ENDOGENOUS CAPACITY EXPANSION FOR A LONG-TERM WORLD COAL MARKET MODEL

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1 – Introduction

Since autumn 2006 European steam coal prices have more than doubled. Between October 2007 and July 2008 steam coal prices rose from 100 United States Dollars [USD] to 170 [USD] per tone (CIF ARA[1]). Similar price evolutions could also be observed in the other fossil fuel markets and in the electric market. With these figures it is quite clear that the development of a model to forecast steam coal prices is interesting and useful to be able to understand the underlying phenomena. The goal of this paper is to present the model we developed. We started by developing a short-term model, named World Coal Model (WCM). The horizon of WCM is between 2000 and 2010. The model is based on a bottom-up approach. The economic concept chosen in our modelling of the global steam coal trade is a production (extraction + transport) organized to response to the demand at a least-cost. The literature devoted to coal models based on a bottom-up approach is not very widespread. The situation is even worse if one looks for a steam coal model at a world scale [1]. The use of linear programming methods is interested in this case because a realistic world coal model requires a huge amount of data leading to large size problems. The outputs of the model are mainly the marginal costs in different areas of the world (Japan, South Africa, Australia, Indonesia, Europe, etc.) and the quantities transported. WCM does not take into account any investment. The production capacities of the different mines located around the world are fixed figures in the data. New capacities cannot be developed even if a huge peak of demand appears or if we extend the time horizon of the model. We enriched the model taking into account investment for theses purposes. We have made the choice to consider endogenous capacities modeling. In this way, the new capacities are simulated. For the first release of the model we wished to keep the same theoretical and computational framework. This implied to keep a determinist and linear optimisation framework.

The paper is divided in 4 main sections. The first section is devoted to a presentation of the short-term model. A qualitative overview is provided to the reader followed by a short description of the equations. The second section is focused on the equations defining the capacity expansion module, the heart of the long-term model. The third section proposes two economical formulations of the long-term model based respectively on the rational price expectation and the adaptive price expectation. In the last section, we develop some mathematical analysis of the capacity expansion module. The mathematical analysis has been performed to test the feasibility of the linear and determinist optimisation approach.

2 – Description of the short-term model

One understands by fundamental a model capable to provide the evolution of the marginal cost on a certain time horizon. In this way, it is not the coal price, which is delivered by this type of model but what constitutes its principal base. This base depends only on the physical constraints and financial constraints (production cost, investment cost, transport cost, discount rate, inflation and exchange rates). The model is based on a bottom-up approach. A linear programming method [2] [3] is used to

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[1] Cost Insurance Freight Amsterdam Rotterdam Antwerpen

calculate the minimum of a cost function consisting of production costs and transport as well as port and loading costs and constrained by production, transport and port capacities.

2.1 - Qualitative description

WCM employs diverse set of parameters, e.g. transport costs and also figures for coal demand, which are attached to the net importing countries. These data have been bought partly from specialized consultancies. Model developers also do research on some of the more readily accessible information as demand structure in Europe, inflation rates or exchange rates.

The horizon covered by WCM is between 2000 and 2010. The model pictures the world coal market with 9 coal supply countries, 10 coal demand countries and 8 "rest of the world" regions which are both coal supply and demand regions.

Figure 1 presents a base case of the model topology.

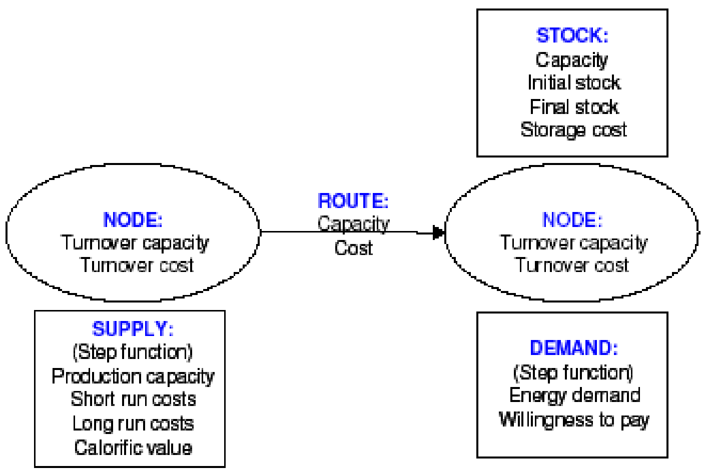

Figure 1: Model topology basic scheme

To link the different nodes 371 routes have been created. They can be classified as:

- Internal routes in countries/regions. They link supply or demand nodes to ports
- International routes for sea transport (36 in total). They link the ports of supply to demand regions
- Virtual routes. They mainly link countries to "Rest of the World" Regions

The outputs of the model are mainly coal prices (marginal costs) from 2000 to 2010 in different areas of the world (Japan, South Africa, Australia, Indonesia, Europe, etc.), the transported volumes per route, the export-import volumes per region. A special emphasis is done on CIF ARA[2] price and FOB[3] Richards Bay (South Africa) and Newcastle (Australia) price. In Figure 2, we present the difference between the variations of the historical API2[4] price and the variations of the price of the model for a scenario in which China does not export anymore.

[2] Cost Insurance Freight Amsterdam Rotterdam Antwerpen

[3] Free On Board

[4] The API2 is a monthly basket index for ARA coal price wiht a basis of 6000 Kcal/kg

Figure 2: Comparison between the variations of the price of the model and the variations of the historical API2 price

2.2 Main equations of the short-term model

For an inelastic demand the cost function in WCM has been written as follows:

$$\sum_t \left[DisF(t) \left[\sum_s C_s^P(t)\ q_s(t) + \sum_R C^R(t)\ q_R(t) + \sum_n \left(\sum_{d(n)} q_{d(n)}(t) + \sum_{rp(n)} q_{rp(n)}(t) + \sum_{stock(n)} ddpos(stock(n),t) \right) C^{TO}(t) + \sum_{stock} q_{stock}(t) C^{Stock}(t) \right] \right]$$

with $DisF(t) = \dfrac{1}{(1+TA)^t}$

Notations:

TA	: Discount Rate
C_s^P	: Production cost for a mine s
q_s	: Produced quantity at a mine s
C^R	: Transport cost for a road R
q_R	: Transported quantity on road R
$q_{d(n)}$	: Demand quantity attached to node n
$q_{rp(n)}$	: Quantity from node n leaving by road R
$ddpos(stock(n))$	: Storage at node n
C^{TO}	: Turnover cost
q_{stock}	: Quantity in the stock

C^{Stock}	: Storage cost
$q_{ra(n)}$	: Quantity transported on route ra(n) (route ending at node n)
$q_{s(n)}$	: Produced quantity at mine s attached to node n
$dpneg(stock(n))$	: Destocking at node n

The constraints are mainly capacity constraints on mines and routes. They are inequality constraints.

$$\forall t, \quad q_s(t) \leq Capacity\_available(t)$$

$$\forall t, \quad q_R(t) \leq Capacity\_route\_R(t)$$

The main equality constraint is the constraint of the flow conservation at a node n. The dual variable attached to this constraint is the marginal cost at node n. Figure 4 illustrates the outgoing and ingoing flows at a node n.

$$\sum_{s(n)} q_{s(n)}(t) + \sum_{ra(n)} q_{ra(n)}(t) + \sum_{stock(n)} dpneg(stock(n),t) = \sum_{d(n)} q_{d(n)}(t) + \sum_{rp(n)} q_{rp(n)}(t) + \sum_{stock(n)} dppos(stock(n),t)$$

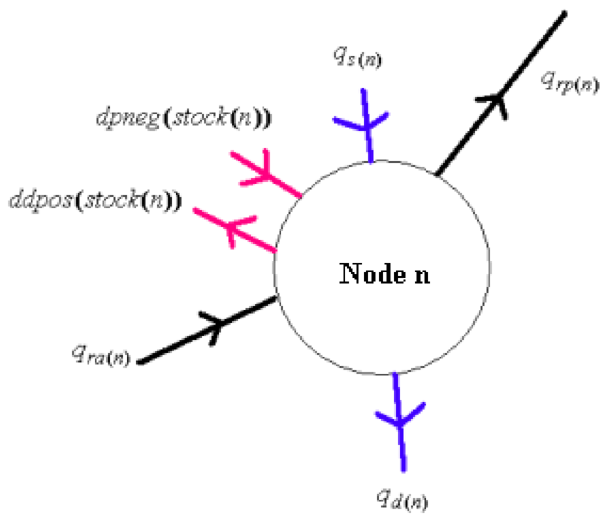

Figure 3: Flows in and out of a node n in order to satisfy the volume equilibrium

3 – Capacity Expansion Module

WCM does not take into account any investment. Two strong assumptions have been made in the short-term model. The coal market is considered as a perfectly competitive market and the mining park is fixed. The first assumption can be kept for a long-term horizon (2035) but a fixed mining park is not realistic. New capacities have to be modeled during time and consequently the future investment. We made the choice to treat the new capacities in an endogenous way. It means that a new capacity is not part of the data but it is considered as a variable calculated in the objective function. In other words, the new capacities are simulated and are part of the outputs. For the first release of the long-term model we wished to keep the same theoretical and computational framework used for the short-term model. That implied to keep a deterministic and linear optimisation framework.

In this section, we present the new set of equations defining the heart of the long-term model, the Capacity Expansion Module (CEM). First we describe the way investments are modelled in the cost function. Afterwards we give details about the new capacities and the reserves by introducing the notion of mine life. For the investment on routes or ports we explain how to compute the annuity assuming infinite life duration. We then develop how we handle an investment not far from the end of the study period. If the investment is done lately in the study period not far from the end of the study horizon then the mine life is longer than the time left between the investment and the end of the study. In this case we have to get our money back after the end of the study.

3.1 – Cost function with investment

The cost function for the long-term model is quite identical to the one of the short-term.

$$\sum_t \Bigg[DisF(t) \Bigg[\sum_s C_s^P(t)\ q_s(t) + \sum_{si} \left(C_{si}^P(t)\ q_{si}(t) + C_{si}^I(t)\ k_{si}(t) \right) + \sum_R C^R(t)\ q_R(t)$$
$$+ \sum_n \left(\sum_{d(n)} q_{d(n)}(t) + \sum_{rp(n)} q_{r(n)}(t) + \sum_{stock(n)} ddpos(stock(n), t) \right) C^{TO}(t)$$
$$+ \sum_{stock} q_{stock}(t) C^{Stock}(t) \Bigg] \Bigg]$$

q_{si}	: Produced quantity of the mine si (si is an invest mine)
C_{si}^P	: Production cost at mine si
C_{si}^I	: Investment cost at mine si
k_{si}	: New capacity at mine si

3.2 – Investment allocation

There are two ways of modeling investment. The first one consists of paying an annuity every year during the total life of the investment. The second one is to pay the total figure of the investment at once. We have chosen to model the investment with the second approach. However, in this case one has to handle the investment that will go over the end of the study period. This case is described in Figure 5. In Figure 5 we have written “Mine Life”. For the rest of the document, we will use the term “Life Span” rather than “Mine Life”. The investment is realized in 2028 and the investment life is 20 years, which means that the investment will run until 2048 that is after the end of the study period (here 2035).

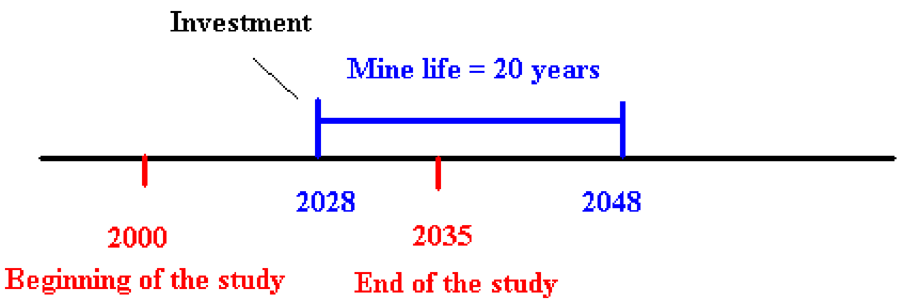

Figure 4: Investment allocation

In the cost function we have to get the money back after 2035. This is why some additional terms have to be added to the cost function. These terms are described in the equation below.

$$\sum_t \left[DisF(t) \left[\sum_{si} \left(\ldots C_{si}^{I}(t) k_{si}(t) - \left(\sum_{k=Study\ End+1}^{t+Lifespan} annuity \frac{DisF(k)}{DisF(t)} \right) k_{si}(t) \right) \ldots \right] \right]$$

The annuity (equation below) is calculated with the discount rate TA, the life span LS of the mine and the investment cost C_{si}^{I} of the mine si. To provide an example with figures, TA could equal 5%, LS equal 20 years and C_{si}^{I} equal 5 US Dollar/tARA/year. If one wants to develop a new capacity k_{si} of 10 million tones per year in 2028 it thus corresponds to a cost of 50 million dollars in the year 2028.

$$annuity = \frac{TA}{1 - \left(\frac{1}{1+TA} \right)^{LS}} C^{I}$$

<u>Remark</u>: As for the mines it is possible to model investments in ports or routes. We consider that annuities are spread over an infinite period of time. That is equivalent to consider that new port or road infrastructure will run for an infinite period of time. This assumption has some impact on the formula providing the annuity.

$$annuity_{\infty} = TA\ C^{I}$$

3.3 – New capacity (notion of life span)

The new capacities found at time t must be kept at time t+1 and for the life span. At time t we model this by adding the sum of the capacities to constraints developed since the beginning of the study period (SS) until time t-1 and the capacity developed at time t. The equation below reflects our remark.

$$\forall t, \qquad q_{si}(t) \quad \leq \quad Capacity\_available\ (t) \quad + \sum_{y=SS}^{t} k_{si}(y)$$

$$SS = Study\ Start$$

To take into account the phase-out of a capacity we have two options.

1. The first approach consists of keeping the capacity in the sum sign (cf. equation above) only for the life span (LS) of the mine. Generally we take a life span of twenty years. If the SS corresponds to 2000 we want the capacity to phase-out after twenty years. That means a capacity developed in 2000 must necessarily phase-out in 2020. That can be realized while making start the index (positive integer y at SS + t – LS + 1). We then observe that for t=20 (2020) we obtain the index y equal to 2000 (SS) plus 20 (t) minus 19 (LS+1) and so the sum starts at 2001. The capacity then developed in 2000 is no longer present after 2020. It is interesting to notice that LS (Life Span) is also used to calculate the annuity.

$$\forall t, \qquad q_{si}(t) \leq Capacity\_available\ (t) + \sum_{y=SS+t-LS+1}^{t} k_{si}(y)$$

$$SS = Start\ Study$$
$$LS = LifeSpan$$

2. The second approach consists of not suppressing the capacity but to reduce the reserve. To do this it is necessary to handle the following equations.

$$\forall t, \qquad q_{si}(t) \leq Capacity\_available\ (t) + \sum_{y=SS}^{t} k_{si}(y)$$

$$\forall t, \qquad \sum_{y=SS}^{t} k_{si}(y) \leq Capacity\_Maximum$$

$$\forall t, \qquad k_{si}(t) \leq Step\_Capacity$$

$$\forall t, \qquad \mathbf{Re}s_{si}(t) = \mathbf{Re}s_{si}(t-1) + k_{si}(t-1)\,LS - q_{si}(t-1)$$

$$with\ \ q_{si},\ \mathbf{Re}s_{si}\ et\ k_{si}\ positive\ \ real\ \ numbers$$
$$SS = Study\ \ Start$$
$$LS = LifeSpan$$

In these equations we consider that the capacity can be developed for each time step without exceeding the value predefined in the data. We refer to this predefined value as Step_Capacity. The sum of the developed capacities cannot exceed a predefined value in the data named Capacity_Maximum. At any time the reserve can increase or decrease. A reserve increases as soon as a new capacity appears and the increase is equal to the product of the new capacity in t/year times the life span (LS). A reserve decreases as soon as some quantities are produced. This is modeled by withdrawing at reserve at time t the quantity produced at time t.

4 – Two economical formulations for a long-term model

From the CEM module representing the base endogenous investment model described just above, we have been studied several economic formulation of our long-term model referring to the macro-economic concepts of either rational price expectation or adaptive price expectation. In the next section, we present two formulations.

4.1 - First Formulation: Rational price expectation (long-term model n°1)

From a macro-economical point of view, the first approach is based on the rational price expectation **[4]** concept initially developed by John Muth [5] in 1961 followed by Robert Lucas [6] in 1972 to explain the links between inflation and unemployment. In this model, the different economic actors have a perfect control of the information, and then they are never mistaken. It is a model not far from the perfect forecast allowing economic actors to thwart the expected effects of a monetary policy for instance. It is especially interesting to notice that the cost minimisation in the CEM module is equivalent to the profit maximisation with a perfect knowledge of the future prices. This approach is shown in Figure 2 (the heart of this approach being the Capacity Expansion Model: CEM). New capacities are created by minimizing a cost function composed of production and investment costs. The production costs are attached to the produced quantities (variable q) while the investment costs are attached to the new capacities (variable k). The variable q is constrained by the existing capacities and the new ones (variable k) that will be found by the model during the calculation.

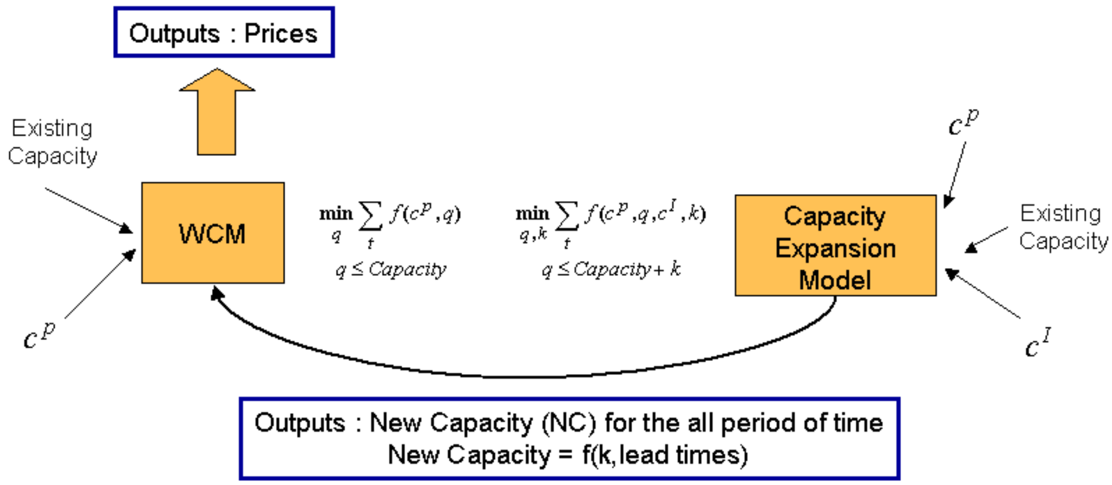

Figure 5: Model n°1

The CEM module produces long-term marginal costs, coal volumes produced and transported and the new capacities build (investment to face the demand). The new capacities calculated by the CEM module can be sent to the WCM module in order to get short-term marginal costs. Lead times can be taken into account either inside the CEM module or outside the CEM module. If lead times are added outside the CEM module this leads to infinite prices in the WCM. It is due to the unavailability of the new capacities (found by the CEM module) in the WCM module. The demand cannot be satisfied. Because we do not have data to take into account an elastic demand, any demand, which is not satisfied, is reflected by an infinite price in the short-term WCM model. To avoid this kind of situation it is suggested to introduce the lead times in the CEM module (inside the maximization problem). In this case we get less realistic prices but we do not have to cope with not satisfied demand and consequently with infinite prices. In this second approach the lead times are less realistic because they are modelled in a perfect foresight way. The deterministic mathematical framework of the linear programming always produces perfect foresight results.

4.2 – Second formulation: Adaptive price expectations (long-term model n°2)

The second economic formulation is based on the adaptive price expectations. This theory has been proposed and developed separately by Milton Friedman [7] and Edmund Pelphs [8] in the sixties to catch the subjacent mechanisms between the unemployment and the inflation. In the framework of the adaptive expectations, the economic actors base their future decisions on what has happened in the past. From the computational point of view, this second approach (Figure 3) is a generalization of the first model. We define an iterative loop over the time between the CEM module (providing the new capacities) and the WCM module (providing the short-term marginal costs). We add two new modules. The Price Expectation Module (PEM module) computes the expected prices for a period between t_1 and t_2 (typically 5 years) with a linear regression. The Capacity Planning Module has been developed to take into account the lead times. One loop of the algorithm starts with the WCM module. With this module we compute a price for a single period in time. This price (marginal cost) is sent to feed into the PEM. The PEM uses a list of historic and calculated prices to forecast the price for the next five years. With this price signal we maximize the profit in the CEM module. In the model n°1 we performed a minimization instead of a maximization. We switch from a cost minimization in model n°1 to a profit maximization in model n°3 because it is a better way to model the behaviour of the coal producers. The CEM gives new capacities by taking into account investments. For the last step of the loop we send the new capacities calculated by the CEM module to the ST WCM module. But before we performed an update of the capacity planning in order to simulate the lead times in a more realistic way.

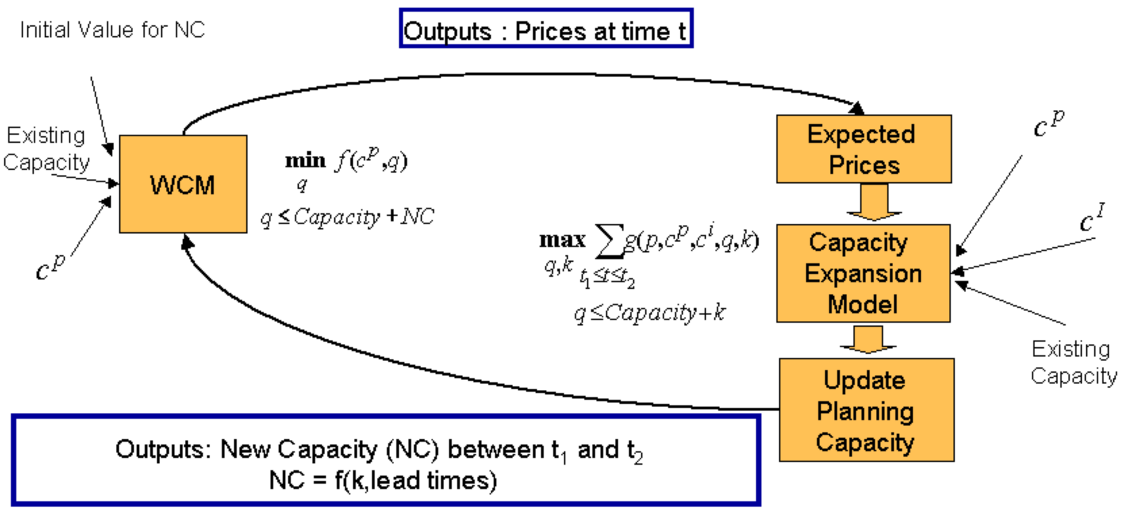

Figure 6: Model n°2

Different demand scenarios have been designed to test the adaptive price expection. Figure 7 shows a demand scenario with some demand spikes. Demand remains flat at 258 mt over all 35 years of the time horizon except of the years 2006, 2008 and 2020 where demand volumes rise by 98 mt, 48 mt and 198 mt respectively (cf. figure 7). The scenario has been used with the adaptive price expectation approach. Short term demand spikes (2006 and 2008) provoke a capacity increase (cf. figure 7). In 2020 capacities do not increase due the ephemerality of the demand spikes as new mine investments do not generate a return on investment.

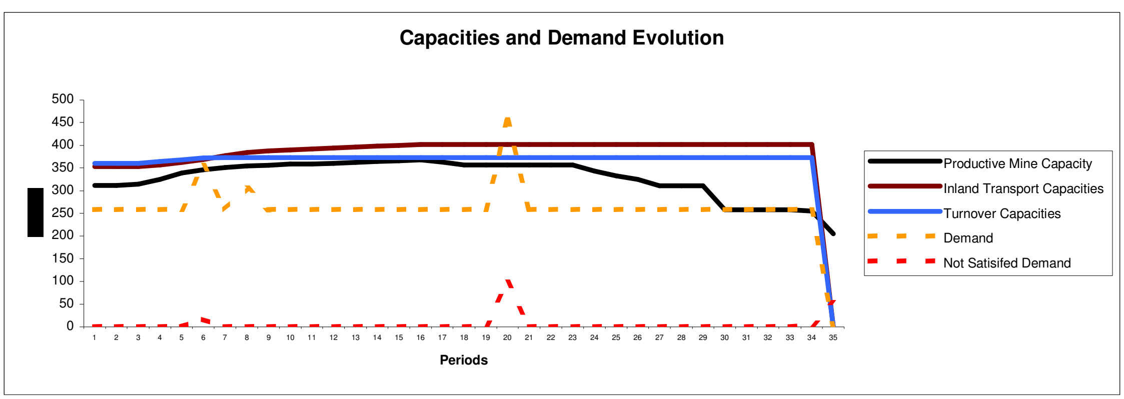

Figure 7: Capacities evolution for a specific demand scenario with some spikes

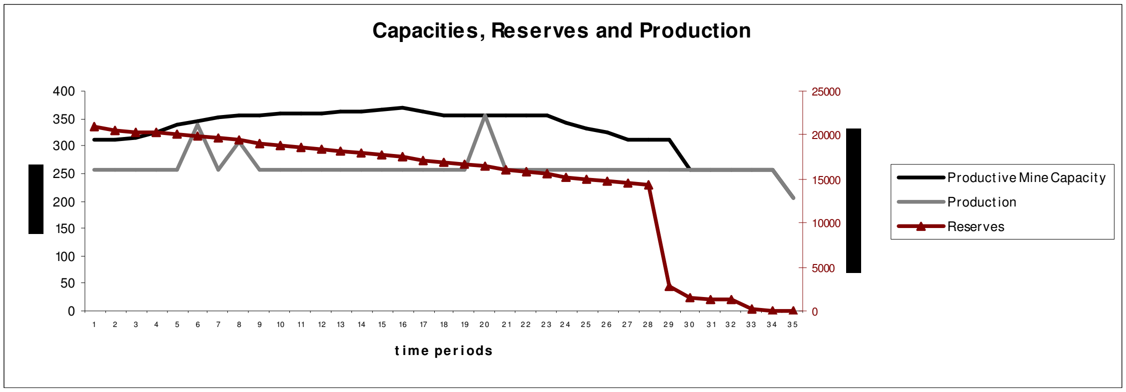

Figure 8: Capacities, production and reserves evolution for a specific demand scenario with some spikes

The model n°2 is more sophisticated in comparison with the model n°1. This approach enables to deliver very realistic results taking into account various phenomena linked to demand behaviour, reserves (cf. figure 8) and lead times. However, this method is difficult to be implemented. This model has been tested on an academic data set. The PEM based on a linear regression to forecast the prices can be contested. It is well known that prices cannot be forecasted with a linear regression. This is due to the nature of a price signal. A price signal following a random walk leads to spurious correlation coefficients in a linear regression approach. We knew this was a weakness but our goal was to demonstrate the upgradability of the model n°1.

Remark: It is interesting to notice that the adaptive price expectation formulation overestimates the investment in comparison with the rational price expectation.

5 – Mathematical analysis of the Capacity Expansion Module in model n°1

It is interesting to study some very special situations in order to be able to write an analytical value of the dual variable (the marginal cost) attached to the balance equation at a node. In the following paragraphs we mainly refer to the Lagrangian which is associated to the primal and dual minimization problem of the capacity expansion module in model n°1 in order to analyze the marginal costs which result from the model. We study the influence of an investment in a mine, route or a port on the marginal cost. We also describe the influence of lead times on the marginal cost.

5.1 – Analysis with only one investor

We can look at the very simple configuration where only one country, Japan for instance, suddenly exhibits two peaks of consumption. To give an example the two peaks could occur in 2017 and 2023. For this scenario we make the assumption that only one mine in South Africa is capable to invest. This mine has a production cost and an investment, which remain constant over time. If we consider t_1 and t_2 as the two times when the peaks of demand appear we can adopt the following notation for the costs with $t_1 = t$ and $t_2 - t_1 = h$:

$$\begin{cases} C_{t_1}^{P} = C_{t_2}^{\ P} = C_t^P = C_{t+h}^P \\ C_{t_1}^{I} = C_{t_2}^{\ I} = C_t^I = C_{t+h}^I \end{cases}$$

With this notation the primal problem to be minimized is written below.

$$
\begin{cases}
\underset{q,k}{Min}\ C_t^P\ q_t + C_t^I\ k_t + \frac{1}{(1+TA)^h}\left[C_t^P\ q_{t+h} + C_t^I\ k_{t+h}\right] \\
D_t = S_t\ (S_t = q_t) \\
D_{t+h} = S_{t+h}\ (S_{t+h} = q_{t+h}) \\
q_t \le k_t \\
q_{t+h} \le k_t + k_{t+h}
\end{cases}
$$

S_t (respectively S_{t+h}) reflect the quantities produced by the mine at time t (respectively at time t+h) and D_t (respectively D_{t+h}) represent the peaks of demand at the same time (respectively at time t+h). Only one mine can invest. Consequently this mine will define the price in Japan at a time t and t+h. This is the reason why we can reduce the cost function to this mine since the other mines will not determine the marginal costs even if they are called. For this very simple example, it is possible to write the Lagrangian attaches to the primal and dual problems. As a start we first associate at each constraint its dual variable.

$$
\begin{cases}
D_t = S_t\ (S_t = q_t) & \Rightarrow\ \alpha_t \\
D_{t+h} = S_{t+h}\ (S_{t+h} = q_{t+h}) & \Rightarrow\ \alpha_{t+h} \\
q_t \le k_t & \Rightarrow\ \beta_t \\
q_{t+h} \le k_t + k_{t+h} & \Rightarrow \beta_{t+h}
\end{cases}
$$

Then we can write the Lagrangian L.

$$
\begin{aligned}
&L(q_t, q_{t+h}, k_t, k_{t+h}) = \\
&\quad C_t^P\ q_t + C_t^I\ k_t + \frac{1}{(1+TA)^h}\left[C_t^P\ q_{t+h} + C_t^I\ k_{t+h}\right] \\
&+(D_t - q_t)\alpha_t + (D_{t+h} - q_{t+h})\alpha_{t+h} + (k_t - q_t)\beta_t + (k_t + k_{t+h} - q_{t+h})\beta_{t+h}
\end{aligned}
$$

We deduct the dual variable by considering that the Lagrangian L is stationary.

$$
\begin{cases}
\alpha_t = C_t^P + C_t^I\left(1 - \frac{1}{(1+TA)^h}\right) \\
\alpha_{t+h} = \frac{C_t^P + C_t^I}{(1+TA)^h} \\
\beta_t = -C_t^I\left(1 - \frac{1}{(1+TA)^h}\right) \\
\beta_{t+h} = -\frac{C_t^I}{(1+TA)^h}
\end{cases}
$$

This result is very important. The dual variables α_t and α_{t+h} correspond to the marginal costs at a time t and t+h if we do not consider the discount rate. We notice that the marginal cost for the last investment exactly equals the production cost plus the investment. On the other hand, the marginal cost for the first investment is equal to the production cost plus only a fraction of the investment. These two results require some comments.

First, the value of the marginal cost at a time t+h (when the last investment was realised) is not intuitive. In a short-term vision, the marginal cost is only equal to the production cost. Here, the investment cost is appearing. For a long-term model, this result is not too surprising. It is exactly what the model should produce. With a deterministic approach and a linear programming model, we link the two time steps. To keep the capacity developed at time t and t+h, we have to write the following equation.

$$q_{t+h} \leq k_t + k_{t+h}$$

This equation tells us that a capacity developed at time step t must be added to the capacity developed at time step t+h. We recall to the reader that we want to keep during time what we have developed. It has consequences on the final marginal costs. In a long-term view, the marginal cost is finally the production cost plus a fraction of the investment and sometimes even the total investment. It is due to the role played by the capacity variables. The quantity variables are constrained by the capacities and then a link between time steps appears. But it is easier for the comprehension to write the Lagrangian and to look at it being stationary (where it derivate is zero) and then to analyze what is the value taken by the dual variable. At the first investment, we notice that the marginal cost is equal to the production plus a fraction of the investment. The value of this fraction is conditioned by the discount rate and the time gap that exists between the first and the second investment. It is possible to give an economical interpretation to these two conditionings. If we split the investment into two cost terms we are able to understand what it is happening when the two investments appear in the framework of a monopolistic market (where only one producer is capable to invest).

$$C_t^I = Term1 + Term2$$

$$with\ Term1 = \frac{C_t^I}{(1+TA)^h},$$

$$with\ Term2 = C_t^I \left(1 - \frac{1}{(1+TA)^h}\right)$$

At time t, the producer invests knowing (deterministic modeling view) that he will have to invest again at time t+h. We then notice that what the producer integrates into the long-term marginal cost at time t is a portion of the investment equivalent to the interests for borrowing the capacity at time t+h but used a time t. He saves the quantity « Term1 ». And « Term2 » is equal to the borrowing value. It is obvious that « Term2 » is equal to the total investment minus the « Term1 » corresponding to the part not paid by the producer.

As an example we can suppose the production equals 5$/t and the investment equals 2$/t/year. We then study two extreme situations. The first one consists of looking at the situation where h is going to zero and the second one where h is going to infinite. It corresponds either to the situation with very close investments and very distant investments.

- First extreme situation (h is going to zero): The two peaks of demand occur very close to each other. So h is small. If h is small the « Term1 » is an absolute value very close to the total investment. Then the « Term2 » is very small in comparison to the investment. And because « Term2 » exactly equals what the producer pays to borrow at time t+h the capacity needed at time t. Then we have a marginal cost which is a little bit bigger than the production cost at time t since the marginal cost is exactly equal to the production cost plus the « Term2 ». For instance if h is equal to 2, we find that « Term2 » is equal to 0.185 and then the marginal cost at time t α_t is equal to 5.185 $.

- Second extreme situation (h is going to infinite): If h is going to infinite that corresponds to a second investment, which is as distant as possible to the first investment. In this situation « Term1 » goes to zero. Consequently, « Term2 » that appears in the marginal cost at time t goes to the total investment. It means that the more the second investment is distant to the first investment the more the borrowing cost increases. At infinite it is equivalent for the producer not to borrow the capacity and to pay directly the total amount attached to the investment. He will then integrate it into the long-term marginal cost.

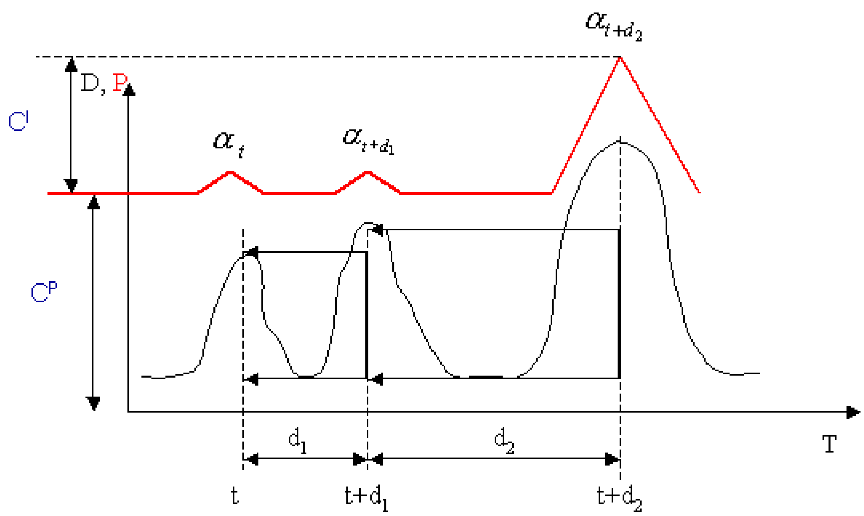

Figure 9: Development of N investments

One can generalize this approach in the framework of a mono actor market (monopolistic market). If N investments are developed (Figure 6) with time intervals d_i with i from 1 to N then we can write the following relation allowing to find the price at time t+d_i.

$$\alpha_{t+d_i} = C^p + C^i - \frac{C^i}{(1+TA)^{d_i+1}}$$

It is important to underline that all the analyses presented has been conducted without stocks.

5.2 – Impact of transport and ports on the marginal cost

All the different considerations exposed on the investment for the mine can be applied to the transport and port investments as well. We can add two components to the variable dual α_t calculated with the Lagrangian. The two components are the investment cost for a road and the investment cost to develop the port infrastructure. The marginal cost can then be written:

$$\alpha_t = C_t^P + (C_t^I + C_t^{IT} + C_t^{ITO})\left(1 - \frac{1}{(1+TA)^h}\right)$$

with C_t^{IT} being the investment for transport and C_t^{ITO} being the investment in a port. In this case the model would have invested in roads, in a mine and in ports to get this value of the marginal cost.

5.3 – Impact of lead times on the marginal cost

The lead times can be taken into account in a linear programming approach as follows.

$$\forall t \leq LT + SS, \qquad q_{si}(t) \quad \leq \quad Capacity\_available\ (t)$$

$$\forall t > LT + SS, \qquad q_{si}(t) \quad \leq \quad Capacity\_available\ (t) \quad + \sum_{y=SS}^{t} k_{si}(y - LT)$$

$$SS = Study\ Start,\ LS = Lead\ Times$$

This way of modelling produces perfect foresight lead times. The capacity will appear before the peak of demand. For instance, if in 2017 a capacity is called and the lead times are 3 years then the capacity will be developed in 2014. The consequence on the long-term marginal cost α_{t+h} can be written as follows:

$$\alpha_{t+h} = \frac{C^P}{(1+TA)^{t+h}} + \frac{C^I}{(1+TA)^{t+h-LT}}$$

The reader could rewrite the Lagrangian with the inequalities defined above to find this price (marginal cost).

6 – Conclusions

The main objective of our work presented in this paper was to propose an economic formulation of our long-term model for the world coal market and then to prove its numerical implementation feasibility.

- To test the feasibility of the linear and determinist optimisation approach, we produced a reduced data set. The reduced data set has been built from the short-term data set. With this reduced data set, we have tested the numerical outputs. The outputs have been compared to some analytical solutions obtained from some mathematical analysis of our model. With this road map, the long-term marginal costs have been economically interpreted for some basic scenarios (only one investor, congestion of certain sea routes, etc.)
- For the economical formulation, we have studied from our base endogenous investment model several economic models referring to the macro-economic concepts of either rational price expectation or adaptive price expectation.

In the future, we would like to go beyond the deterministic framework. Uncertainty on the demand or on the costs for instance have to be studied more closely. For this purpose, we could use the concept of spanning tree optimization. The sensitivity to different parameters could be a second direction of study. The Linear Robust Optimization could be a good candidate.

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