.
  – Jha and Wolak (Stanford, 2013)

**Negative view of virtual transactions**

- The introduction of virtual bidders is analogous to the classical form of biological control in which an animal is introduced into an ecosystem to help manage pests.
- Virtual bids do not improve the performance of the system. Therefore, any profit captured by virtual bidders is purely parasitic.
  – Parsons et al. (MIT, 2015)
Virtual transaction: Price convergence

Monitoring Analytics, external market monitoring institution for PJM market, mentioned that price convergence cannot be a measure for market efficiency.

- The degree of convergence, by itself, is not a measure of the competitiveness or effectiveness of the Day-Ahead Energy Market.
  - Convergence is not the goal of virtual trading, but it is a possible outcome.

- Price convergence does not necessarily mean a zero or even a very small difference in prices between Day-Ahead and Real-Time Energy Markets.
  - It is not a realistic expectation for the convergence in the sense that day-ahead and real-time prices are equal at individual buses or aggregates on a day to day basis.

- Where arbitrage incentives are created by systematic modeling differences between day-ahead and real-time energy market, virtual bids and offers cannot result in more efficient market outcomes.
  - Such offers may be profitable but cannot change the underlying reason for the price difference. The virtual transactions will continue to profit from the activity for that reason.
Past Studies

Several past studies suggest that virtual transactions increase electricity market efficiency with ‘Price Convergence’ as a measure. Price Convergence may not increase welfare of market participants in all cases.

Market Efficiency
- PJM
- MISO
- CAISO

Price Convergence
- Social welfare
- Welfare of market participants
- Giraldo et al.: analysis without a power network

Welfare analysis
- Financial Transmission Right

Virtual Transaction
- Market Manipulation

Financial Product
- Unit Commitment
- Hourly vs 5 min settlement

Market Performance
This Study

This study of welfare impacts of virtual bidding integrates power network with market data and transmission line constraints governed by physical laws.

<table>
<thead>
<tr>
<th>Past Study</th>
<th>This Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>No network case</td>
<td>• Numerical solution</td>
</tr>
<tr>
<td>Power network</td>
<td>• Closed form solution</td>
</tr>
<tr>
<td>Market data</td>
<td>• Transmission line congestion</td>
</tr>
<tr>
<td></td>
<td>• Physical laws governing power flow</td>
</tr>
<tr>
<td></td>
<td>• ISO-NE test case data</td>
</tr>
</tbody>
</table>

- No Network
- Arbitrary Data
- Numerical solution
- N/A
- N/A
- N/A
- ISO-NE test case data
Problem statements

Fundamental objective

• Understand the effect of virtual transactions on electricity market efficiency using a power network model

Research question

• What is the impact of network congestion on welfare shifts caused by the participation of financial virtual traders?

Hypotheses

• $H_0^1$: Having congestion in the network creates no differences in optimal bidding strategy of the financial trader and its welfare impact on market participants

• $H_0^2$: Having optimal virtual bidding in the congested network creates homogeneous welfare impacts across the network
Major findings

We derive major findings of welfare impact on market participants in the case of zero expected demand deviation in various network cases

- Virtual trader makes profit
- Consumer welfare generally decreases
- Producer welfare generally increases

Welfare impacts on market participants by introducing optimal virtual bidding*

- Congestion in the power network amplifies welfare change in the network
  - Consumers lose more
  - Producers gain more
  - Virtual traders gain more

Impact of congested network on the welfare of market participants

- Location relative to a congested transmission line alters the magnitude of welfare impact of market participants
  - Source vs Sink

Locational differences in the welfare impact on market participants

* Note: When the difference in supply slopes in DA and RT is not too large
### Experimental design

<table>
<thead>
<tr>
<th></th>
<th>Uncongested</th>
<th>Congested</th>
<th>Loop flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed form</td>
<td>No network</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Sandbox model</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Three-bus</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>ISO/RTO network</td>
<td>ISO-NE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Closed form solution approach**
- Use single bus case to derive closed form solution for the optimal bidding strategy and its expected welfare impact

**Sandbox approach**
- Analyze two- and three-bus networks using simulation to estimate the welfare impacts of the introduction of virtual transactions in a congested network

**ISO/RTO network approach**
- Use the ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion (2016) as basis for the analysis of a complicated electricity network
No network case: Optimal bidding strategy and expected welfare changes of market participants

Algebraic analysis allows derivation of optimal virtual bidding strategies in specific market environment and determination of expected sign of welfare changes of market participants

- Critical point of expected demand deviation determines the optimal bidding strategy
- Type of virtual products (DECs and INCs) have consistent impact on welfare of market participants
- The overall results are consistent with past numerical analysis (Giraldo et al.)

<table>
<thead>
<tr>
<th>Economic Welfare</th>
<th>$E[\Delta]&lt;\delta^*$</th>
<th>$E[\Delta]=0$</th>
<th>$E[\Delta]&gt;\delta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual</td>
<td>Positive</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>Consumer</td>
<td>Increase</td>
<td>Decrease</td>
<td>Decrease</td>
</tr>
<tr>
<td>Producer</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
</tr>
</tbody>
</table>

**Expected demand deviation**

- **Negative**
- **Zero**
- **Positive**

<table>
<thead>
<tr>
<th><strong>Optimal bidding strategy</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual supply (INC)</td>
</tr>
<tr>
<td>Virtual demand (DEC)</td>
</tr>
</tbody>
</table>

© James Hyungkwan Kim 2018. All rights reserved
Two-bus network

- Bus 1: Load, Generation units
- Bus 2: Load, Generation units

- Fix
  - Demand Uncertainty
  - Expected demand deviation
  - Slopes of supply function

- Change
  - Line capacity

### Two-bus model: Load data

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load (MW)</th>
<th>Distribution of deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>[-10,10]</td>
</tr>
</tbody>
</table>

* Uniform distribution represents the uncertainty of the load deviations. The realization of bus demand is independent to the other bus

### Two-bus model: Generation unit data

<table>
<thead>
<tr>
<th>Bus</th>
<th>DA inverse supply curve parameter (a1)</th>
<th>DA supply curve slope (b1)</th>
<th>RT supply curve slope (b2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Two-bus model: Line data

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>Reactance</th>
<th>Loss</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.1</td>
<td>0</td>
<td>1-unlimited</td>
</tr>
</tbody>
</table>

* Line capacity will be adjusted: uncongested - 95 MW, congest - below 75 MW, partially congested – between 75 and 95 MW. The power flow over line 1-2 without virtual transaction is 86 MW
Three-bus network

Split load version

- Split load version: Split the load at bus 1 in two-bus model between bus 1 and bus 3 in three-bus model
  - Bus 1: Load, Generation units
  - Bus 2: Load, Generation units
  - Bus 3: Load

### Load data

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load (MW)</th>
<th>Distribution of deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200 × 0.8</td>
<td>[-10,10] × 0.8</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>3</td>
<td>200 × 0.2</td>
<td>[-10,10] × 0.2</td>
</tr>
</tbody>
</table>

### Generation unit data

<table>
<thead>
<tr>
<th>Bus</th>
<th>DA inverse supply curve parameter (a1)</th>
<th>DA supply curve slope (b1)</th>
<th>RT supply curve slope (b2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

### Line data

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>Reactance</th>
<th>Loss</th>
<th>Capacity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.0465</td>
<td>0</td>
<td>1-unlimited</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.1</td>
<td>0</td>
<td>Unlimited</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.1</td>
<td>0</td>
<td>Unlimited</td>
</tr>
</tbody>
</table>

* Line capacity will be adjusted: uncongested - 95 MW, congest - below 75 MW, partially congested – between 75 and 95 MW. The power flow over line 1-2 without virtual transaction is 86 MW.
Three-bus network

Split generation version

- Split generation version: Divide the generation supply curve at bus 1 in two-bus model between bus 1 and bus 3 in three-bus model
  - Bus 1: Load, Generation units
  - Bus 2: Load, Generation units
  - Bus 3: Generation units

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load (MW)</th>
<th>Distribution of deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>3</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus</th>
<th>DA inverse supply curve parameter (a1)</th>
<th>DA supply curve slope (b1)</th>
<th>RT supply curve slope (b2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.95</td>
<td>0.475</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.05</td>
<td>0.025</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>Reactance*</th>
<th>Loss</th>
<th>Capacity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.005</td>
<td>0</td>
<td>1-unlimited</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0.005</td>
<td>0</td>
<td>Unlimited</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>Unlimited</td>
</tr>
</tbody>
</table>

* Line capacity will be adjusted: uncongested - 95 MW, congest – below 75 MW, partially congested – between 75 and 95 MW. The power flow over line 1-2 without virtual transaction is 86 MW.
Bilevel programming in multi-bus model

Bilevel programming reflects leader / follower situation in the multi-settlement electricity market (followers) with financial trader (leader). The common example in economics is a Stackelberg game

Bilevel programming

**Leader: Financial trader**
Objective: Maximize expected profit
s.t.

**Follower 1: System operator (DA)**
Objective: Maximize Welfare
s.t.
Balancing constraints
Power flow constraints

**Follower 2: System operator (RT)**
Objective: Maximize welfare
s.t.
Balancing constraints
Power flow constraints

KKT reformulation: Single level programming

**Leader: Financial trader**
Objective: Maximize expected profit
s.t.

**Follower 1: System operator (DA)**
FOC condition
Balancing constraints
Power flow constraints
Complementary conditions

**Follower 2: System operator (RT)**
FOC condition
Balancing constraints
Power flow constraints
Complementary conditions
Welfare Impact: Sandbox models

In all cases, the optimal virtual bids obtain surplus from its transactions. With the zero expected demand deviation, the consumer loses and the producer gains

<table>
<thead>
<tr>
<th>Welfare impact by virtual with zero expected demand deviation*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Welfare</td>
</tr>
<tr>
<td>Network-wise</td>
</tr>
<tr>
<td>Uncongested (= Single bus)</td>
</tr>
<tr>
<td>Congested (=Two bus)</td>
</tr>
<tr>
<td>Congested with loop flow (=Three bus)</td>
</tr>
</tbody>
</table>

• The uncongested network yields results consistent with the algebraic analysis of the single bus case.

• In both two-bus and three-bus models the expected welfare impact is qualitatively identical to the uncongested case

• The impact of virtual bids in congested two-bus and three-bus cases maintains the directional change in the expected welfare of market participants, which is amplified

* Note: When the difference in supply slopes in DA and RT is not too large
Welfare Impact: Sandbox model simulations

<table>
<thead>
<tr>
<th></th>
<th>Frequency of expected welfare increased due to virtual</th>
<th>Magnitude of expected welfare changed due to virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consumer</td>
<td>Producer</td>
</tr>
<tr>
<td>CV-C: Welfare impact due to virtual in congested network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Source</td>
<td>2.3%</td>
<td>97.5%</td>
</tr>
<tr>
<td>Sink</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Other</td>
<td>0.0%</td>
<td>99.5%</td>
</tr>
<tr>
<td>UV-U: Welfare impact due to virtual in uncongested network</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network</td>
<td>0.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Note: Iteration = 100. Source is a bus located at the start point of the predominant power flow adjacent to congested line. Sink is a bus located at the start point of the predominant power flow adjacent to the congested line.

- Having congestion in the network amplifies the welfare impact
- The welfare impacts are different by location of the market participants relative to the congested line(s) as well as the network topology
  - At the sink buses, consumers and society tend to lose more while the producers tend to gain more relative to the source bus due to the introduction of the optimal virtual bidding
ISO-NE Test System: Introduction

Eight-Zone ISO-NE Test System

Why ISO-NE Test System

- Substantially larger than our sandbox models
- Relatively complex, incorporating a number of loops and making loop flow a potentially important phenomenon
- While highly aggregated, it has a clear basis in the real world
- Unlike many other ISO/RTO-scale systems, the basic model is open source and readily accessible for replication purposes

Krishnamurthy, Li, and Tesfatsion, 2016
Welfare Impact: ISO-NE test case

| Frequency of expected welfare increased due to introducing virtual transactions | Magnitude of expected welfare changed due to introducing virtual transactions |
|---|---|---|---|---|---|---|---|---|
| Consumer | Producer | Social | Virtual | Consumer | Producer | Social | Virtual |
| Network | 0% | 100.0% | 9% | 100.0% | -6,121 | 5,108 | -1,012 | 123 |
| Source | 19% | 78% | 45% | 98% | -523 | 930 | 400 | 7 |
| Sink | 6% | 97% | 17% | 100% | -2,229 | 1,680 | -668 | 67 |
| Both | 12% | 83% | 15% | 98% | -888 | 346 | -577 | 18 |
| Other | 0% | 100% | 9% | 100% | -765 | 730 | -127 | 15 |

CV-C: welfare impacts due to introducing virtual transactions in a congested network

UV-U: impact due to virtual in uncongested network

Note: Iterations = 1000. Feasible cases = 130, Infeasible cases = 865, Uncongested cases = 5, Congested lines = 326. (Average congested lines per case: 2.5) Source and sink are buses adjacent to (next to) each congested line. The source is a bus located at the origin of the predominant power flow adjacent to the congested line. The sink is a bus located at the destination of the predominant power flow adjacent to the congested line. “Both” indicates a bus located as sink and source both. Source buses = 190, Sink buses = 186, Both buses = 69.
Conclusion

- Prices throughout the network in both forward and spot markets are changed, and there are welfare transfers among producers, consumers, and virtual traders.

- Results suggest that price convergence occurs with optimal virtual bidding in most cases, which is consistent with existing literature.

- At the network level, having virtual transaction generally decreases the expected consumer and social welfare while increasing the expected producer welfare.

- Having congestion in the network generally amplifies the expected welfare impact of the virtual transaction, the consumer and society lose more and the producer gain more.

- When the network is congested, the expected welfare impact is heterogeneous and depending on where the market participants are located relative to the congested line.
Limitations and potential future work

Potential outlets for the future work can be conducted to address limitations of current study

<table>
<thead>
<tr>
<th>Current</th>
<th>Potential future work</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Granularity of network topology</strong></td>
<td></td>
</tr>
<tr>
<td>• Aggregated zonal level of ISO-NE test case network</td>
<td>• Granular nodal level of observed data of ISO/RTOs with physical characteristics</td>
</tr>
<tr>
<td><strong>Congestion study</strong></td>
<td></td>
</tr>
<tr>
<td>• Congestion and bidding behavior</td>
<td>• Congestion in the transmission generation planning</td>
</tr>
<tr>
<td></td>
<td>• Congestion and energy storage</td>
</tr>
<tr>
<td><strong>Financial product type</strong></td>
<td></td>
</tr>
<tr>
<td>• Virtual supply/demand</td>
<td>• Mixed with different types of financial products</td>
</tr>
<tr>
<td></td>
<td>• Distributed generations</td>
</tr>
</tbody>
</table>
End of Document
Supply curve with and without Virtual

**DA and RT Supply without Virtual**

- **Deterministic**
- **DA demand** (d1) = **RT demand** (d2)
- **Without Virtual**

**RT supply with and without Virtual DEC bid**

- **Stochastic**
- **DA demand** (d1) = **Expected RT demand**
- **With Virtual DEC bid**
Physical laws in the electricity network

Multi-bus network with closed loop is governed by physical laws that manage power flow over the transmission lines.

Loop flow and inverse reactance

**Loop Flow**
- Power moves between buses of the network, it does not follow the shortest delivery path.
- Power flow follows every available parallel path between the injection and withdrawal buses.

**Inverse Reactance**
- Share of power that flows along each path is inversely proportional to the relative reactance of such a path.
- The share is called the Power Transmission Distribution Factor (PTDF).

**Illustrative example**

1. **Loop Flow**
   - 100 MW power moves between buses 1 and 3, it does not follow the shortest delivery path.
   - Power flow follows every available parallel path between the injection and withdrawal buses.

2. **Inverse Reactance**
   - Share of power that flows along each path is inversely proportional to the relative reactance of such a path.
   - The share is called the Power Transmission Distribution Factor (PTDF).

3. **PTDF Calculation**
   - PTDF\(_{1-3}\) = \( \frac{R_{1-2} + R_{2-3}}{R_{1-2} + R_{2-3} + R_{1-3}} \) = \( \frac{2R}{3R} \) = \( \frac{2}{3} \)
   - PTDF\(_{1-2,2-3}\) = \( \frac{R_{1-3}}{R_{1-2} + R_{2-3} + R_{1-3}} \) = \( \frac{1R}{3R} \) = \( \frac{1}{3} \)
Experimental design: Sandbox model

<table>
<thead>
<tr>
<th>Heterogeneous Impact</th>
<th>Congested</th>
<th>Uncongested</th>
<th>Impact of congestion</th>
<th>Impact of virtuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two bus</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three bus</td>
<td>Virtual vs No Virtual</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Analyze two- and three-bus networks using simulation to estimate the welfare impacts of the introduction of virtual transactions in a congested network.
  - The two-bus network is the simplest network that includes transmission.
  - The three-bus network is the simplest network that can incorporate a loop, and hence where Kirchhoff’s Laws governing loop flows in an electrical network apply

- Compare the welfare impact between virtual and no virtual cases to estimate the impact of introducing optimal virtual bidding in the network

- Compare the welfare impact between different buses in the network to estimate the heterogeneous welfare impact over the network
ISO-NE Test System

- Real electricity networks are far more complicated than our sandbox models, with more buses and transmission lines and many possible patterns of congestion.
- The financial trader needs to design an optimal bidding strategy for virtual products to maximize the profit out of the complexities of the network and the potential for congestion from a holistic perspective.
- The optimal bidding strategy in the more complicated network may involve multiple bids at alternative nodes, and hence, the overall welfare impact will likely be different.
- The principle question at hand is whether the lessons regarding virtual trader behavior and the impacts on the market participant welfare extend from the simple networks to the larger, more complex network. To facilitate addressing this question, we employ a stylized version of the network of the ISO-NE in a simulation context under alternative loadings.
Welfare Impact: Averages across simulations and comparison

Welfare impacts of virtual and congested network

Two- and Three Bus Simulation

ISO-NE Test Case Simulation

Heterogeneous welfare impacts of virtual in congested network

Two- and Three Bus Simulation

ISO-NE Test Case Simulation
Virtual bids and cleared by type of parent organization

Majority of market participants bidding virtual products are financial organization. In fact, it is a lower boundary considering financial arms under physical companies (e.g. BETM under NRG is considered physical)

<table>
<thead>
<tr>
<th>Category</th>
<th>Total Virtual Bid MWh</th>
<th>Total Virtual Cleared MWh</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent</td>
<td>Percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial</td>
<td>85,003,436</td>
<td>31,248,242</td>
<td>84,330,236</td>
<td>33,711,497</td>
</tr>
<tr>
<td></td>
<td>54.8%</td>
<td>35.5%</td>
<td>60.8%</td>
<td>44.8%</td>
</tr>
<tr>
<td>Physical</td>
<td>70,131,528</td>
<td>56,700,299</td>
<td>54,433,944</td>
<td>41,594,291</td>
</tr>
<tr>
<td></td>
<td>45.2%</td>
<td>64.5%</td>
<td>39.2%</td>
<td>55.2%</td>
</tr>
<tr>
<td>Total</td>
<td>155,134,964</td>
<td>87,948,540</td>
<td>138,764,180</td>
<td>75,305,788</td>
</tr>
<tr>
<td></td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

PJM INC and DEC bids

<table>
<thead>
<tr>
<th>Category</th>
<th>Total Virtual Bid MWh</th>
<th>Total Virtual Cleared MWh</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent</td>
<td>Percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial</td>
<td>1,198,418,888</td>
<td>282,808,931</td>
<td>1,180,634,460</td>
<td>293,713,948</td>
</tr>
<tr>
<td></td>
<td>96.0%</td>
<td>93.6%</td>
<td>98.1%</td>
<td>96.0%</td>
</tr>
<tr>
<td>Physical</td>
<td>49,564,960</td>
<td>19,231,146</td>
<td>23,152,092</td>
<td>12,250,315</td>
</tr>
<tr>
<td></td>
<td>4.0%</td>
<td>6.4%</td>
<td>1.9%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Total</td>
<td>1,247,983,848</td>
<td>302,040,077</td>
<td>1,203,786,552</td>
<td>305,964,263</td>
</tr>
<tr>
<td></td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

PJM UTC bids

<table>
<thead>
<tr>
<th>Category</th>
<th>Total Up to Congestion Bid MWh</th>
<th>Total Up to Congestion Cleared MWh</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Percent</td>
<td>Percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial</td>
<td>1,180,634,460</td>
<td>293,713,948</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>98.1%</td>
<td>96.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physical</td>
<td>23,152,092</td>
<td>12,250,315</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9%</td>
<td>4.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,203,786,552</td>
<td>305,964,263</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100.0%</td>
<td>100.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Monitoring Analytics, 2016 and 2017
Real-time hourly LMP minus day-ahead hourly LMP in PJM

Expected demand deviation in real time market is close to zero
Welfare calculation when RT demand is identical to DA demand with DECs

- It is the case when DA demand is identical to RT demand with DECs. Without the financial trader, the electricity market settles as single spot market where \(d_1=d_2=g_1=g_2\). Hence the consumer surplus is \(A+B\), and the producer surplus is \(C\). With the financial trader, the consumer surplus is \(A\), the producer surplus is \(B+C+F+G+H\) and financial trader loss is \(F+G+H\). When DA demand and RT demand are identical with INCs, consumer surplus decreases, producer surplus increases and financial trader surplus becomes negative.

- Calculating economic surplus without DECs, DA payment by the consumer is \(d_1\cdot LMP_1\), the area of \(C+D\), RT balance payment by the consumer is none, since \(d_2\) is equivalent to \(d_1\), and consumer value is \(d_2\cdot RP\), the area of \(A+B+C+D\). The total consumer surplus is the area of \(A+B\), \(-(C+D)+(A+B+C+D)\). DA payment to the producer is \(g_1\cdot LMP_1\), the area of \(C+D\), RT balance payment to the producer is none, since \(g_2\) is equivalent to \(g_1\), and the generation cost is under the area of supply curve until \(d_2\), \(D\). The total producer surplus is the area of \(C\), \((C+D)-(D)\).

- Based on the changed generation and price by DECs, DA payment by the consumer is \(d_1\cdot LMP_1\) \(\prime\), the area of \(B+C+D\), RT balance payment by the consumer is \((d_1-d_2)\cdot LMP_2\), which is none since the \(d_2\) is equivalent to \(d_1\), and consumer value is \(d_2\cdot RP\), the area of \(A+B+C+D\). The total consumer surplus is the area of \(A\), \(-(B+C+D)+(A+B+C+D)\). DA payment to producer is \(g_1\cdot LMP_1\) \(\prime\), the area of \(B+C+D+F+G+H+I\), RT balance payment to producer is \(-(g_1\cdot g_2)\cdot LMP_2\), the area of \(H\), which is negative when \(g_2\) is less than \(g_1\), and the generation cost is under the area of RT supply curve until \(d_2\), \(D\). The total producer surplus is the area of \(B+C+F+G+H\), \((B+C+D+F+G+H+I)-(I)-(D)\).

- The financial trader surplus is \(F+G\), which is also negative in this case. DA payment to the financial trader is INCs\(\cdot LMP_1\) \(\prime\), the area of \(F+G+H\). RT payment by the financial trader is INCs\(\cdot LMP_2\), the area of \(H\). DECs are settled as they bid virtual demand. The total financial trader surplus is the area of \(F+G+H\), \(-(F+G+H+I)+(I)\) which is negative. Consequently, bidding DECs strategy would not be favored by financial trader expecting RT demand is identical to DA demand.
ACOPF and DCOPF

• The Alternating Current Optimal Power Flow (ACOPF) is an optimization problem that models how electricity flows across the electric grid following Kirchhoff’s laws, and it is used to economically dispatch generation in real time in the least cost manner. The ACOPF optimization problem is, however, a tough problem to solve because it is a nonlinear, non-convex optimization problem with constraints that include trigonometric functions. The non-linearity that these equations create significantly complicates the optimization problem (Hedman, Oren, and O’Neill 2011). Thus, it is common to use a linear approximation of the ACOPF problem.

• The most common linear approximation of the ACOPF problem is known as the Direct Current Optimal Power Flow (DCOPF) problem. The DCOPF problem uses a simplification of the physical attributes of an electricity network that complicate the ACOPF problem. The linearization is based on the assumption that resistance and line losses are insignificant, the per-unit voltage magnitude at each bus is 1 p.u, and uses linear approximations to the sine and cosine functions about the point where the angle, θ, is equal to zero with cos(θ) = 1 and sin(θ) = θ respectively. With these approximations, the DCOPF problem is a linear program, resulting in a problem that is much easier to solve than the non-convex ACOPF problem.

• Formulations of the DCOPF problem often eliminate the phase angle variables and express the network constraints following Kirchhoff’s laws regarding Power Transfer Distribution Factors, or PTDFs (Hedman, Oren, and O’Neill 2011). The PTDF formulation models flow on lines based on power injections and withdrawals at each bus.

• The optimal power flow method solves the system operator’s market welfare maximization problem using Direct Current Optimal Power Flow (DCOPF) with the physical constraints such as Kirchhoff’s laws, line constraints and energy balancing. Following the general practice of academia and industry, this study adopts Direct Current Optimal Power Flow (DCOPF) formulation for modeling the DA and RT markets.

• It is a convention to express voltage magnitude in per-unit (p.u) terms that indicates the local value of specific bus relative to the nominal value. For example, when we set 100kV equals 1.00 p.u, the voltage magnitude at Bus 1 can be 1.02 p.u that is 102kV.
In order to generate random cases to run the simulation, we use the two-bus network for the base case to generate random parameters to create a diverse set of network configurations. First, the random scalers from the uniform distribution of [0.3, 3] will multiply the loads in the bus 1 and bus 2, respectively (Load ('1') = 200 * Uniform[0.3, 3], Load ('2') = 200 * Uniform[0.3, 3]). The MW of the load in bus 1 can be greater or lesser than the MW of the load in bus 2. After generating random load, we run the case without virtual trader and without line capacity limits, which is an uncongested case and measure the power flow on the line from bus 1 to bus 2.

Based on the generated specification of the two-bus network, three versions of three-buses are generated: load divide, generation unit divide and load and generation unit divide. The load divide version is generated by separating the load at bus 1. The load in bus 1 becomes the loads in bus 1 multiplied the random scaler from the uniform distribution of [0.1, 1] (Load('1')_LoadDivide = Load ('1') * uniform[0.1,1]). The load in bus 3 is generated by subtracting the random scaler from 1 and multiply it to the load in bus 1 (Load('3')_LoadDivide = Load ('1') * (1-uniform[0.1,1]))

The generation divide version is based on the generated specification of the two-bus network. The generation units in bus 1 are separated into bus 1 and bus 3. The supply function slope in DA in bus 1 in the generation unit divide model becomes the supply function slope in DA in bus 1 in the two-bus model multiply the random scaler from the uniform distribution of [0.1, 1] (b1('1')_GenDivide = b1 ('1') * uniform[0.1,1]). The supply function slope in DA in bus 1 in the gentian unit divide model becomes the subtracting the random scaler from 1 and multiply the supply function slope in DA in bus 1 in the two-bus model (b1('3')_GenDivide = b1 ('1') * (1-uniform[0.1,1])).

The load and generation divide three-bus version is generated by combining the load divide and generation divide version. Hence, the one case from the two-bus model and three cases from the three-bus model, becomes a set network cases are generated to be compatible with each other. The market outcomes, such as price and the overall MW of virtual bids in the network, and the expected welfare impacts are identical among the four compatible cases when uncongested.

The magnitude of line reactance in the generated three-bus network cases is adjusted so that they can have the same MW of power flow from bus 1 to bus 2 in Day-Ahead market without virtual trader. The congested case is defined by setting the line capacity as 80% of power flow on line from 1 bus to bus 2 in Day-Ahead market without virtual trader and congested in Real-Time market.
We generate multiplicative scale factors independently for each zone and transmission capacity so they can be adjusted independently. First, the random scalars from a uniform distribution on [0.8, 1.2] are generated independently and multiplied by the loads in the 8 zones. The multiplicative factors are applied to the entire empirical distribution of RT loads and then the mean of the distribution is calculated to serve as the DA load. The MW of the load distribution at any bus can be greater or less than the MW of the base case.

After generating random loads, we solve the DA and RT dispatch problems without virtual trading and without line capacity limits. This gives results for an uncongested case, and provides DA and RT generation and power flows as well as the expected welfare of the consumer, producer, virtual and society taking into account the DA market and the RT market. Using this information, the congested cases are generated by randomly generating line capacity.

We generate the random multiplicative factors from the uniform distribution of [0.5, 2]. Then, multiply the generated factors independently to the base line capacity, MW of power flow in DA with the virtual transaction in the uncongested network. If one or more lines are congested in RT with virtual and the optimization problem is feasible, we measure the expected welfare impacts for consumers and generators in both the uncongested and the congested cases by comparing expected welfare between introducing optimal virtual bids and without virtual bids. If the DA or any of the RT dispatch problems is infeasible or the network is not congested in any of the RT dispatch problems with the adjusted line capacities, the case is omitted from the simulation results.
Two-Bus (No loop flow) and Three Bus (Loop flow) simulation

To compare the impact of having loop flow in the network, we demonstrate the case of two-bus, without loop flow and three-bus, with loop flow

<table>
<thead>
<tr>
<th></th>
<th>Consumer</th>
<th>Producer</th>
<th>Social</th>
<th>Virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV-C: Welfare impact due to virtual in a congested network</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two</td>
<td>-2103</td>
<td>439</td>
<td>-1664</td>
<td>15</td>
</tr>
<tr>
<td>Three: Gen divide</td>
<td>-2123</td>
<td>444</td>
<td>-1679</td>
<td>15</td>
</tr>
<tr>
<td>Three: Load divide</td>
<td>-2256</td>
<td>503</td>
<td>-1753</td>
<td>13</td>
</tr>
<tr>
<td>Three: Gen + Load divide</td>
<td>-2123</td>
<td>473</td>
<td>-1651</td>
<td>14</td>
</tr>
</tbody>
</table>

The magnitude of expected welfare changed due to virtual

: overall network
Pasting RT inverse supply function

Let

- \( g_{DA} \) = DA generation
- \( g_{RT} \) = RT generation

\( P_{RT}(g_{RT}) \) = the pasted inverse supply function

\( P_{DA}(g) \) = the DA inverse supply function (\( P_{DA}(0) = 0 \))

\( P_{P}(g) \) = the peaker only inverse supply function (\( P_{P}(0) = 0 \))

\[
P_{RT}(g_{RT}) = P_{DA}[\min(g_{RT},g_{DA})]
+ P_{P}[\max(g_{RT}-g_{DA},0) + \max(g_{DA}-C_{B},0)]
- P_{P}[\max(g_{DA}-C_{B},0)]
\]

If \( g_{RT} < g_{DA} \) then \( P_{RT} = P_{DA} \).

If \( g_{RT} > g_{DA} < C_{B} \) then \( P_{RT} = P_{DA}(g_{DA}) + P_{P}[g_{RT}-g_{DA}] \).

If \( g_{RT} > g_{DA} > C_{B} \) then \( P_{RT} = P_{DA}(g_{DA}) + P_{P}[g_{RT}-C_{B}] - P_{P}[[g_{DA}-C_{B}] \).

Note that as \( g_{RT} \) approaches \( g_{DA} \), \( P_{RT} \) is continuous in every case.
Consumer and producer welfare change and slopes

Certain extreme sets of parameters for DA and RT slope relative to the DA demand and the range of uncertainty may change the qualitative welfare impact of market participants due to the different functional form.

**Consumer welfare change**

- Optimal virtual bid decreases consumer welfare with zero expected demand deviation when the slope in RT is not substantially steeper than the slope in DA, \( \frac{b_1 - b_2}{b_2} < \frac{18d_1}{5\alpha} \), rearranged as \( \frac{b_2}{b_1} > \frac{5x}{1800+5x} \), when \( \alpha = x\% \) of \( d_1 \).

**Producer welfare change**

- Optimal virtual bid increases producer welfare with zero expected demand deviation when the slope in RT is not substantially steeper than the slope in DA, \( \frac{b_1 - b_2}{b_2} < \frac{9d_1}{2\alpha} \), rearranged as \( \frac{b_2}{b_1} > \frac{2x}{900+2x} \), when \( \alpha = x\% \) of \( d_1 \).
Components for Multi-bus model configuration

Out of interested components to configure market environment, we selected moving components to be used in the stylized multi bus model to have preliminary results

**Moving components**
- Range of demand uncertainty
- Transmission Line capacity
- Expected demand deviation

**Reserved moving components**
- Number of financial trader: Single trader
  - Regional heterogeneity of the competitiveness of virtual bids
  - PJM has 11,000 locations/24hr to bid
- Expected demand deviation: Zero
  - Ideal scenario for system operator
  - Financial trader have expected profit

**Used moving components**
- Range of demand and uncertainty
- Transmission line capacity (congestion)
ISO-NE test case data

Transmission Line Benchmark Values for the ISO-NE Test System

<table>
<thead>
<tr>
<th>From Zone</th>
<th>To Zone</th>
<th>Resistance (ohms)</th>
<th>Reactance (per unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>NH</td>
<td>19.09</td>
<td>0.05</td>
</tr>
<tr>
<td>VT</td>
<td>NH</td>
<td>16.6</td>
<td>0.04</td>
</tr>
<tr>
<td>VT</td>
<td>WCMA</td>
<td>24.9</td>
<td>0.06</td>
</tr>
<tr>
<td>WCMA</td>
<td>NH</td>
<td>14.28</td>
<td>0.03</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>WCMA</td>
<td>13.28</td>
<td>0.03</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>NH</td>
<td>10.46</td>
<td>0.02</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>SEMA</td>
<td>4.98</td>
<td>0.01</td>
</tr>
<tr>
<td>WCMA</td>
<td>CT</td>
<td>4.98</td>
<td>0.01</td>
</tr>
<tr>
<td>WCMA</td>
<td>RI</td>
<td>10.79</td>
<td>0.03</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>RI</td>
<td>6.64</td>
<td>0.02</td>
</tr>
<tr>
<td>CT</td>
<td>RI</td>
<td>10.62</td>
<td>0.03</td>
</tr>
<tr>
<td>SEMA</td>
<td>RI</td>
<td>3.32</td>
<td>0.01</td>
</tr>
</tbody>
</table>
## ISO-NE test case data

### Load Data Summary

<table>
<thead>
<tr>
<th>Name</th>
<th>Avg</th>
<th>S.D</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>1,343</td>
<td>87</td>
<td>1,172</td>
<td>1,521</td>
</tr>
<tr>
<td>WCMA</td>
<td>1,337</td>
<td>102</td>
<td>1,090</td>
<td>1,609</td>
</tr>
<tr>
<td>VT</td>
<td>683</td>
<td>42</td>
<td>598</td>
<td>786</td>
</tr>
<tr>
<td>SEMA</td>
<td>3,715</td>
<td>294</td>
<td>2,870</td>
<td>4,493</td>
</tr>
<tr>
<td>RI</td>
<td>923</td>
<td>74</td>
<td>707</td>
<td>1,107</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>1,730</td>
<td>137</td>
<td>1,353</td>
<td>2,173</td>
</tr>
<tr>
<td>NH</td>
<td>2,068</td>
<td>174</td>
<td>1,576</td>
<td>2,535</td>
</tr>
<tr>
<td>ME</td>
<td>2,855</td>
<td>221</td>
<td>2,291</td>
<td>3,541</td>
</tr>
</tbody>
</table>
## ISO-NE test case data

### Generator Capacity by Type and Zone

<table>
<thead>
<tr>
<th>Zone</th>
<th>Capacity (MW)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Base</td>
<td>Peaker</td>
</tr>
<tr>
<td>CT</td>
<td>5,694.6</td>
<td>2,860.4</td>
<td>2,834.2</td>
</tr>
<tr>
<td>WCMA</td>
<td>1,227.7</td>
<td>144.4</td>
<td>1,083.3</td>
</tr>
<tr>
<td>VT</td>
<td>620.2</td>
<td>620.2</td>
<td>0.0</td>
</tr>
<tr>
<td>SEMA</td>
<td>2,962.7</td>
<td>684.7</td>
<td>2,278.0</td>
</tr>
<tr>
<td>RI</td>
<td>5,026.1</td>
<td>1,099.5</td>
<td>3,926.6</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>NH</td>
<td>2,247.6</td>
<td>1,339.4</td>
<td>908.2</td>
</tr>
<tr>
<td>ME</td>
<td>5,321.4</td>
<td>311.8</td>
<td>5,009.6</td>
</tr>
</tbody>
</table>
## ISO-NE test case data

### Generator Supply Functions and Estimated Parameters by Type and Zone

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter_a</th>
<th>Parameter_b</th>
<th>Type</th>
<th>Parameter_a</th>
<th>Parameter_b</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>Exponential</td>
<td>5,715</td>
<td>0.0008</td>
<td>Quadratic</td>
<td>42</td>
</tr>
<tr>
<td>WCMASH</td>
<td>Exponential</td>
<td>13,061</td>
<td>0.0025</td>
<td>Exponential</td>
<td>22,917</td>
</tr>
<tr>
<td>VT</td>
<td>Quadratic</td>
<td>11</td>
<td>0.0002</td>
<td>N/A</td>
<td>0</td>
</tr>
<tr>
<td>SEMASS</td>
<td>Exponential</td>
<td>13,174</td>
<td>0.0011</td>
<td>Quadratic</td>
<td>14</td>
</tr>
<tr>
<td>RI</td>
<td>Exponential</td>
<td>5,274</td>
<td>0.0009</td>
<td>Exponential</td>
<td>7,749</td>
</tr>
<tr>
<td>NEMA/BOST</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>NH</td>
<td>Exponential</td>
<td>5,037</td>
<td>0.0010</td>
<td>Quadratic</td>
<td>14</td>
</tr>
<tr>
<td>ME</td>
<td>Exponential</td>
<td>18,294</td>
<td>0.0007</td>
<td>Exponential</td>
<td>22,301</td>
</tr>
</tbody>
</table>
Bilevel formulation: sandbox models

Upper-level: Virtual trader

\[
\begin{align*}
\text{max} & \quad \sum_s \sum_b \text{Prob}(s) \cdot V_b (\text{LMP1}_b - \text{LMP2}_{bs}) \\
\text{LMP1}_b &= \lambda_1 - \sum_i \sum_j \text{PTDF}_{ijb} \left( \pi_{1+}^{ij} - \pi_{1-}^{ij} \right) \quad \forall b \\
\text{LMP2}_{bs} &= \lambda_2 - \sum_i \sum_j \text{PTDF}_{ijb} \left( \pi_{2+}^{ij} - \pi_{2-}^{ij} \right) \quad \forall bs \\
\end{align*}
\]

\(s.t\)

Lower-level: Day-Ahead market

\[
\begin{align*}
\text{max} & \quad \sum_b \text{RP}_b \cdot d_{1b} - \sum_b \int_0^{g_{1b}} \left( -\frac{a_{1b}}{b_{1b}} + \frac{g}{b_{1b}} \right) dg \\
\text{s.t} & \quad \sum_b (g_{1b} - (d_{1b} - \text{V}_b)) \geq 0 \quad \lambda_1 \geq 0 \\
& \quad \sum_b \text{PTDF}_{ijb} (g_{1b} - (d_{1b} - \text{V}_b)) \leq F_{ij}^{\text{max}} \quad \pi_{1+}^{ij} \geq 0 \quad \forall ij \\
& \quad - \sum_b \text{PTDF}_{ijb} (g_{1b} - (d_{1b} - \text{V}_b)) \leq F_{ij}^{\text{max}} \quad \pi_{1-}^{ij} \geq 0 \quad \forall ij \\
\end{align*}
\]

Lower-level: Real-Time market

\[
\begin{align*}
\text{max} & \quad \sum_s \sum_b \text{Prob}(s) \cdot \text{RP}_b \cdot d_{1b} - \sum_s \sum_b \text{Prob}(s) \cdot \left( \int_0^{\min(g_{1b}, g_{2bs})} \left( -\frac{a_{1b}}{b_{1b}} + \frac{g}{b_{1b}} \right) dg + \int_{g_{1b}}^{\max(g_{1b}, g_{2bs})} \left( -\frac{a_{2b}}{b_{2b}} + \frac{g}{b_{2b}} \right) dg \right) \\
\text{s.t} & \quad \sum_b (g_{2bs} - d_{2bs}) \geq 0 \quad \lambda_2 \geq 0 \quad \forall s \\
& \quad \sum_b \text{PTDF}_{ijb} (g_{2bs} - d_{2bs}) \leq F_{ij}^{\text{max}} \quad \pi_{2+}^{ij} \geq 0 \quad \forall ij \\
& \quad - \sum_b \text{PTDF}_{ijb} (g_{2bs} - d_{2bs}) \leq F_{ij}^{\text{max}} \quad \pi_{2-}^{ij} \geq 0 \quad \forall ij \\
\end{align*}
\]
KKT reformulation: sandbox models

Upper-level: Virtual trader
\[
\max \sum_s \sum_b \text{Prob}(s) \cdot V_b (\text{LMP}_1 - \text{LMP}_2)
\]
\[
\text{LMP}_1 = \lambda_1 - \sum_i \sum_j \text{PTDF}_{ijb} (\pi_1^+ - \pi_1^-)
\]
\[
\text{LMP}_2 = \lambda_2 - \sum_i \sum_j \text{PTDF}_{ijb} (\pi_2^+ - \pi_2^-)
\]
\[
\begin{align*}
\text{s.t} \\
\text{Lower-level: Day-Ahead market} \\
- \frac{a_1}{b_1} + \frac{g_1}{b_1} - \lambda_1 + \pi_1^+ \sum_i \sum_j \text{PTDF}_{ijb} - \pi_1^- \sum_i \sum_j \text{PTDF}_{ijb} = 0 \\
- \sum_b (g_1 - (d_1 - V_b)) + S_\lambda_1 = 0 \\
\sum_b \text{PTDF}_{ijb} (g_1 - (d_1 - V_b)) - F_{ij} \max + S_\pi_1^+ = 0 \\
- \sum_b \text{PTDF}_{ijb} (g_1 - (d_1 - V_b)) - F_{ij} \max + S_\pi_1^- = 0 \\
0 \leq S_\lambda_1 \perp \lambda_1 \geq 0 \\
0 \leq S_\pi_1^+ \perp \pi_1^+ \geq 0 \\
0 \leq S_\pi_1^- \perp \pi_1^- \geq 0
\end{align*}
\]
\[
\text{Lower-level: Real-Time market} \\
\max (- \frac{a_1}{b_1} + \frac{g_2}{b_1}, - \frac{a_2}{b_2} + \frac{g_2}{b_2})
\]
\[
\begin{align*}
- \lambda_2 + \pi_2^+ \sum_i \sum_j \text{PTDF}_{ijb} - \pi_2^- \sum_i \sum_j \text{PTDF}_{ijb} = 0 \\
- \sum_b (g_2 - d_2) + S_\lambda_2 = 0 \\
\sum_b \text{PTDF}_{ijb} (g_2 - d_2) - F_{ij} \max + S_\pi_2^+ = 0 \\
- \sum_b \text{PTDF}_{ijb} (g_2 - d_2) - F_{ij} \max + S_\pi_2^- = 0 \\
0 \leq S_\lambda_2 \perp \lambda_2 \geq 0 \\
0 \leq S_\pi_2^+ \perp \pi_2^+ \geq 0 \\
0 \leq S_\pi_2^- \perp \pi_2^- \geq 0
\end{align*}
\]
Bilevel formulation: ISO-NE test case

**Bilevel Formulation**

**Upper-level: Virtual trader**

\[
\text{max } \sum_s \sum_b \text{Prob}(s) \cdot V_b (LMP_{1b} - LMP_{2bs})
\]

\[
LMP_{1b} = \lambda_1 - \sum_i \sum_j \text{PTDF}_{ijb} (\pi_{1^+} - \pi_{1^-}) \quad \forall_b
\]

\[
LMP_{2bs} = \lambda_2 s - \sum_i \sum_j \text{PTDF}_{ijb} (\pi_{2^+} - \pi_{2^-}) \quad \forall_{bs}
\]

s.t

**Lower-level: Day-Ahead market**

\[
\text{max } \sum_b R_P \cdot d_{1b} - \sum_b\int_0^{g_{1b}(aq)} a_{1b(aq)} \cdot g + b_{1b(aq)} \cdot g^2 \, dg - \sum_{b(ae)}\int_0^{g_{1b}(ae)} a_{1b(ae)} (e^{-b_{1b(ae)}} - 1) \, dg
\]

s.t

\[
\sum_b (g_{1b} - (d_{1b} - V_b)) \geq 0 \quad \lambda_1 \geq 0
\]

\[
\sum_b \text{PTDF}_{ijb} (g_{1b} - (d_{1b} - V_b)) \leq F_{ij}^{\text{max}} \quad \pi_{1^+} \geq 0 \quad \forall_{ij}
\]

\[
- \sum_b \text{PTDF}_{ijb} (g_{1b} - (d_{1b} - V_b)) \leq -F_{ij}^{\text{max}} \quad \pi_{1^-} \geq 0 \quad \forall_{ij}
\]

\[
g_{1b} \leq G_b^{\text{max}} \quad \mu_{1^+} \geq 0 \quad \forall_b
\]

\[
-g_{1b} \leq -G_b^{\text{min}} \quad \mu_{1^-} \geq 0 \quad \forall_b
\]
Bilevel formulation: ISO-NE test case

Lower-level: Real-Time market

\[
\begin{align*}
\text{max} & \quad \sum_s \sum_b \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
& - \sum_s \text{Prob}(s) \cdot \left( \sum_{b(aq)} \int_0^{\min(g_1 b(aq), g_2 b(aq))} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \right) \\
& + \sum_{b(ae)} \int_0^{\min(g_1 b(ae), g_2 b(ae))} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
& + \sum_{b(pq)} \int_0^{\max(g_2 b(pq), g_1 b(pq), \text{Base}_{b(pq)})} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
& - \sum_{b(pq)} \int_0^{\max(g_1 b(pq), \text{Base}_{b(pq)})} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
& + \sum_{b(pe)} \int_0^{\max(g_2 b(pe), g_1 b(pe), \text{Base}_{b(pe)})} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
& - \sum_{b(pe)} \int_0^{\max(g_1 b(pe), \text{Base}_{b(pe)})} \text{Prob}(s) \cdot \text{RP}_b \cdot d_1 \nu_b \\
\text{s.t} & \quad \sum_b (g_2 b_s - d_2 b_s) \geq 0 \\
& \quad \sum_b \text{PTDF}_{ijb} (g_2 b_s - d_2 b_s) \leq F_{ij}^{\text{max}} \\
& \quad - \sum_b \text{PTDF}_{ijb} (g_2 b_s - d_2 b_s) \leq F_{ij}^{\text{max}} \\
& \quad g_2 b_s \leq G_b^{\text{max}} \\
& \quad -g_2 b_s \leq -G_b^{\text{min}} \\
\end{align*}
\]

\[
\lambda_{2s} \geq 0 \quad \forall s \\
\pi_{2ijs}^+ \geq 0 \quad \forall ijs \\
\pi_{2ijs}^- \geq 0 \quad \forall ijs \\
\mu_{2bs}^+ \geq 0 \quad \forall bs \\
\mu_{2bs}^- \geq 0 \quad \forall bs
\]
KKT reformulation: ISO-NE test case

**Upper-level: Virtual trader**
max \( \sum_s \sum_b \text{Prob}(s) \cdot V_b (\text{LMP}_1 - \text{LMP}_2) \)
\[
\begin{align*}
\text{LMP}_1 &= \lambda_1 - \sum_i \sum_j \text{PTDF}_{ijb} \left( \pi_{1ij}^+ - \pi_{1ij}^- \right) \\
\text{LMP}_2 &= \lambda_2 - \sum_i \sum_j \text{PTDF}_{ijb} \left( \pi_{2ij}^+ - \pi_{2ij}^- \right)
\end{align*}
\]
\( \forall b \)
\( \forall b \)

s.t

**Lower-level: Day-Ahead market**
\[
\begin{align*}
\text{a}_1 b(aq) \cdot g_1 b + \beta_1 b(aq) \cdot g_1 b(aq)^2 + a_1 b(ae)(e^{g_1 b(ae)} \cdot b_1 b(ae) - 1) \\
- \lambda_1 + \pi_{1ij}^+ \sum_i \sum_j \text{PTDF}_{ijb} - \pi_{1ij}^- \sum_i \sum_j \text{PTDF}_{ijb} + \mu^+_b - \mu^-_b &= 0 \\
- \sum_b (g_1 b - (d_1 b - V_b)) + S_\lambda_1 &= 0 \\
\sum_b \text{PTDF}_{ijb} (g_1 b - (d_1 b - V_b)) - F_{ij}^\text{max} + S_\pi_{1ij}^+ &= 0 \\
- \sum_b \text{PTDF}_{ijb} (g_1 b - (d_1 b - V_b)) - F_{ij}^\text{max} + S_\pi_{1ij}^- &= 0 \\
g_1 b - G_b^\text{max} + S_\mu^+_b &= 0 \\
-g_1 b + G_b^\text{min} + S_\mu^-_b &= 0 \\
0 \leq S_\lambda_1 \perp \lambda_1 \geq 0 \\
0 \leq S_\pi_{1ij}^+ \perp \pi_{1ij}^+ \geq 0 \\
0 \leq S_\pi_{1ij}^- \perp \pi_{1ij}^- \geq 0 \\
0 \leq S_\mu^+_b \perp \mu^+_b \geq 0 \\
0 \leq S_\mu^-_b \perp \mu^-_b \geq 0
\end{align*}
\]
KKT reformulation: ISO-NE test case

**Lower-level: Real-Time market**

\[
\begin{align*}
(a_1 b_{aq}) \cdot \min(g_1 b_{aq}, g_2 b_{aq}s) + b_1 b_{aq} \cdot \min(g_1 b_{aq}, g_2 b_{aq}s)^2 + a_1 b_{ae} \left( e^{\min(g_1 b_{ae}, g_2 b_{ae}s)} \cdot b_{1b(ae)} - 1 \right) + (a_2 b_{pq}) \cdot (\max(g_2 b_{pq}s - g_1 b_{pq}, 0) + \max(g_1 b_{pq} - B_{aseb}(pq), 0)) + \beta_2 b_{pq} \cdot (\max(g_2 b_{pq}s - g_1 b_{pq}, 0) + \max(g_1 b_{pq} - B_{aseb}(pq), 0))^2
\end{align*}
\]

\[
\begin{align*}
-(a_2 b_{pq}) \cdot \max(g_1 b_{pq} - B_{aseb}(pq), 0) + \beta_2 b_{pq} \cdot \max(g_1 b_{pq} - B_{aseb}(pq), 0)^2 + a_2 b_{pe} \left( e^{\max(g_2 b_{pq}s - g_1 b_{pq}, 0) + \max(g_1 b_{pq} - B_{aseb}(pq), 0)} \cdot b_{2b(pe)} - 1 \right) - a_2 b_{pe} \left( e^{\max(g_1 b_{pq} - B_{aseb}(pq), 0)} \cdot \beta_2 b_{pe} - 1 \right)
\end{align*}
\]

\[
\begin{align*}
-\lambda_2 s^+ + \pi_2^+ \sum_i \sum_j PTDF_{ijb} - \pi_i^+ \sum_i \sum_j PTDF_{ijb} + \mu_1^{+} - \mu_1^{-} = 0 & \quad \forall b_s
\end{align*}
\]

\[
\begin{align*}
-\sum_b (g_2 b_{bs} - d_{2b}) + S_2 \lambda_2 s = 0 & \quad \forall s
\end{align*}
\]

\[
\begin{align*}
\sum_b PTDF_{ijb} (g_2 b_{bs} - d_{2b}) - F_{ij}^{max} + S_2 \pi_2^+ = 0 & \quad \forall ij
\end{align*}
\]

\[
\begin{align*}
-\sum_b PTDF_{ijb} (g_2 b_{bs} - d_{2b}) - F_{ij}^{max} + S_2 \pi_2^- = 0 & \quad \forall ij
\end{align*}
\]

\[
\begin{align*}
g_1 b_{bs} - G_{bs}^{max} + S_2 \mu_1^{+} = 0 & \quad \forall b_s
\end{align*}
\]

\[
\begin{align*}
-g_1 b_{bs} + G_{bs}^{min} + S_2 \mu_1^{-} = 0 & \quad \forall b_s
\end{align*}
\]

\[
\begin{align*}
0 \leq S_2 \lambda_2 s & \perp \lambda_2 s \geq 0 & \forall s
\end{align*}
\]

\[
\begin{align*}
0 \leq S_2 \pi_2^{+} & \perp \pi_2^{-} \geq 0 & \forall ij
\end{align*}
\]

\[
\begin{align*}
0 \leq S_2 \pi_2^{-} & \perp \pi_2^{+} \geq 0 & \forall ij
\end{align*}
\]

\[
\begin{align*}
0 \leq S_2 \mu_1^{+} & \perp \mu_1^{-} \geq 0 & \forall b_s
\end{align*}
\]

\[
\begin{align*}
0 \leq S_2 \mu_1^{-} & \perp \mu_1^{+} \geq 0 & \forall b_s
\end{align*}
\]
Notation

Indices and sets

- **b, i, j**: Set of buses in the electricity network
- **b(ae)**: Subset of buses having exponential inverse supply function for all generators
- **b(aq)**: Subset of buses having quadratic inverse supply function for all generators
- **b(pe)**: Subset of buses having exponential inverse supply function for peakers
- **b(pq)**: Subset of buses having quadratic inverse supply function for peakers
- **ij**: Set of connected arc lines in the electricity network
- **s**: Set of positive mass points in the empirical distribution of real-time demand

Parameters

- **d_{1b}**: Demand in Day-Ahead (DA) market at bus b
- **d_{2bs}**: Demand in Real-Time (RT) market at bus b in scenario s
- **\Delta s_{bs}**: Demand deviation in RT market at bus b in scenario s
- **Prob(s)**: Probability of scenario s
- **a_{1b}**: Supply function parameter of aggregated generators in DA market at bus b
- **b_{1b}**: Supply function slope of aggregated generators in DA market at bus b
- **a_{2b}**: Supply function parameter of aggregated generators in RT market at bus b
- **b_{2b}**: Supply function slope of aggregated generators in RT market at bus b
- **PTDF_{ijb}**: Power transfer distribution factor matrix
- **G_{b_{max}}, G_{b_{min}}**: Maximum and minimum capacity of aggregated generators located at bus b
- **F_{ij_{max}}**: Maximum capacity on line ij
- **RP**: Reservation price of consumer demand
- **LMP_{1b}**: Locational marginal price in DA market at bus b
- **LMP_{2bs}**: Locational marginal price in RT market at bus b in scenario s
### Notation

#### Variables

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_b$</td>
<td>Cleared virtual bids on bus b</td>
</tr>
<tr>
<td>$f_{1ij}$</td>
<td>Power flowing in DA market on line ij</td>
</tr>
<tr>
<td>$f_{2ijs}$</td>
<td>Power flowing in RT market on line ij in scenario s</td>
</tr>
<tr>
<td>$g_{1b}$</td>
<td>Power generated in DA market at bus b</td>
</tr>
<tr>
<td>$g_{2bs}$</td>
<td>Power generated in DA market at bus b in scenario s</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Lagrange multiplier of system balance equation in DA</td>
</tr>
<tr>
<td>$\lambda_{2s}$</td>
<td>Lagrange multiplier of system balance equation in RT in scenario s</td>
</tr>
<tr>
<td>$\lambda_{1b}$</td>
<td>Lagrange multiplier of bus balance equation in DA at bus b</td>
</tr>
<tr>
<td>$\lambda_{2bs}$</td>
<td>Lagrange multiplier of bus balance equation in RT at bus b in scenario s</td>
</tr>
<tr>
<td>$\pi_{1ij}^+, \pi_{1ij}^-$</td>
<td>Lagrange multiplier of line capacity equation in DA on line ij</td>
</tr>
<tr>
<td>$\pi_{2ijs}^+, \pi_{2ijs}^-$</td>
<td>Lagrange multiplier of line capacity equation in RT on line ij in scenario s</td>
</tr>
<tr>
<td>$\mu_b^+, \mu_b^-$</td>
<td>Lagrange multiplier of generation limit equation in DA at bus b</td>
</tr>
<tr>
<td>$\mu_{bs}^+, \mu_{bs}^-$</td>
<td>Lagrange multiplier of generation limit equation in DA at bus b in scenario s</td>
</tr>
<tr>
<td>$S_{\lambda 1}$</td>
<td>Slack variable for $\lambda 1$</td>
</tr>
<tr>
<td>$S_{\lambda 2s}$</td>
<td>Slack variable for $\lambda 2s$</td>
</tr>
<tr>
<td>$S_{\lambda 1b}$</td>
<td>Slack variable for $\lambda 1b$</td>
</tr>
<tr>
<td>$S_{\lambda 2bs}$</td>
<td>Slack variable for $\lambda 2bs$</td>
</tr>
<tr>
<td>$S_{\pi 1ij}^+$</td>
<td>Slack variable for $\pi 1ij^+$</td>
</tr>
<tr>
<td>$S_{\pi 1ij}^-$</td>
<td>Slack variable for $\pi 1ij^-$</td>
</tr>
<tr>
<td>$S_{\pi 2ijs}^+$</td>
<td>Slack variable for $\pi 2ijs^+$</td>
</tr>
<tr>
<td>$S_{\pi 2ijs}^-$</td>
<td>Slack variable for $\pi 2ijs^-$</td>
</tr>
<tr>
<td>$S_{\mu 1b}^+$</td>
<td>Slack variable for $\mu 1b^+$</td>
</tr>
<tr>
<td>$S_{\mu 1b}^-$</td>
<td>Slack variable for $\mu 1b^-$</td>
</tr>
<tr>
<td>$S_{\mu 2bs}^+$</td>
<td>Slack variable for $\mu 2bs^+$</td>
</tr>
<tr>
<td>$S_{\mu 2bs}^-$</td>
<td>Slack variable for $\mu 2bs^-$</td>
</tr>
</tbody>
</table>