THE GAME THAT DRIVES THE LNG TRAIN

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1. Introduction

This paper focuses on the game major oil companies play when selecting regasification terminals sites for importing liquefied natural gas (LNG) along coast lines of North America and delivering regasified gas into the North America energy market. This process is complicated and extensive for each participating firms in terms of capital and human investments. Furthermore, fierce competition exists among firms getting LNG cargos and serving demand areas. This paper puts a game-theory lens on the decision making process for firms and explores the strategic elements of the competition.

2. Literature Review

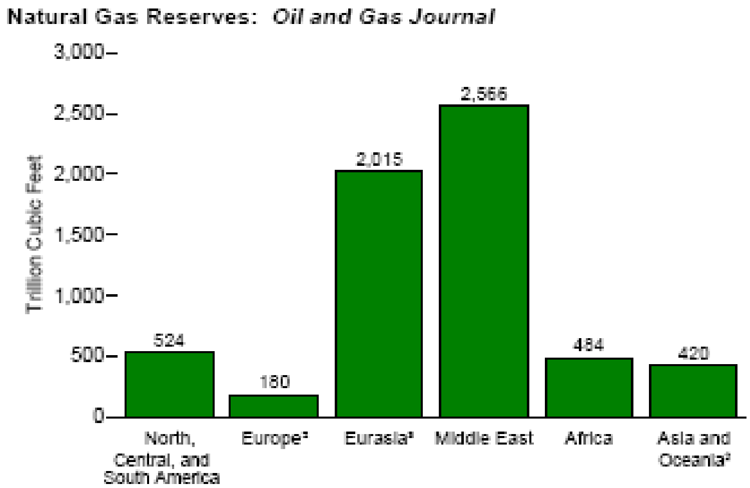
The basis of the model used in this paper was introduced by Bullow et al. [7] who describes a game of two markets and two firms. Farrell and Shaprio [8] studied a game on quantity decisions with one market and n firms where decisions are simultaneous and products are homogenous. Labbe and Hakimi [9] consider a two state location quantity simultaneous game with m markets and two firms with linear demand. Sakar et al. [10] extended the model to a two-stage static and simultaneous game with m markets and n firms in a network. They only considered a case with a fixed number of firms entering in the market, i.e., the quantity offered by each firms is strictly positive. Rhim et al. [11] extended the work by considering free entry (simultaneous and sequential) with symmetric cost capacity and quantity decisions. In these studies, cases applied are small, mostly symmetric and theoretical based only. Saiz and Hendrix [12] extends the studies based on Sarkar et al. and Rhim et al. to a two stage location-quantity game with m markets and n firms in a network space. Free entry is possible and firms can be asymmetric.

This paper extends from the previous works in two aspects: Firms have an outside choice of not entering at all and also are given a second chance to choose a location in a sequential period. There are two periods in the model and each period includes a two stage location-quantity game for all firms. Firms have a choice to choose the location and enter the markets in the first period of time based on the expectation of the future market. Once a firm enters, they cannot relocate. On the other hand, firms that elect to stay out of the first period time given they have a second chance to select the location again in second period. Since location choices in each period are simultaneous, the number of firms entering the market is unknown beforehand.

3. Industry Background

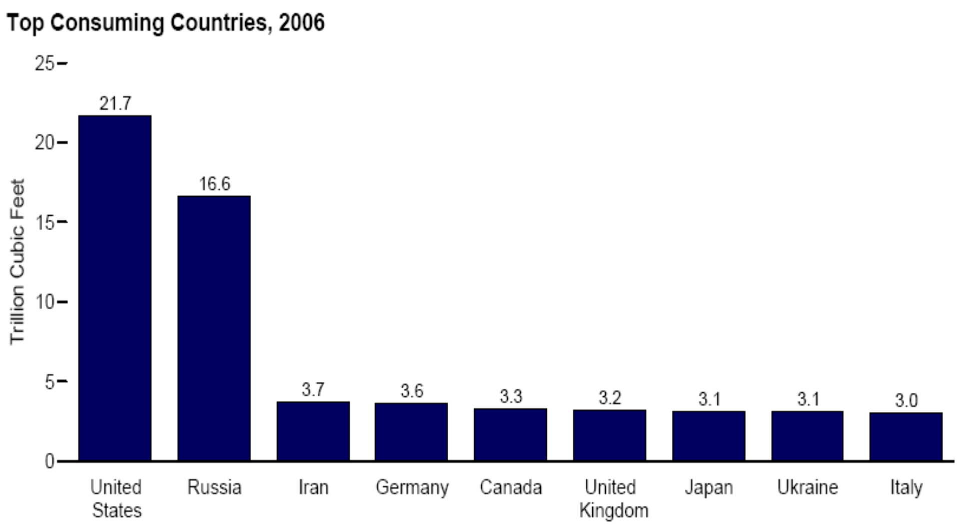
A common misconception about natural gas is that natural gas supplies are quickly depleting. However, this could not be further from the truth. The price spikes in 1970s, more recently in winter of 2000, and even this past summer were not signals indicating that we are depleting natural gas reservoirs. These price spikes were not caused by waning natural gas resources; rather there were the result of other forces at work in marketplace. In fact, analysts predict there are vast amounts of natural gas waiting to be tapped. However, the bigger challenge is, geographically-wise. The demand and supply of energy do not perfectly line up. For example, as a major high-demand energy market, Japan requires 3.1 trillion cubic feet natural gas every year, while it barely produces any gas or oil domestically. Japan instead relies heavily on imports of oil and gas. LNG is therefore principally used for transporting natural gas from remote supply area to markets. A majority of the world's LNG supply comes from countries with abundant natural gas reserves. These countries include Algeria, Australia, Brunei, Indonesia, Libya, Malaysia, Nigeria, Oman, Qatar, and Trinidad and Tobago. There are 60 LNG receiving terminals located worldwide. Japan, South Korea, the United State and a number of European Counties import LNG.

Figure 1



Source: EIA

Figure 2



Source: EIA

With natural gas consumption exceeding 60,000 mmcfd (Million metric cubic feet per day) and accounting for over a fifth of global demand, the U.S. is by far the largest and most developed natural gas market in the world. The U.S. is also the largest gas producer in the world. Although most of the 16% of natural gas we consume in the U.S. is delivered by pipeline from Canada, there is a growing volume of natural gas coming to the U.S. in liquid form from overseas. Following moderate growth in gas demand of around 2% per annum through the 1990s, stagnant and declining production has led to price volatility and the year 2000 peaks in U.S. gas demand. Despite the recent slowdown in the economy, natural gas is well established within the US overall energy matrix, accounting for over a fifth of total energy demand. Gas demand in the US is expected to continue growing for the next decade. Major growth drivers come from gas-fired power generation. As a result, it is likely that U.S. imports of LNG also will need to increase.

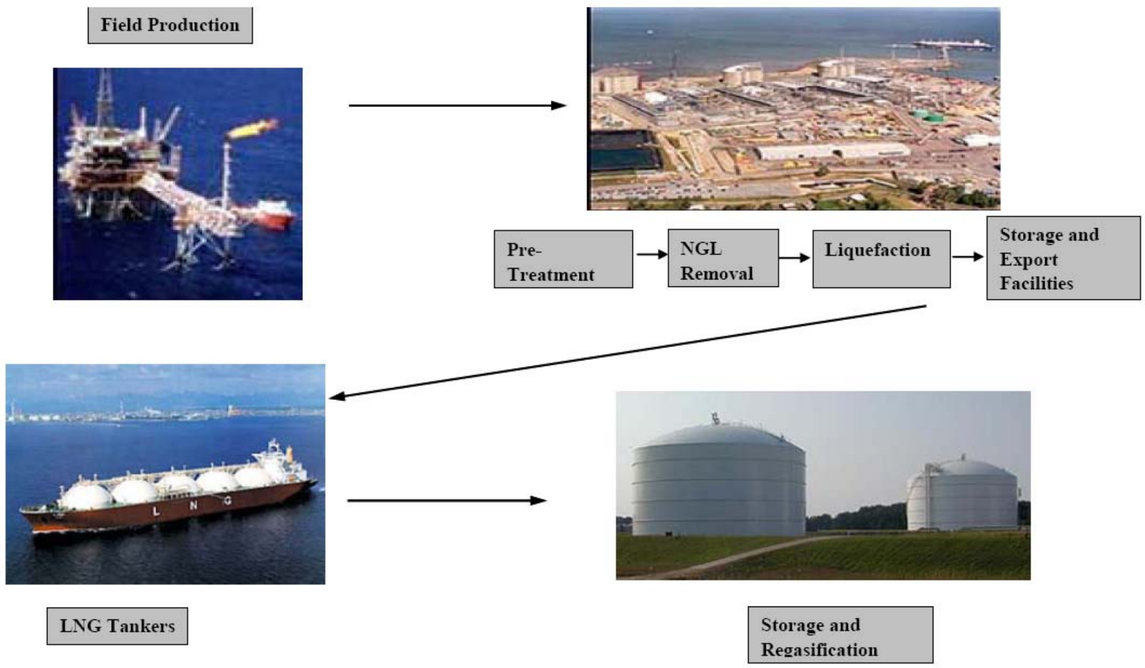
The U.S. has the largest and most developed gas pipeline network in the world. With continuous investments, the network expands to connect new sources of supply to demand areas. The US has multiple pipeline connections with Canada and Mexico for imports and exports purposes, of which many are bidirectional due to the competitive and dynamic nature of the North American market. From an LNG perspective, US is both an exporter and importer of LNG, having a liquefaction plant at Kenai in Alaska and eight existing regasification terminals located on the East and Gulf coasts. During the past few years, US regasification capacity has increased significantly via expansions at some of the more established terminals and with the commissioning of new facilities – Northeast Gateway, Sabine Pass and Freeport – in 2008. In addition the US has indirect access to regas capacity on the West Coast via the Coast Azul terminal in the Northern Baja California which was commissioned in 2008. Access is permitted via open and closed access regimes, although all of the new terminals operate on a closed access base. Under the open access regime, the owner of the regas facility must offer third party access and publish information about the services available and associated tariffs. Cove Point, Elba Island and Lake Charles all operate on an open access basis because they were commissioned at a time when they were owned and operate by interstate pipeline companies. This meant the terminals were simply an extension of pipeline infrastructure leading to their rates and services being regulated. However, while access at these terminals is open in principle, most of the capacity is secured under long term contracts. In 2002, the FERC decided to remove regulatory barriers and open access requirements to allow terminal operators to provide regas services under negotiated rather than regulated rate and thereby encourage new LNG regas terminal development.

US LNG imports grew in the 1970s before collapsing due to a combination of market conditions of rising oil prices and US regulatory regime. Plentiful US domestic gas production and low gas prices then restricted the commercial viability of LNG imports for 20 years and resulted in the closure of three of the four LNG terminals. However, LNG imports started pick up in the late 1990s and start of the 2000s. Regasfication terminals have been reactivated and LNG imports surpassed the levels sustained in the 1970s for the first time in 2003. Deliveries vary from year to year and are dependent on the availability of supply and the relative prices in the US market compared to the European market and Asia market. Imports for LNG reached new heights for the US market due to the soft demand in Europe and Asia, however imports are expected to be significantly lower in 2008. Traditionally, the US has been dependent on Algeria for the its LNG supply, but the start up of the Atlantic LNG project in Trinidad in 1999 has provided a source of LNG much closer to the US terminals. A number of contracts are in place for the supply of LNG from Trinidad to the US market. Additionally, given its proximity, the US receives a significant portion of spot cargos from Trinidad. Another recent trend has been the growth of imports from Egypt and Nigeria.

4. The LNG Game

As the LNG market is rapidly expanding on a global level playing a more and more important role in energy market, it is important to understand the game of the LNG world. There are two major types of players. Countries like Qatar, which owns the natural resources of oil and gas are LNG suppliers. The ownership of the natural resource gives them the dominant decision maker position in the market. The commercial development of LNG is a style called value chain, which means LNG suppliers first confirm the downstream buyers and then sign 20–25 year contracts with strict terms and structures for gas pricing. Only when the customers were confirmed and the development of a Greenfield project is deemed economically feasible can the sponsors of an LNG project invest in their development and operation. These energy companies usually are responsible for the marketing of LNG in different continents. They are the second type of players in the LNG game.

LNG Value Chain



Quick LNG Conversions:
1 Bcfd of natural gas = 7.5 million metric tons per annum (mmtpa) of LNG
1 cubic meter = 35.3 cubic feet
1 metric ton LNG = 2.2 cubic meters of LNG
A 138,000 m3 LNG carrier holds ~3.1 bcfe of LNG
1 knot = 1.15 mph

The upstream infrastructure needed for LNG production and transportation is an LNG plant consisting of one or more LNG trains, each of which is an independent unit for gas liquefaction. The largest LNG train in operation is now in Qatar. Until recently it was the Train 4 of Atlantic LNG in Trinidad and Tobago with a production capacity of 5.2 million metric ton per annum (mmtpa), followed by the SEGAS LNG plant in Egypt with a capacity of 5 mmtpa. The Qatar gas II plant, under construction by QP and ExxonMobil, will have a production capacity of 7.8 mmtpa for each of its two trains. LNG is loaded onto ships and delivered to a regasification terminal. Regasification terminals are usually connected to a storage and pipeline distribution network to distribute natural gas to demand markets, or a local distribution company (LDCs). The construction of an LNG plant costs at least USD 1.5 billion per 1 mmtpa capacity, a receiving terminal costs USD 1 billion per 1 bcf/d (billion cubic feet per day) throughput capacity, and LNG vessels cost USD 0.2–0.3 billion.

Major international oil companies such as BP, ExxonMobil, Royal Dutch Shell, BG Group; and national oil companies (NOCs) such as Pertamina, Petronas are active players. In 2005, Japan imported 58.6 million tons of LNG, representing some 30% of the LNG trade around the world that year. Japan, Korea and Taiwan are three major buyers that purchase approximately two-thirds of the world's LNG demand. In addition, Spain imported some 8.2 mmtpa in 2006, making it the third largest importer. France also imported similar quantities as Spain. In the early 2000s, as more players took part in investments, both in downstream and upstream, and new technologies were adopted, the prices for construction of LNG plants, receiving terminals and vessels have fallen, making LNG a more competitive means of energy distribution, but increasing material costs and demand for construction contractors have driven up prices in the last few years. The standard price for a 125,000 cubic meter LNG vessel built in European and Japanese shipyards used to be USD 250 million. When Korean and Chinese shipyards entered the race, increased competition reduced profit margins and improved efficiency, reducing costs by 60%. Costs in US dollar terms also declined due to the devaluation of the currencies of the world's largest shipbuilders, Japan and Korean. Since 2004, ship costs have increased due to a large number of orders increasing demand for shipyard slots. The per-ton construction cost of an LNG liquefaction plant fell steadily from the 1970s through the 1990s, with costs reduced by approximately 35%. However, recently, due to materials costs, lack of skilled labor, shortage of professional engineers, designers, managers and other white-collar professionals, the cost of building liquefaction and gasification terminals have doubled.

The high degree of capital investment required for the LNG business makes a secure market for the product a strategic imperative .Historically, this has occurred through direct take-or-pay contracts with end-market consumers (the prime example being the Japanese utilities). So, with strong balance sheets, global partnerships, access to reserves and a diversified equity market for natural gas volumes around the world, the integrated oils have been continue to be well positioned to take advantage of the emerging LNG opportunity.

This paper focus is on the siting decisions for liquefied natural gas regasification terminal by energy companies in North American market and the major players for this game are major oil companies. The siting game among firms which are planning to build regas terminals are characterized here as a location-then-quantity game. The companies choose a location for the terminal to receive their LNG cargos. They then deliver natural gas into different markets for end-users or trading partners through pipelines. They compete in markets (one or many) through delivered quantities considering transportation cost and active competitors in the market. Price of each market is determined by the market conditions and the total amount supplied by all participating companies. The quantity competition is called the Cournot competition in Economics. This is when oligopoly companies compete through quantity in the market, which is an accurate description of the natural gas market. The baseline model is a simple location-then-quantity game. Firms are choosing a location to set up the terminal simultaneously, then determine the quantity of supply for each market. After setting up the baseline model, in following sectors, more dimensions are integrated into the baseline model to gain extra insights of the firms' behavior in the decision making process of siting a regas terminal.

5. Model Setup

The baseline model is a simple one period game introduced as a building block for the extension.

The baseline model describes a two-stage non-cooperative game. In the first stage of the game, firms make a simultaneous decision about where to locate a regasification facility along the coastline of North America, (i.e., each firm chooses a location-strategy without knowledge of the strategy chosen by the other firms). This is a simplification compared to the reality, but yet a reasonable one. However, the time lag from the announcement of one LNG terminal to the online date is usually at least two to three years. Many uncertainties around the environmental regulatory approval process and firm financial investment flows can easily derail the process. Announcements of LNG terminals are therefore not always a sure signal of entry. In the second stage of the game, firms decide contracted quantity to be delivered to these facilities and how much to supply to each market.

The model on quantity decisions and location choice is described by the following notation. Firms are denoted by an index $i \in N = \{1, . . . , n\}$ and markets are denoted by an index $h \in M = \{1, . . . , m\}$ each demanding a quantity of a good, depending on its price. The demand is fulfilled by the supply of a quantity Q_{ih} from the facility of firm i to market h. Instead of using the traditional hoteling location model assumption of continuous market space, the assumption of the existence of a finite number of markets is used instead. The firms will choose one of the five markets to be the location of the regas facility. There are only limited ports along the coast line in the U.S., which can be considered as a site for a LNG regas terminal. Also, the location of market is a broad concept. It does not represent the exact location of the terminals, which are usually on the costal. Instead, it implies the location of the market hub or zone that the firm chooses to be located close by. For example, for Elba Island, it is obvious that the market chosen here is the Transco Zone 4. Hence, multiple terminals may be selected to enter the market from the same area or hub around where they are geographically located. A perfect example is the cluster of regas terminals in Gulf of Mexico along the coast line of Texas and Louisiana. These terminals are considered to be located in the same market node. They may enter the pipeline network from different points, but they are serving one broad common market area, where they are all priced based one common or similar market price curves. For this version of the model, each supply firm can open a facility at only one of the locations. This is a simplification for the model, because companies can and actually have invested in multiple regas terminals. For example, Shell has Cove Point and Elba Island, and Conoco Phillips has regas access to Freeport and Golden Pass.

The market space is formalized as follows. Let $G = (V, E)$ be an undirected graph with V and E as its sets of nodes and edges, respectively, $|V| = m$. Given two nodes, $v_i, v_j \in V$, $d(v_i, v_j)$ is the length of a shortest (with respect to the sum of edge lengths) path on G connecting v_i and v_j. There are m markets located each at one node on the network; there are n firms that open a facility each at one node. Let $x_i \in V = \{v_1, \ldots, v_m\}$ be the location decision by firm i on the network. The quantity decision matrix Q for all firms and all markets is given by:

$$\underline{Q} = \begin{pmatrix} Q_{11} \cdots Q_{1h} \cdots Q_{1m} \\ \cdots\cdots\cdots\cdots\cdots \\ Q_{i1} \cdots Q_{ih} \cdots Q_{im} \\ \cdots\cdots\cdots\cdots\cdots \\ Q_{n1} \cdots Q_{nh} \cdots Q_{nm} \end{pmatrix}$$

where the sum of a row is the quantity supply by firm i over all firms:

$$\mathrm{s_i} = \sum_{h=1}^{m} Q_{ih}$$

where the sum of a column is the total quantity supplied to market h:

$$q_h = \sum_{n=1}^{n} Q_{ih}$$

The price $p_h(q_h)$ at market h is assumed to follow a simple linear function here:

$p_h(q_h) = \max\{0, \alpha_h - \beta_h q_h\}, \quad q_h \geq 0$ with price parameters $\alpha_h \geq 0, \beta_h > 0.$

The n firms interact over two stages. In the first stage, firms simultaneously choose the locations of their facilities, $x_i, i = 1...n,$ vector $X = (x_1, ..., x_n)$ gives the location of the firms. In the second stage, depending on the location decisions x_i, firms choose quantities Q_{ih} to be supplied to markets, which results in the quantity decision matrix Q.

The profit firm i wants to maximize is denoted by $\pi_i(x_i, \underline{Q})$. A strategy for firm i at market h, $[x_i, Q_{ih}]$, comprises of a choice of x_i for stage 1 and a choice of Q_{ih} for stage 2; a strategy $[x_i, Q_{ig}]$, for all markets, where Q_{ig} denotes the row vector of the full quantity matrix.

The game is solved backwards. First the second stage is solved. Firm i chooses optimally the vector of quantities $Q_{ig} = (Q_{i1},...,Q_{im})$, based on what the others deliver and depending on the chosen location x_i:

$$Q^*_{ig} = \arg\max_{Q_{ig}} \pi_i(x_i, \underline{Q}^*(X))$$

After determining the optimal quantity supplied for each market given location choice, then going back to the first stage, where firm i chooses a location strategy x_i^* such that:

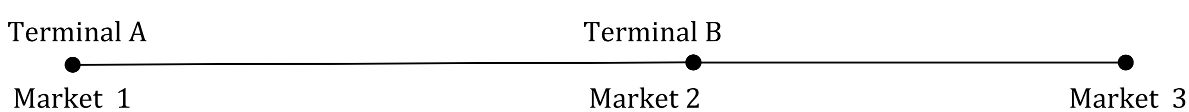
$$x_i^* = \arg\max_{x_i} \pi_i(x_i, \underline{Q}^*(X)).$$

The key drivers of difference in the behaviors for firms are their cost structures. There are three types of costs for a firm:

- Opening cost
- Production cost = Shipping cost and regas cost
- Transportation cost

The cost of establishing a facility by firm i at x_i is $w(x_i) \geq 0$. In this model, the opening cost is geographically specific. Given the availability of existing infrastructure and local regulatory status, the cost of constructing a regas terminal can vary significantly. If needed, an expansion of pipeline from the regas terminal to the main pipeline network will be built to transport the imported gas to the grid. This opening cost can also be firm specific given more detail information on each firms' existing asset portfolio and transportation capacity contracts.

The location x_i of the facility of firm i also determines its marginal production cost $c_i(x_i)$. In addition, the unit transportation cost between the location x_i of the facility of firm i and location v_h of market h, is represented by $t_{ih} = T(d(x_i, v_h))$, where T is increasing and concave in the distance. This is consistent to the long-haul discounts for pipeline transportation going from the Gulf to Northeast market. In addition, pipelines are constrained by their capacity, and there are bottlenecks along pipeline going into the demand markets in the Northeast. To reflect this concern, a penalty is imposed for the firm transporting gas through a competing LNG terminal. For example, firm A decides to deliver to the largest market node, which is located on the other end of the network, and there are firm B's terminal located between A's terminal and the market. Therefore, not only A has to pay unit transportation cost to deliver to market, but also it will pay a premium on transportation unit cost for the distance between Terminal B and market node.



The transportation cost for firm 1 to deliver gas from Terminal A to Market 3 is:

$$t_{13} = T(d(v_1, v_2)) + \eta * T(d(v_2, v_3))$$

where there is an extra cost for passing by the other competitors, η>1. This assumption is reasonable, as it characterizes the constraints of the pipeline capacity for delivering the LNG to demand markets.

The cost structure of firms are given simple numbers to represent the ranking of the production cost of major players of the regas market. Since firms' cost structures are far too complicated and cannot be condensed into a simple number, the cost matrix here focuses on showing a clear ranking of costs, which will be the basis of quantity decisions. The production cost here is the price that the firm pays at the regas terminal in order to receive the cargo. Although the possible diversion of cargos was not explicitly included; however, unexpected changes of unit costs can deter a firm's decision to supply gas to markets, which mimic an intermitted supply pattern resulted from cargo diversion. The diversion of cargo is not due to lack of supply, but the failure to provide the most competitive price for the cargo.

The total cost of the location and supply decision of firm i is therefore given by

$$TC_i(x_i, Q_{ig}) = \sum_{h=1}^{m} t_{ih} Q_{ih} + c_i(x_i) \sum_{h=1}^{m} Q_{ih} + w(x_i)$$

$$= \sum_{h=1}^{m} (t_{ih} + c_i(x_i)) Q_{ih} + w(x_i)$$

$$= \sum_{h=1}^{m} TCu_{ih} Q_{ih} + w(x_i)$$

$$where \quad TCu_{ih} = t_{ih} + c_i(x_i)$$

Profit is denoted by i and defined as

$$\pi_i(x_i, \underline{Q}) = \sum_{h=1}^{m} p_h(q_h) Q_{ih} - TC_i(x_i, Q_{ig}) .$$

Firms determine quantities for the markets to maximize profit. Substituting the price relation of the market into profit function gives

$$\pi_i(x_i, \underline{Q}) = \sum_{h=1}^{m} \max\left\{ \alpha_h - \beta_h \sum_{j=1}^{n} Q_{jh}, 0 \right\} Q_{ih} - TC_i(x_i, Q_{ig})$$

The Nash equilibrium is the solution concept used in the quantity-stage of the game. From optimization function for second stage, the Nash elements of the quantity matrix can be determined by an iterative process. Nash equilibrium quantities shipped by firm *i* to market *h* follow form the first order condition optimizing $\pi_i(x_i, \underline{Q})$:

$$Q_{ih}^* = \max \left\{ 0, \frac{\alpha_h - \beta_h \sum_{j=1, j \neq i}^{n} Q_{jh}^* - t_{ih} - c_i(x_i)}{2\beta_h} \right\}$$

To solve for the exact quantities delivered to each market, we introduce the concept of A_h and $\overline{A}_h$, the set of firms delivering to market h, and the firms not delivering to h respectively:

$$\begin{cases} Q_{ih}^* > 0 \;\; for\, i \in A_h \\ Q_{ih}^* = 0 \;\; for\, i \in \overline{A}_h \end{cases}$$

The following proposition 1 provides the equilibrium quantity for each firm $i \in A_h$.

Proposition 1. Let A_h be the set of firms which supply market h, $|A_h| = k$. The positive equilibrium quantities are given by:

$$Q_{ih}^{*} = \frac{\alpha_h - k_h\left(t_{ih} + c_i(x_i)\right) + \sum_{j \in A_h \setminus j \neq i}\left(t_{jh} + c_j(x_j)\right)}{\left(k_h + 1\right)\beta_h}$$

$$with\, Q_{ih}^{*} > 0\, \forall i \in A_h.$$

Q_{ih}^{*} depends on production and transportation cost of the active suppliers in market h. Consequently the total quantity supplied to market h is

$$q_h^{*} = \sum_{j \in A_h} \frac{k_h\alpha_h - \sum_{j \in A_h}\left(t_{jh} + c_j(x_j)\right)}{\left(k_h + 1\right)\beta_h}$$

which means that higher average marginal cost and transportation costs decreases the total quantity supplied to the market. The optimal price at each market can be derived as:

$$p_h^{*} = \frac{1}{k_h + 1}\left(\alpha_h + \sum_{j \in A_h}\left(t_{jh} + c_j(x_j)\right)\right)$$

Optimal prices at each market rise with average marginal cost and transportation cost over the firms actively supplying the market.

Furthermore, equilibrium quantities are given by

$$Q_{ih}^{*}=\frac{\alpha_h-k_h\left(t_{ih}+c_i(x_i)\right)+\sum_{j\in A_h\setminus j\neq i}\left(t_{jh}+c_j(x_j)\right)}{\left(k_h+1\right)\beta_h}$$

$$=\frac{\alpha_h-\left(k_h+1\right)TCu_{ih}+\sum_{j\in A_h}TCu_{jh}}{\left(k_h+1\right)\beta_h}$$

$$=\frac{\alpha_h+\sum_{j\in A_h}TCu_{jh}}{\left(k_h+1\right)\beta_h}-\frac{\left(k_h+1\right)TCu_{ih}}{\left(k_h+1\right)\beta_h}$$

$$=\frac{p_h^{*}}{\beta_h}-\frac{TCu_{ih}}{\beta_h}$$

$$=\frac{p_h^{*}-TCu_{ih}}{\beta_h}$$

At equilibrium $Q_{ih}^{*}>0\ \ for\, i\in A_h$, $(p_h^{*}-TCu_{ih})/\beta_h>0$ such that $p_h^{*}>TCu_{ih}$.

Proposition 2. The relation between the firm with the highest total unit costs in the active set, $i\in A_h$, with any firm which is not entering market $j\in \bar{A}_h$, is

$$TCu_{ih}<\frac{\alpha_h+\sum_{j\in A_h}TCu_{jh}}{\left(k_h+1\right)}\leq TCu_{jh}\,.$$

Firms are ordered on the basis of total unit costs and they will only enter the market if the market clearing price covers their variable cost, which is the total unit cost of production plus transportation.

Hence the total quantity supplied by each firm is:

$$s_i=\sum_{i=1}^{n}Q_{ih}^{*}=\sum_{i=1}^{n}\frac{\alpha_h-k_h\left(t_{ih}+c_i(x_i)\right)+\sum_{j\in A_h\setminus j\neq i}\left(t_{jh}+c_j(x_j)\right)}{\left(k_h+1\right)\beta_h}$$

Total cost for each firm is:

$$TC_i=\sum_{h=1}^{m}\left(t_{ih}+c_i(x_i)\right)\frac{\alpha_h+k_h\left(t_{ih}+c_i(x_i)\right)+\sum_{j\in A_h\setminus j\neq i}\left(t_{jh}+c_j(x_j)\right)}{\left(k_h+1\right)\beta_h}+w(x_i)$$

The final payoff for each firm given location vector X is:

$$\pi_i(X) = \sum_{h=1}^{m}\left(p_h^* - \left(t_{ih} + c_i(x_i)\right)Q_{ih}^* - w(x_i)\right)$$

$$= \sum_{h=1}^{m}\left(\frac{\left[\alpha_h + \sum_{j\in A_h}\left(t_{jh} + c_j(x_j)\right) - \left(k_h+1\right)\left(t_{ih} + c_i(x_i)\right)\right]^2}{\left(k_h+1\right)^2 \beta_h} - w(x_i)\right)$$

$$= \sum_{h=1}^{m}\left(\frac{\left[\alpha_h + \sum_{j\in A_h\setminus\{i\}}\left(t_{jh} + c_j(x_j)\right) - n\left(t_{ih} + c_i(x_i)\right)\right]^2}{\left(k_h+1\right)^2 \beta_h} - w(x_i)\right)$$

$$\sum_{h=1}^{m}\beta_h (Q_{ih}^*)^2 - w(x_i)$$

Given the optima of the second stage, focus is on the first stage of game in term of location decision. Considering the equilibrium supply quantity in the second stage, each firm maximizes the profit function by selecting a location on the network, which has the highest optimal profit. The strategy $X^* = (x_1^*, \ldots x_n^*)$ is a Nash Equilibrium if for each firm i, x_i^* is the best response to the strategies specified by the n-1 other firms:

$$\pi_i\left(x_i^*,\ \underline{Q}^*(X^*)\right) \geq \pi_i\left(x_i^* \underline{Q}^*(\breve{X})\right)\ with\ \breve{X} = (x_1^*, \ldots, x_i, \ldots, x_n^*)\ \forall x_i$$

For every feasible strategy x_i . That is x_i^* solves

$$\max \pi_i\left(x_i\ ,\ \underline{Q}^*(\breve{X})\right)$$

<u>Sequential Choices</u>

Based on the results from one single period model, this section extends the game to a total of two periods. Firms have the choice to choose a location considering the current market conditions or postpone the decision to the next period. The market conditions are unknown to firms and will only be revealed at the beginning of each stage. Here, the concept of stage can be thought as each regime change in the market. It is not constrained to a regular time period such as a month or a year. Instead, it can be thought as a reasonably stable period of time in the market, where firms are confident about the foreseeable future. In a natural gas market, this can be a medium term of 2-4 years time for a firm which is considering a business development project like a regas terminal.

Without repeating the formulas and calculations for the extended model, the description for the baseline model in the previous section can serve as a detailed base for the discussion here. Several additional parameters are introduced for the extended model.

At T =1, the current market conditions in each market are revealed to the firms:

$$\alpha^{T=1} = \{\alpha_1, ...\alpha_m\}^{T=1} \ and \ \beta^{T=1} = \{\beta_1, ...\beta_m\}^{T=1}$$
$$w^{T=1} = \{w_1, ...w_m\}^{T=1}$$

Firm *i* has an expectation of the future market in the second period time , which is based on firm's current knowledge of the future, denoted as κ_i and the observed market condition in current period :

$$Exp\left[(\alpha,\beta,w)^{T=2} \mid \kappa_i, T=1\right] =$$
$$Exp\left[\left|\{\alpha_1,...\alpha_m\}^{T=2}, \{\beta_1,...\beta_m\}^{T=2}, \{w_1,...w_m\}^{T=2}\right| (\alpha,\beta,w)^{T=1}, \kappa_i\right]$$

The investor firms competing for locating regas terminals are all very sophisticated players and possess comprehensive market information and knowledge. It is reasonable to assume that the firms are forward looking. Firms are considering the expected outcome for next market regime when determining the current location choice. In this paper, all firms assume to have the same expectation of the future at current time. This is reasonable for energy industry, as firms all have strong analytical teams and all have access to a few pioneering consulting firms and forecasting services. Although firms have their modified views on many different aspects of the market, they usually come to a consensus in the industry about the general direction of the market.

In addition, with the introduction of multiple periods, there is a value adjustment for opening cost for each market. If there are incumbent facilities for the second period, denoted as γ_h . When γ_h >1, it represents the crowding-out affect that opening cost for new entry is higher when there is incumbent firms in the market already. This can be considered as additional entry barrier. When γ_h <1, it represents the learning benefit for the later entrants, where there is knowledge sharing of the construction and operation of the facility from the existing players. For example, to construct the first LNG regas terminal, some challenges may include getting approval from local government and acceptance from the community. However, as a result of the first terminal, the second entrant can enjoy the established regulatory policies which were initiated by the previous terminal investor.

Since the future market conditions are unknown, firms have to decide enter or not enter based on current market conditions and their expectation of future market conditions. In each period, backward induction is used to solve the game, like the baseline model. The main difference is at first period of time. Firms do not only consider location choices in current period, but also in the next period. Given the location choices of both periods, firms will estimate the expected profit for both periods. The decisions of accessing and delivering to each market are the same here as in the baseline model. Firms will then choose the best location given the associated optimal expected profit for both periods.

First, solve the first period. When T = 1, the game is solved backwards like the baseline model. Based on current observations and expectations, firms first optimally choose the vectors of quantities for both periods given a location choice:

$$Q_{ig}^* = \arg\max_{Q_{ig}} \pi_i(x_i, \underline{Q}^*(X))$$

Where

$$Q_{ig} = \left((Q_{i1},...,Q_{im})^{T=1}, (Q_{i1},...,Q_{im})^{T=2} \middle| (\alpha,\beta,w)^{T=1}, \kappa_i \right)$$ and

$$X = \left(X^{T=1}, Exp(X^{T=2} \left| (\alpha,\beta,w)^{T=1}, \kappa_i) \right. \right)$$

After determining the optimal quantity supplied for each market given location choice, go back to the first stage, where firm i chooses a location strategy x_i^* such that:

$$\begin{aligned}
&x_i^* = \arg\max_{x_i} Exp\left[\pi_i(x_i, \underline{Q}^*(X)) \middle| (\alpha,\beta,w)^{T=1}, \kappa_i, \gamma \right] \\
&where \\
&Exp\left[\pi_i(x_i, \underline{Q}^*(X)) \middle| (\alpha,\beta,w)^{T=1}, \kappa_i, \gamma \right] \\
&= \pi_i^1(x_i^1, \underline{Q}^*(X^1)) + \delta Exp\left[\pi_i^2(x_i^2, Exp\underline{Q}^*(X^2)) \middle| (\alpha,\beta,w)^{T=1}, \kappa_i, \gamma \right] \\
&\delta > 0 \\
&x_i^* = (x_i^{1*}, x_i^{2*}) \\
&\underline{Q}^*(X) = \left(\underline{Q}^{1*}(X^1, X^2), Exp\left[\underline{Q}^{2*}(X^1, X^2) \right] \right)
\end{aligned}$$

δ is the discount factor for the future. If firms are not forward looking and only able to observe the current market condition, the equilibrium result for the first stage of siting game should be the same as the equilibrium resulted from the baseline model. Firms' cost structure for T =1 is known but there is uncertainty of next period's cost structure. This can be just another expected term added in the calculation for the first period.

In second period, when nature reveals the market condition, firms that have not yet have chosen a location will be given a second chance to choose. Firms that have chosen are not given a chance to relocate. Once location choices are made, firms will again simultaneously choose the quantity supplied to each markets. The game in the second stage is solved like the baseline model, except certain firms' location choices are pre-determined as they have chosen already. If there are still firms left without making a location choice from the first stage, that firm (or those firms) will have a second chance here, and they only enter the market whenever the market conditions in the future stages are revealed to be profitable for entries. If the market condition did not improve to be profitable for firms which have not chosen, these firms will never enter, as they did not at the previous stage. As firms make entry decisions depending on their cost structure of producing or delivering gas, more efficient players with lower costs will enter the market first, while others wait till the conditions become more profitable for entering.

Also, in reality the firms play the quantity game respectively. Although it is possible to introduce a multi-stage cournot quantity game here, a one stage quantity game will serve to illustrate the purpose. If the focus of the analysis here is the location decision, the gain of a multi-stage quantity game is assessed at the beginning of every period based on firms' expectations. Hence, it can be rewritten as one constant term in the formula given firm's expectation as if it is one stage cournot game.

$$\begin{aligned}
\sum_{t=1}^{T} E\left(\Pi_{t=1}\right) &= \sum_{t=1}^{T} E\left(\Pi_{t=1} \mid p\right) \\
&= \sum_{t=1}^{T} E\left(\Pi_{t=1} \mid \{p_1, \ldots, p_T\}\right)
\end{aligned}$$

Since the focus of this paper is to solve the model numerically for illustration of the market dynamics, the Saize and Hendrix [12] algorithm is introduced that systematically enumerates all location possibilities for which equilibrium quantities are computed. Afterwards, it tries to detect which location vectors correspond to a Nash Equilibrium by checking whether it is better for a firm to relocate its facility given everything else remains constant. For the extended model, the same method is used, except there are more location combinations to calculate as both periods are considered when T =1.

6. Numerical Illustrations

The reality only shows one story, and one story only, which is a result of orchestrating through many fundamental drivers in the market place. It is challenging to understand the intricacy of all drivers at once. The purpose of this model is to exploit the dynamics observed in the LNG market, by providing a simplified virtual space where the researcher is able to gain a better understanding for works of each variable. Hence, the following sector focuses on numerical illustrations of the model. Assumptions are made to mirror certain behaviors and observations in the natural gas market; scenarios are designed to investigate the impacts of a particular market fundamental driver. Based on the framework set up in the previous section, scenarios are elaborated here for both the baseline model and the extended model.

Scenario 1. – Varying Transportation Tariff

There are three major pipelines extending from Gulf of Mexico to major demand area ins the northeastern US: William's Transco pipeline (10, 500 miles), El Paso's Tennessee Gas Pipeline (14,200 miles) and Spectra Energy's Texas Eastern Pipeline (9,040 miles), tapping supply regions in the Gulf of Mexico, Texas, Appalachia, and Canada and serving gas to markets across the Midwest and mid-Atlantic regions, including major metropolitan centers such as New York and Boston. There are major markets located along these pipelines where trading and distributing. As mentioned in the previous sector, although LNG terminals are constrained to locate along the coastline for receiving cargos, gas from each terminal has to be transported first onto these pipelines in order to deliver to major market areas. In this model, there are five markets, m = {1, 2, 3, 4, 5}, and the increasing in value of $(\alpha, \beta, w)_h$ $for\ h = 1...5$ reflects the increasing gas demand and price volatilities. α represents the price when quantity is zero in the market. Note that in the reality, when the quantity approaches zero, the price will go to extremely high value, theoretically, approaching infinity. With a positive α, the higher it is, the stronger demand pull there is in the market. Market 5 has the highest α, where it is supported by the strongest demand. β is the slope of the demand curve in the case of linear demand function. The higher β is, the greater downward price impact from the marginal unit of product supplied to the market. In other words, gas prices in the market are more responsive to quantity changes. In addition, it is more expensive to setup a regas terminal in the northeast compared to gulf states, because there are more regulatory and environmental challenges when the location is closed to major metropolitan areas.

In the reality, there is capacity limitation on a pipeline, and the cost of transportation increases dramatically when the utilization rate of a pipeline approaches its capacity limit. In recent years, the three major pipelines are constantly under a high utilization rate, especially in winter seasons. When the pipeline network is relatively full, it is much more expensive to deliver the gas to the market. With the development of unconventional gas production newly found in Texas and Louisiana, there is increasing challenge for displacement of LNG supply in the future. Transportation cost is certainly a concern for firms when choosing location of the regas terminal. Scenario 1 investigates the effect of transportation tariff on firms' location choices. Assuming the production costs are identical for all firms. As the tariff rate goes from most expensive configuration to the least expensive one, the firms' equilibrium locations move from market 4 to market 1. Based on the equilibrium quantity choices for each firm, the market focus are on the larger demand area. However, the most expensive opening cost prevents firms all locating at market 5. As the transportation cost gets cheaper, firms choose to locate to the markets with lower opening cost. In the case where tariff rate is 1/6, there are three pure strategy equilibria. Since the firms are identical in term of cost structures, the equilibria are always symmetric. Note when the transportation cost decreases, gas supplied to the markets increases. This is true holding other factors constant and there is no constraint on availability of supply in this model. In reality, often there is no cargo arriving in the U.S., due to more competitive price offered in European or Asian markets. So, holding everything else constant, when the transportation cost increases, firms have incentive to locate closer to the largest demand area; when the transportation cost decreases, firms tend to locate further away in this network setting in order to save on initial capital cost as well as operating cost.

Scenario 1: Basic Model - Varying Cost of Transportation

Number of Firms: 5

Market	Alpha	Beta	Opening Cost
1	200	1	1000
2	500	2	2000
3	1000	3	3000
4	1500	4	4000
5	2000	5	5000

Firm Cost Structure Matrix

Firm/Market	1	2	3	4	5
1	10	11	12	13	14
2	10	11	12	13	14
3	10	11	12	13	14
4	10	11	12	13	14
5	10	11	12	13	14

Configuration of Transportation Tariff	Nash Location X*	Corresponding Nash Quantity for firm i					Corresponding Profit for firm i
2	(4, 4, 4, 4, 4)	(21.167	37.250	53.722	61.958	65.567)	44732 ;
	(3, 3, 3, 3, 3); (4,	(28.000	39.833	54.889	61.583	65.600);	
1	4, 4, 4, 4)	(26.167	38.917	54.278	61.958	65.900)	46683; 45621
1/3	(3, 3, 3, 3, 3)	(30.222	40.389	54.889	61.861	66.044)	47331
	(2, 2, 2, 2, 2); (3,	(31.083	40.750	54.806	61.833	66.050);	
1/4	3, 3, 3, 3)	(30.500	40.458	54.889	61.896	66.100)	48405; 47413
	(1, 1, 1, 1, 1); (2,	(31.667	40.694	54.815	61.875	66.111);	
	2, 2, 2, 2); (3, 3,	(31.222	40.750	54.852	61.903	66.133);	49496 , 48518,
1/6	3, 3, 3)	(30.778	40.528	54.889	61.931	66.156)	47495

Scenario 2 – Varying Price Responsiveness in Markets

Price responsiveness in the market changes as the economic conditions change. When there is strong demand, there is plenty of liquidity in the market to support additional supply of gas and price impact from extra supply is minimal. However, on the other hand, in a market with little liquidity, any additional supply can have a significant impact on the price in the market.

Keeping the same network setup as in scenario 1 and holding all the market conditions as well as transportation costs constant, when there is higher price responsiveness in the market in general, the competition is more fierce. Because there is a greater reduction in market price from every additional quantity of gas supplied to the market. Firms act more cautiously with their supply decisions to markets. When the price impact is softer, firms produce significantly more. In term of location choices, as a result of the stronger pull from demand, firms also move towards the market where they deliver to most. Lowering price responsiveness in market implies a stronger demand as well as greater liquidity in the market. Note that when beta is halved, the quantity supplied more than double for market 5, whereas the quantity supplied less than doubles for market 1. This scenario reveals that, given the same growth rate of demand, markets with large consumption levels grow faster. On the other hand, with general slowdown of demand growth for gas, this can also indicate that there will be larger percentage decline in gas delivered in major demand area in northeast compared to southern states in the US.

Scenario:2 Basic Model - Varying Price Responsiveness

Number of Firms:		3
Market	**Alpha**	**Opening Cost**
1	200	1000
2	500	2000
3	1000	3000
4	1500	4000
5	2000	5000

Firm Cost Structure Matrix

Firm/Market	1	2	3	4	5
1	10	11	12	13	14
2	10	11	12	13	14
3	10	11	12	13	14

Tariff Rate = 1

Beta Configuration	Corresponding Nash Location X*	Total Quantity supplied to markets				
(2, 4, 6, 8, 10)	(3 3 3)	63.000	89.625	123.500	138.563	146.850
(1, 2, 3, 4, 5)	(3 4 3) (3 3 4) (4 3 3)	120.500	176.500	245.167	278.250	295.600
(.5, 1, 1.5, 2, 2.5)	(4 4 4)	235.500	350.250	488.500	557.625	593.100

Scenario 3 – Varying Cost Structures of Firms

Firms are not identical in reality. In fact, they vary significantly in terms of their cost structures and cost structure is key driver to shape the business model and strategic behavior of a firm. Keeping market conditions as well as transportation costs constant in scenario 3, the focus shifts to the cost structures of firms. Instead of being equally productive, firms are configured to have different cost structures. Location choices are not only determined by the absolute value of shipping costs and regas charges, but even more by the relativity of cost structures among firms. When the cost structure of bringing LNG into US is similar among competitors, this increases competitiveness of the game. As a result, it is less profitable for each firm. When the cost structure among firms vary significantly, due to the difference in cost structure, the strategic focus of each firms differ: most cost-efficient players will focus on choosing the largest demand market area to serve based on the attractiveness of markets, whereas the strong demand pull can means too much competition to be profitable for a less cost efficient firm, as the priority of the high-cost firms are to avoid competition for survival more than competing in the spotlight. As a result, a high-cost firm chooses to locate away from the competition and concentrate in supplying the nearby markets. There are more market segmentations observed with more sparsely distributed cost structure among firms. When the difference of firms cost structure gets narrower, competition increases and pulls the equilibrium location choices closer together. Also, when there is a higher cost structure, firms tend to move down to locations where there is lower cost of constructing a LNG terminal.

The configuration is the cost scale between firms. For example, when the cost configuration is (1, 5, 5), it implies that firm 1 has cost as the default value, and firm 2 and firm 3's costs are 5 times higher than their default values. As the cost structures of firms vary, we notice changes of location choices in the equilibria. In addition, at first, the result when cost structures are identical that firms are not located in the same market. That is because market 4 is always more attractive, when the transportation costs are low enough to allow firms to move closer. However, in this market setup, the competition for all firms to locate in market 4 will not be optimal, as the competition of locating too close to the demand center, as well as higher capital cost, will eat away too much profit. Therefore, firms choose not to locate in the same market as the optimal choice, even though they have the identical cost structure.

Scenario 3 Basic Model - Varying Cost Structure

Number of Firms: 3

Market	Alpha	Beta	Opening Cost
1	400	2	1000
2	1000	4	2000
3	2000	6	3000
4	3000	8	4000
5	4000	10	5000

Firm Cost Structure Matrix

Firm/Market	1	2	3	4	5
1	10	11	12	13	14
2	10	11	12	13	14
3	10	11	12	13	14

Tariff Rate = 1

Cost Configuration	Corresponding Nash Location X*
(1, 1, 1)	(3 4 3) (3 3 4) (4 3 3)
(1, 1, 5)	(3 3 3)
(1, 5, 5)	(3 3 3)
(1, 1, 10)	(3 3 2)
(1, 10, 10)	(3 2 2)

From here onwards, the scenarios extend to the two period game. Global energy markets are changing constantly, and there are even occasional seismic-size shifts. However, with great challenge trying to predict a recession, business plans are designed with the BAU (Business-As-Usual) mindset. One important assumption here is that future realization aligns with the firms' expectation. Since firms are making a decision on location choices for both periods at beginning of time, if the expectation becomes reality, the firms will simply carry out the original plan. This is just a simplification for the purpose of this paper, or pure good faith in fundamental forecasting. The focus of interest is to understand the impact of forward-looking vision on the location choices at beginning of the first period compared to decisions made when knowing there is no second chance.

Scenario 4 – Crowding Effect

When firms have the choice to enter now or later, the order of entry matters. Usually, firms are concerned about first-mover advantage facing competition; however, there is great deal of uncertainty in the operation details and the market dynamics. Firms will hence tend to wait to learn from first movers. When varying γ and holding everything equal, this scenario explore how firms impact each other's entry decision.

When γ>1, this is the crowd-out effect in the market. It reflects the increasing input cost as well as much more regulatory challenges to setup additional LNG regas terminals in the same area for environmental concerns. The firms have incentive to enter the market even bearing a negative profit for the first period to avoid the increasing entry barrier in the second period. Therefore, there is an extra push of entering the market as earlier as economic viable. When $\gamma<1$, this is a learning benefit for later entrants from experience of incumbents. In LNG market, with great capital investment and high risk associated with long term contracts, firms tend to be cautious in entering the market. In the configuration of $\gamma<1$, the least efficient firm waited to enter at the second period of the game. The saving on the entry cost by waiting and learning is greater than the profit that firm would make by entering in the first period. Note in the last configuration, when $\gamma=1.5$, since the crowding effect is so severe, the later entrant just chose to stay a different location instead to avoid the additional increase in opening cost.

Scenario 4: Varying Gamma

Number of Firms: **3**

Market	Alpha	Beta	Opening Cost
1	400	2	1000
2	1000	4	2000
3	2000	6	3000
4	3000	8	4000
5	4000	10	5000

Firm Cost Structure Matrix

Firm/Market	1	2	3	4	5
1	10	11	12	13	14
2	10	11	12	13	14
3	100	110	120	130	140

Time Changing Variables	
Alpha_delta	2
Beta_delta	2
w_delta	1
Gamma	1
ProCost_delta	1
Discount	**1**

Changes (Replacing Default Value	**Corresponding Nash Location X***
Gamma = 0.5	(3 4 0; 3 4 3); (4 3 0; 4 3 3)
Gamme = 1.2	(3 3 0; 3 3 3)
Gamma = 1.5	(3 3 0; 3 3 1)

Scenario 5: Varying Market Outlook

Number of Firms:			**3**
Market	**Alpha**	**Beta**	**Opening Cost**
1	200	1	1000
2	500	2	2000
3	1000	3	3000
4	1500	4	4000
5	2000	5	5000

Firm Cost Structure Matrix

Firm/Market	**1**	**2**	**3**	**4**	**5**
1	10	11	12	13	14
2	10	11	12	13	14
3	100	110	120	130	140

Time Changing Variables	
Alpha_delta	2
Beta_delta	2
w_delta	1
Gamma	0.5
ProCost_delta	1
Discount	**1**

Changes (Replacing Default Value	**Corresponding Nash Location X***	**Entries to markets**
		T = 1 : 1 2 3 4 5 ; T = 2: 1 2 3 4 5
Alpha_delta =0. 1	(3, 3, 3; 3, 3, 3)	(3 3 3 3 3 ; 0 2 2 2 3)
Alpha_delta =0. 5	(4, 4, 3; 4, 4, 3)	(3 3 3 3 3 ; 2 3 3 3 3)
Alpha_delta =2	(4, 4, 3; 4, 4, 3)	(3 3 3 3 3 ; 3 3 3 3 3)

Scenario 5 – Market Outlook

Historical LNG prices, particularly in the Asia-Pacific region, have been contractually linked to oil and/or refined product prices, we expect that a significant increase in LNG deliveries to supplier-owned terminals by integrated LNG players and the general emergence of deregulated U.S. and European LNG import markets (which are linked to their own domestic natural gas markets) will cause a general shift in worldwide LNG pricing towards gas-linked arrangements and away from the oil-linked structures over time. Hence the long term market outlook for prices of LNG is a one important factor that firms consider when choosing LNG regas locations.

All firms have long term market outlooks, and that is the expectation of market conditions and price movements in the future. For a capital intensive project like an LNG regas terminal with 30-year LNG contracts, the long term fundamental market outlook is crucial. When there are two periods in the game, firms have incentive to evaluate the expected profit given current choice based on their market outlooks. When there is bullish outlook for the economy, there is incentive in entering the market to secure a competitive position. Firms will enter even bearing a negative profit at beginning. However, when there is a pessimistic outlook for the economy, there is fear to invest. In the past five years, there was a strong growth in energy market. There were many proposed LNG regas terminals, although only a few were built in the end. At the turn of recession, the freeze-up in credit market and softer global demand for energy will deter investments in regas terminals in the future.

Scenario 5 gives an example that different market expectation can change firms' behavior. When the market outlook is very pessimistic, firms tends to withdraw from certain markets and focus only on the large demand areas.

However it is interesting to observe that with the severe recession ahead firms will not move the location choice closer to the demand centers. This is the fact due to the capital commitment up front for constructing a facility closer to demand center costs more and the expected rate of return is not covering the investment well when the economy's future is looking weak.

7. Conclusion

In order to describe the siting game for LNG regas terminal, a competitive location and quantity "a la Cournot" game has been described in this paper to study the oligopolistic competition between n>2 heterogeneous firms. Firms have to decide where to locate a facility and then decide on how much to supply to all or some of m>2 spatially separated markets from these facilities. Furthermore, the model is also extended into t=2 periods, where firms decide to enter in the first or second period, which allows forward looking vision impact firm's marginal behavior. This model focuses on industry implications by using numerical examples to illustrate impacts of parameters on equilibrium locations and quantities. Through numerical illustration, it implies that market fundamental drivers, like market price responsiveness and long term market outlook, and firm's own cost structures and operation model can have material influence on the strategic decision of locating regas terminals.

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