

# A LONG-TERM GROWTH MODEL WITH ENDOGENOUS BIOPHYSICAL AND ECONOMIC STATES

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## Overview

Long-term economic growth models often assume that energy resources and technology are not constraints on the economy. Energy transition scenario models often assume that economic growth will not constraint an energy transition. Both types of models often neglect fundamental dynamics and the influence of debt and subsequent interest payments. This paper discusses a newly-developed dynamic long-term growth model that endogenously links biophysical (population, resources) and economic (debt, wages, capital) states, in a stock-flow consistent manner for resources (energy, matter) and money. The model helps explain very important and broad-scale historical macroeconomic trends such as 1) the tremendous decline in energy spending relative to net output during the fossil energy and industrial transition, and 2) the post-1970s decline in wage share for Western economies.

## Methodology

The contributions of this work are the combining of biophysical and economic models, and the specification of 2 productive sectors to clearly describe feedbacks from resource-related parameters and depletion. The modeling approach combines two fundamental underlying models in a post-Keynesian framework. First, the biophysical model of (Motesharrei *et al.*, 2014) provides the fundamental feedbacks among resource depletion, resource consumption, and population change. There are no endogenous states. Second, we use the endogenous money “Minsky” economic model developed by Keen (1995) and Keen (2013). Keen’s approach adds debt to the employment-wage dynamics of Goodwin (1967). Endogenous states are wages, debt and labor, capital or employment with exogenous states of labor productivity and population growth. By combining the models we completely endogenize population by keeping population per Motesharrei *et al.* (2014) and eliminating population per Keen/Goodwin.

There are two productive sectors, extraction (e) and goods (g), each modeled with output in physical units (resources and goods) and multiplied by price to convert to monetary units. The extraction sector operates capital for the purpose of extracting nature. The goods sector produces consumer and investment goods. For each sector, the rate of change of debt is modeled as investment minus profit ( $dD_i/dt = I_i - \Pi_i$ ). Investment as a share of sector value added,  $\kappa_i(\Pi_i/V_i)$ , is multiplied by sector value added,  $V_i$ , to equal total sector investment,  $I_i$ . The causality assumed in this model is that first, firms choose investment based on past profits. Second, with assumed production functions for gross output for each sector, intermediate demands and net output are calculated. Finally, households then consume the residual net output both the goods and extraction sectors (e.g.,  $C_i = Y_i - I_i$ ). Banks earn a profit by lending money at a higher rate of interest,  $r_L$ , than for which they pay to household depositors,  $r_M$ .

**Table 1. The input-output representation of the model indicating productive sectors’ intermediate consumption, value added, and net output.**

				Net Output		Total output
		Goods	Extraction	Consumption	Investment	
Value Added	Goods	$P_{ga}a_{gg}X_g$	$P_{ga}a_{ge}X_e$	$C_g$	$I_e + I_g$	$P_gX_g$
	Extraction	$P_{ea}a_{eg}X_g$	$P_{ea}a_{ee}X_e$	$C_e$	---	$P_eX_e$
	Profit	$\Pi_g$	$\Pi_e$			
	Wages	$wL_g$	$wL_e$			
	Interest	$r_L D_g$	$r_L D_e$			
	Depreciation	$P_g\gamma K_g$	$P_g\gamma K_e$			
	Total output	$P_gX_g$	$P_eX_e$			

**There are two critical resource feedbacks included in the model, and these represent the novelty of the approach. First**, we require extracted natural resources to operate each type of capital (e.g., capital requires energy to operate) as intermediate input. **Second**, we require resources input to invest in new capital, and this is part of the intermediate consumption of the goods sector. The intermediate resources consumption per gross output of the extraction sector is  $a_{ee} = (\eta_e K_e C U_e) / (\delta K_e C U_e y) = \eta_e / \delta y$ , where  $\eta_e$  is a consumption technology parameter,  $\delta$  is a extraction technology parameter,  $y$  is the amount of resources in nature (not extracted),  $K_e$  is the amount of extraction capital, and  $C U_e$  is the capacity utilization of extraction capital. Thus,  $\delta y$  is a capital productivity factor, and as the remaining nature ( $y$ ) is depleted, the capital productivity declines. This concept provides a feedback that makes nature extraction increasingly

costly the closer it gets to depletion. The intermediate resources consumption per gross output of the goods sector is  $a_{eg} = (\eta_g K_g C U_g) / (K_g C U_g / v_g) + ((I_g + I_e) / P_e) / (K_g C U_g / v_g) = \eta_g v_g + (I_g + I_e) / (P_e X_g)$ . The per unit intermediate goods consumption for both sectors,  $a_{gg}$  and  $a_{ge}$ , are assumed constant.

The prices of sector outputs are solved using the following Equations (1) – (3). Prices are derived by equating total gross monetary output of each sector,  $P_i X_i$ , to that sector's value added,  $V_i$ , plus its intermediate expenditures. The left hand side of Equations (1) and (2) is a modified value added,  $\tilde{V}$ , to account for price interaction between investment ( $I_e$  and  $I_g$ ) and depreciation expense ( $= \gamma K_i P_g = \text{depreciation rate} \times \text{capital stock} \times \text{price of goods}$ ).

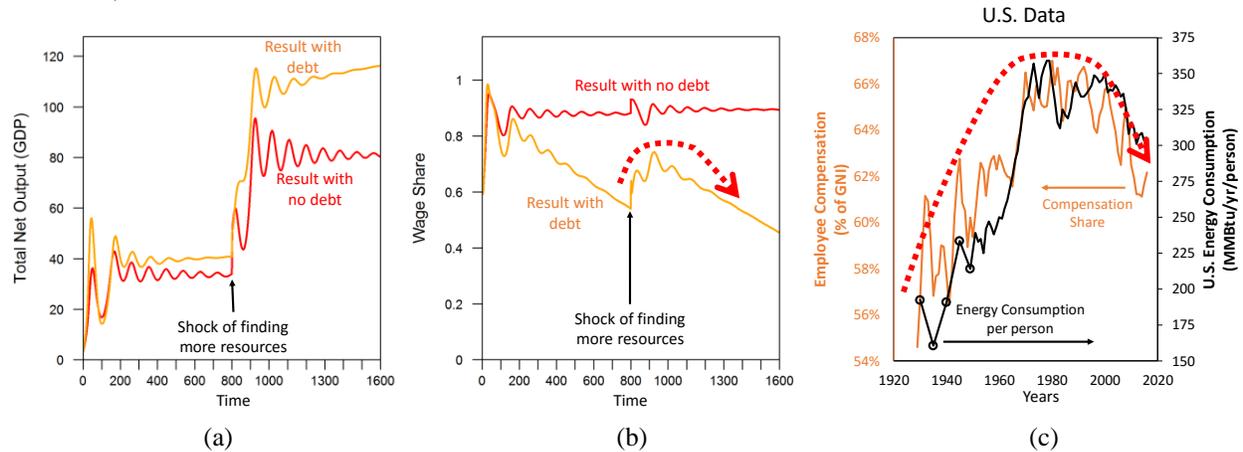
$$\begin{bmatrix} \Pi_g + wL_g + rD_g + I_g + I_e \\ \Pi_e + wL_e + rD_e \end{bmatrix} = \begin{bmatrix} X_g & 0 \\ 0 & X_e \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} X_g & 0 \\ 0 & X_e \end{bmatrix} \begin{bmatrix} a_{gg} & \eta_g v_g \\ a_{ge} & \eta_e / \delta y \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} - \begin{bmatrix} X_g & 0 \\ 0 & X_e \end{bmatrix} \begin{bmatrix} \gamma K_g / X_g & 0 \\ \gamma K_e / X_e & 0 \end{bmatrix} \begin{bmatrix} P_g \\ P_e \end{bmatrix} \quad (1)$$

$$\tilde{V} = \hat{X}P - \hat{X}\tilde{A}^T P - \hat{X}\Gamma P = [\hat{X}(1 - \tilde{A}^T - \Gamma)P] \quad (2)$$

$$P = [\hat{X}(1 - \tilde{A}^T - \Gamma)P]^{-1} \tilde{V} \quad (3)$$

## Results

Example results in Figure 1 are from simulating an energy “shock” of having a 100% larger maximum resource potential (resource is modeled as a regenerative stock, e.g., forest). Figure 1(a) shows increased economic net output when more resources are found (resource extraction goes up nearly 5 times, not shown). Figure 1(b) shows a temporary increase in wage share as the economy adjusts to the new resource extraction rate, but only applicable when the investment assumption forces firms to acquire debt by investing more than their profits. Figure 1(c) uses U.S. data to show that compensation share (wages, salaries, and benefits) also rose temporarily when energy consumption was rising quickly, but compensation share stagnated when energy consumption per capita stagnated in the 1970s, before both metrics declined after 2000.



**Figure 1. Model simulation results of (a) economic net output and (b) wage share compared when additional resources are assumed found at time 800. (c) The U.S. data on compensation share is consistent with the model result (assuming firm debt accumulates) that wage share increases when resource (energy) extraction increases (modeled extraction rate not shown), and vice versa.**

## Conclusions

Despite the simplicity of this model, with 2 productive sectors as well as a banks and household consumers (monetary flows amount the latter two not shown due to space), it displays the ability to describe important trends and feedbacks among debt (and interest payments), profits, wages (and wage share), and the rate of resources extraction. This is due to a rigorous approach to stock-flow consistent modeling for both resources and money that tracks the critical resources requirement as an input to both operate capital and build new capital. Future work will include additional types of resources sectors (fossil and renewable flows such as solar power), government and taxation, and the equity in firms.

## References

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